Ralph's Sanitization[1]

As the manager of a firm, Ralph commits to provide audited reports on productivity. These reports provide implicit (and perhaps, explicit) incentives for Ralph to serve as a faithful steward of the firm. Current (time 0) ownership of the firm involves three (representative) parties: Ralph is risk averse with utility function $U(w) = -\exp[-0.1w]$ where w is wealth and owns $\varepsilon = 10\%$, investor 1 is risk neutral and owns $\frac{1-\varepsilon}{2} = 45\%$, and investor 2 is risk neutral and owns the remaining 45%. Due to his proximity to operations Ralph acquires private information and supplies a report at the end of period 1. At that time trade occurs. For portfolio balancing purposes (including satisfaction of liquidity needs), Ralph sells his shares to investor 1 at a price determined by investor 1. Then, investor 2 is free to exchange (buy or sell) any fraction of the firm's shares with investor 1 at the price determined by investor 1 in her trade with Ralph. At the end of the second period, the firm is liquidated and the owners receive their payout. The timeline is below.

time 0	time 1	time 2
the firm is organized with ownership allocated as Ralph ε shares, investor 1 $\frac{1-\varepsilon}{2}$ shares, investor 2 $\frac{1-\varepsilon}{2}$ shares	Ralph acquires private information, provides audited report & trade occurs (Ralph sells ε shares to investor 1, investor 2 exchanges α shares with investor 1)	firm is liquidated and investors receive their distributions

timeline

For simplicity, there are three equally likely states of nature (s_i, $i = 1, 2, 3$) and the final associated total payoffs from the firm to its owners are $0, 1,000, 2,000$. This is common knowledge to all participants (owners). Ralph privately learns interval information as follows. With probability $\theta_1 = 1 - \theta_2 - \theta_3$, Ralph learns nothing. Hence, the interval covered is $\{s_1, s_2, s_3\}$. With probability θ_2, Ralph learns that the state is either good with interval $\{s_2, s_3\}$ or bad with state partition $\{s_1\}$. And, with probability θ_3, Ralph learns that the state is good with state partition $\{s_3\}$ or bad with interval $\{s_1, s_2\}$.

Full disclosure leads to the following information structure over reports and

[1] This example is drawn from ideas developed in Shin, 1994, "News management and the value of firms," *RAND Journal of Economics*, 25(1), 58-71.

states. That is, the joint likelihood of reports and states is

$\Pr(y,s)$	s_1	s_2	s_3	$\Pr(y)$
$y_1 = \{s_1, s_2, s_3\}$	$\theta_1 p_1$	$\theta_1 p_2$	$\theta_1 p_3$	θ_1
$y_2 = \{s_2, s_3\}$	0	$\theta_2 p_2$	$\theta_2 p_3$	$\theta_2 (p_2 + p_3)$
$y_3 = \{s_3\}$	0	0	$\theta_3 p_3$	$\theta_3 p_3$
$y_4 = \{s_1\}$	$\theta_2 p_1$	0	0	$\theta_2 p_1$
$y_5 = \{s_1, s_2\}$	$\theta_3 p_1$	$\theta_3 p_2$	0	$\theta_3 (p_1 + p_2)$
$\Pr(s)$	$p_1 = \frac{1}{3}$	$p_2 = \frac{1}{3}$	$p_3 = \frac{1}{3}$	1

Joint likelihoods for full disclosure reports and states

However, Ralph may faithfully (truthfully) report without fully disclosing all of his private information. For instance, Ralph may employ a sanitization report strategy in which revealed good news is reported but revealed (to Ralph) bad news (y_4 or y_5) is suppressed. This reduces the message space to three possible interval reports (y_1, y_2, y_3). The joint likelihood of sanitized reports and states is

$\Pr(y,s)$	s_1	s_2	s_3	$\Pr(y)$
$y_1 = \{s_1, s_2, s_3\}$	$\theta_1 p_1 + \theta_2 p_1 + \theta_3 p_1$	$\theta_1 p_2 + \theta_3 p_2$	$\theta_1 p_3$	$\theta_1 + \theta_2 p_1 + \theta_3 (p_1 + p_2)$
$y_2 = \{s_2, s_3\}$	0	$\theta_2 p_2$	$\theta_2 p_3$	$\theta_2 (p_2 + p_3)$
$y_3 = \{s_3\}$	0	0	$\theta_3 p_3$	$\theta_3 p_3$
$\Pr(s)$	$p_1 = \frac{1}{3}$	$p_2 = \frac{1}{3}$	$p_3 = \frac{1}{3}$	1

Joint likelihoods for sanitized reports and states

Equilibrium strategies for the three parties are as follows. Ralph prefers the sanitization report strategy to full disclosure. Investor 1 prices the firm's shares at the expected value of the period two payoff conditional on Ralph's report, $E[x \mid y_i]$. Given investor 1's equilibrium pricing of the firm's shares, investor 2 is indifferent toward the fraction α traded.

Part A

Required:

1. Suppose $\theta_1 = \theta_2 = \theta_3 = \frac{1}{3}$. Determine Ralph's payoff associated with each possible report if he follows the full disclosure strategy and investor 1 prices the firm's shares at their equilibrium value. Determine Ralph's expected value of utility for each of the possible reports, $E_y[U(w \mid y_i)]$, and certainty equivalent associated with the full disclosure strategy.

2. Continue with $\theta_1 = \theta_2 = \theta_3 = \frac{1}{3}$. Determine Ralph's payoff associated with each possible report if he follows the sanitization disclosure strategy and investor 1 prices the firm's shares at their equilibrium value. Determine Ralph's expected value of utility for each of the possible reports, $E_y[U(w \mid y_i)]$, and certainty equivalent associated with the sanitization disclosure strategy.

3. Continue with $\theta_1 = \theta_2 = \theta_3 = \frac{1}{3}$. Given that Ralph is committed to audited performance reports, does Ralph prefer full disclosure or sanitized disclosure?

4. Continue with $\theta_1 = \theta_2 = \theta_3 = \frac{1}{3}$. Suppose investor 2 plans to buy $\alpha = 5\%$ of the shares outstanding from investor 1. If investor 1 prices the firm's shares following Ralph's report at their equilibrium value, does investor 2 have any incentive to deviate from the planned $\alpha = 0.05$? Suppose investor 1 prices the stock below its equilibrium value, what fraction α will investor 2 pursue? Suppose investor 1 prices the stock above its equilibrium value, what fraction α will investor 2 pursue?

5. Suppose $\theta_1 = 0$ ($\theta_2 = \theta_3 = \frac{1}{2}$). In other words, Ralph is always privately informed and this is common knowledge. Is there a substantive distinction between full disclosure and sanitization? How does Ralph's expected utility (certainty equivalent) in this fully informed setting compare with Ralph's expected utility (certainty equivalent) in the infrequently informed $\left(\theta_1 = \theta_2 = \theta_3 = \frac{1}{3}\right)$ setting?

6. Suppose Ralph is not committed to audited performance reporting and the firm's shares are priced at their equilibrium value, does Ralph prefer a sanitization reporting strategy with equilibrium price $E[x \mid y_i]$ or no reports with equilibrium price $E[x]$ prior to selling his shares? What does this suggest about the relative importance of valuation versus evaluation for accounting? In other words, is the role of accounting more substantive in a world of pure exchange or a world of production?

Part B

Now, Ralph considers an alternative reporting strategy (he remains committed to reporting for stewardship purposes). Ralph recognizes that auditing and accounting policy-making supply asymmetric pressure for earlier recognition of poor outcomes and later recognition of good outcomes. Accordingly, Ralph contemplates the mirror image of the sanitization reporting strategy. That is, a conservative reporting strategy in which privately observed bad outcome intervals are reported but good outcome intervals are suppressed.

Everything remains as above except we add this additional disclosure strategy. Again, the message space involves three possible interval reports (y_1, y_4, y_5). The joint likelihood of conservative reports and states is

$\Pr(y, s)$	s_1	s_2	s_3	$\Pr(y)$
$y_1 = \{s_1, s_2, s_3\}$	$\theta_1 p_1$	$\theta_1 p_2 + \theta_2 p_2$	$\theta_1 p_3 + \theta_2 p_3 + \theta_3 p_3$	$\theta_1 + \theta_2 (p_2 + p_3) + \theta_3 p_3$
$y_4 = \{s_1\}$	$\theta_2 p_1$	0	0	$\theta_2 p_1$
$y_5 = \{s_1, s_2\}$	$\theta_3 p_1$	$\theta_3 p_2$	0	$\theta_3 (p_1 + p_2)$
$\Pr(s)$	$p_1 = \frac{1}{3}$	$p_2 = \frac{1}{3}$	$p_3 = \frac{1}{3}$	1

Joint likelihoods for conservative reports and states

Mean preserving spreads To help sort out reporting strategies Ralph contemplates second order stochastic dominance and mean preserving spreads. Ralph recognizes risk measurement is similar to information system ranking. That is, there exists no general way to rank information systems because fineness is the only generally consistent ranking but it's incomplete. Likewise, mean preserving spread is the only general measure of risk but it also is incomplete. Mean preserving spread implies the following. Consider two gambles, A and B, that have the same mean, all risk averse individuals (those with a concave utility function) prefer A to B if B is a mean preserving spread of A. B is a mean preserving spread of A, or equivalently, A second order stochastically dominates B if the gambles have the same mean and B's payoffs have greater spread than A's.

Stochastic dominance The most common (useful) forms of stochastic dominance are first and second order. First order stochastic dominance implies any individual with increasing utility in the payoffs prefers A to B if A first order stochastically dominates B. First order stochastic dominance means the cumulative distribution function (cdf) for B lies on or above the cdf for A everywhere. Let $f(x)$ denote the probability mass function for a discrete random variable x (or density function for a continuous random variable). Then, the cdf is

$$F(x_k) = \sum_{i=1}^{k} f(x_i) \quad k \in (1, \ldots, n)$$

Consider an example. Suppose gamble A has probability mass associated with its payoffs

x	0	1	2
$f\left(x^A\right)$	$\frac{1}{3}$	$\frac{1}{3}$	$\frac{1}{3}$

and gamble B has probability mass associated with its payoffs

x	-2	-1	0
$f\left(x^B\right)$	$\frac{1}{4}$	$\frac{1}{2}$	$\frac{1}{4}$

Clearly, with increasing preference for x, A is preferred to B. Check first order stochastic dominance. The cdf's for gambles A and B are

x	-2	-1	0	1	2
$F\left(x^A\right)$	0	0	$\frac{1}{3}$	$\frac{2}{3}$	1
$F\left(x^B\right)$	$\frac{1}{4}$	$\frac{3}{4}$	1	1	1

and A first order stochastically dominates gamble B as $F\left(x^B\right) \geq F\left(x^A\right)$ for all x. The less preferred outcomes are consistently more likely under gamble B than gamble A. While a powerful tool, Ralph knows first order stochastic dominance is relatively rare.

Second order stochastic dominance speaks to risk aversion (hence, it is more specialized than first order stochastic dominance). Second order stochastic dominance is to first order what first order is to the probability mass (or density) function. That is, it is based on the area of the cdf.

$$areaF\left(x_k\right) = \begin{array}{ll} \sum_{i=1}^{k} F\left(x_i\right)\left(x_{i+1} - x_i\right), & k \in (1, \ldots, n-1) \\ areaF\left(x_{n-1}\right) + F\left(x_n\right) x_n, & k = n \end{array}$$

Consider an example. Suppose gamble A has probability mass associated with its payoffs

x	0	1	2
$f\left(x^A\right)$	$\frac{1}{3}$	$\frac{1}{3}$	$\frac{1}{3}$

and gamble B has probability mass associated with its payoffs

x	-1	1	3
$f\left(x^B\right)$	$\frac{1}{4}$	$\frac{1}{2}$	$\frac{1}{4}$

Check first and second order stochastic dominance. The cdf's and sums for gambles A and B are

x	-1	0	1	2	3
$F\left(x^A\right)$	0	$\frac{1}{3}$	$\frac{2}{3}$	1	1
$F\left(x^B\right)$	$\frac{1}{4}$	$\frac{1}{4}$	$\frac{3}{4}$	$\frac{3}{4}$	1
$areaF\left(x^A\right)$	0	$\frac{1}{3}$	1	2	3
$areaF\left(x^B\right)$	$\frac{1}{4}$	$\frac{1}{2}$	$\frac{5}{4}$	2	3

Neither gamble first order stochastically dominates. However,gamble A second order stochastically dominates gamble B as the gambles have the same means and $areaF\left(x^B\right) \geq areaF\left(x^A\right)$ for all x. That is, gamble B is a mean preserving spread of A. Hence, gamble A is preferred to gamble B by all risk averse individuals.[2]

Required:

1. Continue with $\theta_2 = \theta_3 = \frac{1}{3}$. Determine Ralph's payoff associated with each possible report if he follows the conservative disclosure strategy and investor 1 prices the firm's shares at their equilibrium value. Determine Ralph's expected value of utility for each of the possible reports, $E_y\left[U\left(w \mid y_i\right)\right]$, and certainty equivalent associated with the conservative disclosure strategy.

2. Compare the conservative disclosure strategy with the full disclosure, sanitization disclosure, and no disclosure strategies based on Ralph's expected utility (or certainty equivalent).

3. From Ralph's perspective, are any of these disclosure strategies a mean preserving spread of another disclosure strategy?

4. Compare the variance of Ralph's payoffs for the sanitization and conservative disclosure strategy. Is variance of payoffs a general measure of risk? (Hint: recall the report strategies involve the same expected payoff, does variance of payoffs coincide with Ralph's preferences?)

[2] Also, if x^B is a mean preserving spread of x^A in the sense $x^B \overset{d}{=} x^A + z$ where $E\left[z \mid x^A\right] = 0$ for all x^A and $\overset{d}{=}$ means equal in distribution, then x^A second order stochastically dominates x^B. For the above example,

x_B	x_A	$z\left(x_A = 1\right)$	$\Pr\left(z\left(x_A = 1\right)\right)$
-1	1	-2	$\frac{1}{4}$
1	1	0	$\frac{1}{2}$
3	1	2	$\frac{1}{4}$

5. What does this strategic disclosure setting suggest about the relative importance of valuation (pure exchange) versus evaluation (production) for accounting?