Ralph's random probability and variance

This is a continuation of Ralph's uncertain variance. Everything is the same except Ralph views the proposition probability p (that the state is favorable) as a random variable (in other words, Ralph is uncertain about the probability assignment for the proposition). Accordingly, Ralph believes $E[\log p] = E[\log 1-p] = -1.96191$ (geometric mean $\exp(-1.96191) = 0.14059$) or, in other words, p has a beta distribution with concentration parameters $a = b = 0.3$.

Ralph knows Bayesian (sum and product rules) updating of the proposition probability based on conditionally normally distributed evidence (with a given variance) is

$$\begin{aligned}
& f(p \mid y = 24; \sigma) \\
= & \frac{f(p, y = 24; \sigma)}{f(y = 24; \sigma)} \\
= & \frac{[pf(y = 24 \mid s = f) + (1-p) f(y = 24 \mid s = u)] p^{a-1} (1-p)^{b-1}}{\int_0^1 [pf(y = 24 \mid s = f) + (1-p) f(y = 24 \mid s = u)] p^{a-1} (1-p)^{b-1} dp}
\end{aligned}$$

where $f(y \mid s)$ is a normal density function and the denominator is

$$f(y = 24; \sigma) = \beta(1+a, b) f(y = 24 \mid s = f) + \beta(a, 1+b) f(y = 24 \mid s = u)$$

and $\beta(a, b) = \frac{\Gamma(a)\Gamma(b)}{\Gamma(a+b)}$. Also,

$$\begin{aligned}
& E[p \mid y = 24; \sigma] \\
= & \frac{\frac{1+a}{1+a+b}\beta(1+a, b) f(y = 24 \mid s = f) + \frac{a}{1+a+b}\beta(a, 1+b) f(y = 24 \mid s = u)}{f(y = 24; \sigma)}
\end{aligned}$$

When the variance or standard deviation σ is also uncertain with kernel $\frac{1}{\sigma}$ Bayesian updating of the proposition probability is

$$\begin{aligned}
& f(p \mid y = 24; 1 < \sigma < 3) \\
= & \frac{f(p, y = 24; 1 < \sigma < 3)}{f(y = 24; 1 < \sigma < 3)} \\
= & \frac{\int_1^3 \frac{1}{\sigma} [pf(y = 24 \mid s = f) + (1-p) f(y = 24 \mid s = u)] p^{a-1} (1-p)^{b-1} d\sigma}{\int_0^1 \int_1^3 \frac{1}{\sigma} [pf(y = 24 \mid s = f) + (1-p) f(y = 24 \mid s = u)] p^{a-1} (1-p)^{b-1} d\sigma dp}
\end{aligned}$$

where the numerator is

$$\begin{aligned}
f(p, y = 24; 1 < \sigma < 3) &= p^a (1-p)^{b-1} \frac{\left[F\left(\frac{24-\mu_f}{3}\right) - F\left(\frac{24-\mu_f}{1}\right)\right]}{\mu_f - 24} \\
&\quad + p^{a-1} (1-p)^b \frac{\left[F\left(\frac{24-\mu_u}{3}\right) - F\left(\frac{24-\mu_u}{1}\right)\right]}{\mu_u - 24}
\end{aligned}$$

$F()$ is the cumulative standard normal distribution function, μ_j is the mean of y conditional on state j and the denominator is

$$\begin{aligned} f(y=24; 1<\sigma<3) &= \beta(1+a,b)\frac{\left[F\left(\frac{24-\mu_f}{3}\right)-F\left(\frac{24-\mu_f}{1}\right)\right]}{\mu_f-24} \\ &+\beta(a,1=b)\frac{\left[F\left(\frac{24-\mu_u}{3}\right)-F\left(\frac{24-\mu_u}{1}\right)\right]}{\mu_u-24} \end{aligned}$$

And,

$$\begin{aligned} &E[p \mid y=24; 1<\sigma<3] \\ &= \left\{ \begin{array}{c} \frac{1+a}{1+a+b}\beta(1+a,b)\frac{\left[F\left(\frac{24-\mu_f}{3}\right)-F\left(\frac{24-\mu_f}{1}\right)\right]}{\mu_f-24} \\ +\frac{a}{1+a+b}\beta(a,1=b)\frac{\left[F\left(\frac{24-\mu_u}{3}\right)-F\left(\frac{24-\mu_u}{1}\right)\right]}{\mu_u-24} \end{array} \right\} / f(y=24; 1<\sigma<3) \end{aligned}$$

Suggested:

1. Verify via maximum entropy probability assignment the kernel for p is $p^{-0.7}(1-p)^{-0.7}$.

2. Derive the density function for proposition probability given the evidence $f(p \mid y=24; \sigma)$ and expected value $E[p \mid y=24; \sigma]$ if σ is assigned 1 or if σ is assigned 3.

3. Derive the density function for proposition probability given the evidence $f(p \mid y=24; 1<\sigma<3)$ and expected value $E[p \mid y=24; 1<\sigma<3]$ if σ is assigned density $\frac{1}{\sigma}$ with support between 1 and 3.

4. How do these results compare with those in Ralph's uncertain variance?