## Ralph's Project Information

Ralph manages two projects and believes managing is about information. In particular, synergy in an uncertain world is derived from considering informational interactions amongst productive activities (an information variation on economies of scope). Consider the following cases of joint probability assignment where $s_{j}$ refers to the (future) state, $a_{j}\left(b_{j}\right)$ refers to (recent past) outcome $j$ on project $a(b)$ for $j=1,2$.

Case 1.

|  | $a_{1}, b_{1}$ | $a_{2}, b_{1}$ | $a_{1}, b_{2}$ | $a_{2}, b_{2}$ | $\operatorname{Pr}\left(s_{j}\right)$ |
| :---: | :---: | :---: | :---: | :---: | :---: |
| $s_{1}$ | 0.15 | 0.1 | 0.1 | 0.15 | 0.5 |
| $s_{2}$ | 0.1 | 0.15 | 0.15 | 0.1 | 0.5 |

Case 2.

|  | $a_{1}, b_{1}$ | $a_{2}, b_{1}$ | $a_{1}, b_{2}$ | $a_{2}, b_{2}$ | $\operatorname{Pr}\left(s_{j}\right)$ |
| :---: | :---: | :---: | :---: | :---: | :---: |
| $s_{1}$ | 0.2 | 0.05 | 0.05 | 0.2 | 0.5 |
| $s_{2}$ | 0.05 | 0.2 | 0.2 | 0.05 | 0.5 |

Case 3.

|  | $a_{1}, b_{1}$ | $a_{2}, b_{1}$ | $a_{1}, b_{2}$ | $a_{2}, b_{2}$ | $\operatorname{Pr}\left(s_{j}\right)$ |
| :---: | :---: | :---: | :---: | :---: | :---: |
| $s_{1}$ | 0.249 | 0.001 | 0.001 | 0.249 | 0.5 |
| $s_{2}$ | 0.001 | 0.249 | 0.249 | 0.001 | 0.5 |

Suggested:

1. For each case, determine the (future) state probabilities given (past) outcome for project $a$ only and outcome for project $b$ only. (Hint: utilize the sum and product rules of Bayes' theorem.)
2. For each case, determine the (future) state probabilities given (past) outcome for both projects $a$ and $b$.
3. Is informational synergy associated with any of these cases? Suppose this is a continuation of Ralph's Long-run Frame but in a two state-two project setting? Do you require background knowledge of the payoffs to assess the expected gain from the information? Why?
4. Suppose equally likely (future) returns on projects $a$ and $b$ are

$$
A=\left[\begin{array}{ccc} 
& s_{1} & s_{2} \\
a & 0.9 & 1.1 \\
b & 1.1 & 0.9
\end{array}\right]
$$

Determine mutual information for each case where Ralph takes care to create (contractual) relations with others that accommodates scalable "projects". Determine maximum expected logarithmic returns where project weights sum to one (as with mutual information) but are bounded between zero and one (not scalable). Compare this (not scalable) quantity with mutual information (scalable).
5. What are the implications of no arbitrage, spanning, and scalability (construction of Arrow-Debreu assets) for Ralph's stewardship? How might the long-run view adopted by traditional accounting practice aid Ralph?

