Ralph's Private Information Challenge

Ralph recognizes private information presents challenges for both of accounting's primary roles: valuation and performance evaluation.

A. Valuation.

Ralph wants to purchase a family heirloom from Alice. The heirloom has private value to Alice denoted v. Alice knows v; Ralph only knows v is uniformly distributed between 0 and 100. Whatever v is, the value of the heirloom to Ralph is $1.5v$. Alice knows all of this about Ralph.

Suggested:

1. Who should own the heirloom, Ralph or Alice?

2. Suppose the trade encounter between the two individuals proceeds as follows. Ralph offers to purchase the heirloom at price P. If Alice agrees, Ralph pays P in exchange for the heirloom. If Alice declines, the game ends, and Alice keeps the heirloom.
From Ralph's perspective, what is the expected value (to Ralph) of the heirloom before discussing with Alice?
Conversely, suppose Ralph offers $P > 0$ and Alice accepts the offer; what is the expected value (to Ralph) of the heirloom conditional on acceptance of the offer?
If Ralph pays P for the heirloom, at what price will Ralph's accountants value the heirloom?

3. What is the equilibrium of this game? Why does no trade take place despite the fact Ralph is known to value the object more than Alice?

4. Suppose Ralph hires an auditor to inspect the heirloom before an offer is made. The auditor reports the value of the heirloom is either high ($50 \leq v \leq 100$) or low ($0 \leq v < 50$). How does the auditor change the game? Does socially efficient trade occur with greater likelihood?

B. Performance evaluation.

Alice wishes to hire Ralph to manager a production activity with uncertain payoffs. Ralph and Alice agree about the uncertainty and Ralph's potential influence on the payoffs. If Ralph provides input a_H the favorable payoff to Alice is more likely; however Ralph dislikes input a_H more than input a_L (for simplicity, there are only two possible inputs). The payoffs, probabilities and Ralph's personal cost are as follows.

	Favorable	Unfavorable	Ralph's personal cost
Payoff excluding Ralph's compensation (x)	20	0	
Probability(state $\mid a_H$)	0.7	0.3	$C(a_H) = 1$
Probability(state $\mid a_L$)	0.5	0.5	$C(a_L) = 0$

In addition, Ralph's next best alternative employment opportunity pays him $U_0 = 2$. Ralph's employment contract assures him that his payoff will be non-negative regardless of the outcome (Ralph cannot be asked to pay for poor outcomes out of his own pocket). Rather, as the owner, Alice bears all production/investment costs.

Alice knows that to employ Ralph he must be paid so that his expected compensation less his personal cost is at least equal to his alternative employment opportunities (here, a wage of 2). This is frequently denoted the individual rationality (IR) or participation (P) condition or constraint.

$$\Pr(\text{favorable} \mid a)\, I(x=20) + \Pr(\text{unfavorable} \mid a)\, I(x=0) - C(a) \geq U_0$$

where $\Pr(\ )$ = probability of outcome given input a (either a_H or a_L);

$I(x)$ = compensation based on outcome x;

$C(a)$ = Ralph's personal cost for input a.

Further, since Alice is unable to monitor Ralph, Alice needs to provide incentives to motivate Ralph to supply input a_H, if desired. (Input a_H is desired if it provides a more favorable outcome including the cost of compensating Ralph; this needs to be checked.) In particular, Ralph's expected utility (here, the expected value of compensation less Ralph's personal cost of the input) under a_H must be at least as great as his expected utility under a_L. This is denoted the incentive compatibility (IC) condition or constraint.

$$\Pr(\text{favorable} \mid a_H)\, I(x=20) + \Pr(\text{unfavorable} \mid a_H)\, I(x=0) - C(a_H) \geq$$
$$\Pr(\text{favorable} \mid a_L)\, I(x=20) + \Pr(\text{unfavorable} \mid a_L)\, I(x=0) - C(a_L)$$

(Note: when Ralph is indifferent assume he will act in Alice's best interest.)

Suggested:

1) Suppose that Alice is able to observe Ralph's input. Determine the optimal contract for Alice to offer Ralph. Since Ralph's input is observable a constant wage works. What is the smallest amount acceptable to Ralph? What is Alice's expected payoff after Ralph's compensation?

2) Now, suppose that Alice is busy and unable to monitor /observe Ralph's input. Find an optimal performance-based contract. Does an outcome-based contract that pays Ralph 5 for an outcome of 20 and 0 for an outcome of 0 motivate Ralph to accept Alice's employment opportunity? Does the contract provide incentive for Ralph to provide input a_H? What is Alice's expected payoff after Ralph's compensation?

3) Under the conditions in 2, what is the maximum value to Alice of verifying Ralph's unobservable input? What does this suggest about the demand for audited accounting reports?

4) Suppose Alice believes Ralph's personal cost for input H is $C(a_H) = 1$ but it is actually $C(a_H) = 2$. What are the implications for Alice of supplying the explicit incentive contract identified above?

5) Repeat 1-4 if Ralph is risk averse with utility function $U(I,C) = -\exp[-0.2(I-C(a))]$. That is, replace the non-negative payment (limited liability) restriction above with risk aversion. (Hint: the above performance-based payments are to be revised.)