Ralph's partnership[1]

Alice, Bob, and Ralph are contemplating a partnership in which they borrow 600 @ 2% (with no chance of default as any deficit is made up out of personal assets) and return $b = 612$ to the lender at the end of one period. These proceeds are invested in productive assets that return $y = 1,000,\ 2,000$, or $-1,000$ with equal likelihood. All preferences are negative exponential, $U(x) = -\exp[-\rho x] = -\exp\left[-\frac{x}{\tau}\right]$ with Alice's risk tolerance $\tau_A = \frac{1}{\rho_A} = \frac{1}{0.005}$, Bob's $\tau_B = \frac{1}{\rho_B} = \frac{1}{0.0002}$, and Ralph's $\tau_R = \frac{1}{\rho_R} = \frac{1}{0.0001}$.

Suggested:

1. If individually they could arrange the same financing terms as above, would any of them pursue this investment opportunity on their own? (Hint: determine Alice's, Bob's, and Ralph's certainty equivalent.)

Efficient risk sharing is Pareto efficient or optimal (no one can be made better of without harming another individual). A sharing rule defined as $\alpha_A + \alpha_B + \alpha_R = 1$ maps out an efficient frontier by solving the following constrained optimization problem for each state by varying α_A, α_B, and α_R.

$$\begin{array}{cc} \max\limits_{x_A, x_B, x_R} & \alpha_A U_A(x_A) + \alpha_B U_B(x_B) + \alpha_R U_R(x_R) \\ s.t. & x_A + x_B + x_R = y - b \end{array}$$

2. Solve the above program for each state when the sharing rule assigns weight proportional to each partner's risk tolerance, $\alpha_A = \frac{\tau_A}{T}, \alpha_B = \frac{\tau_B}{T}, \alpha_R = \frac{\tau_R}{T}$ where $T = \tau_A + \tau_B + \tau_R$. (Hint: solver will need some help with starting values. Try $x_A = \frac{\tau_A}{T}x, x_B = \frac{\tau_B}{T}x, x_R = \frac{\tau_R}{T}x$. Alternatively, solve the first order conditions directly.) Is the allocation to each potential partner linear, $x_j = \beta_{j0} + \beta_{j1}(y - b)$ for $j = A, B$, or R, across the states? If so, what properties describe β_{j1}? Determine each partners expected utility and certainty equivalent for this arrangement. Is the investment attractive to each individual when a partnership is formed along these lines?

3. How would efficient arrangements change if Ralph is risk neutral?

[1] This example draws from Kreps,1990, *A course in microeconomic theory*, Princeton University Press, 169-174.