Ralph's optimal accruals

This is a continuation of Ralph's estimate. Ralph is concerned that his process description is too static and cash flows are more variable than he supposed. On reflection, Ralph believes the cash flow process is better described by the following *DGP*.

$$y_t = m_t + e_t$$
$$m_t = m_{t-1} + \varepsilon_t$$

where m_t is the unknown cash flow mean at time t and e_t and ε_t are independent, zero mean random variables each with variance $\sigma^2 I$. In other words, for Ralph's observed cash flow data the *DGP* (where observable outcomes/variables are on the left-hand side and functions of the unknowns are moved to the right-hand side of the equations) is

$$y_1 = m_1 + e_1$$
$$0 = m_1 - m_2 + \varepsilon_2$$
$$y_2 = m_2 + e_2$$
$$0 = m_2 - m_3 + \varepsilon_3$$
$$y_3 = m_3 + e_3$$

Or, more compactly, we have

$$y = Xm + \eta$$

$$\text{where } y = \begin{bmatrix} y_1 \\ 0 \\ y_2 \\ 0 \\ y_3 \end{bmatrix}, X = \begin{bmatrix} 1 & 0 & 0 \\ 1 & -1 & 0 \\ 0 & 1 & 0 \\ 0 & 1 & -1 \\ 0 & 0 & 1 \end{bmatrix}, m = \begin{bmatrix} m_1 \\ m_2 \\ m_3 \end{bmatrix}, \text{and } \eta = \begin{bmatrix} e_1 \\ \varepsilon_2 \\ e_2 \\ \varepsilon_3 \\ e_3 \end{bmatrix} \sim (0, \sigma^2 I).$$

Required:

1. Find the *OLS* estimator m for Ralph's cash flow data. What are the weights on y_1, y_2, and y_3 for each of the estimators m_1, m_2, and m_3?

2. Find the variance of the *OLS* estimator for m based on Ralph's cash flow data. (Hint: this is a 3 x 3 matrix with variances along the diagonal and covariances on the off-diagonals.)

3. Could the estimator for m_3 describe accruals as an estimator of future cash flows? (Hint: consider accelerated amortization.)