Ralph's MLE Accruals

Continue with the setting in Ralph's Optimal Accruals but build a frame beginning with m_0.

$$m_0 = \mu + \varepsilon_0, \quad \varepsilon_0 \sim N\left(0, a\sigma^2\right)$$

Then, cash flow information relates to the quantity of interest, m_3, as follows.

$$\begin{array}{rcl} \mu & = & m_3 - \varepsilon_0 - \varepsilon_1 - \varepsilon_2 - \varepsilon_3 \\ y_1 & = & m_3 - \varepsilon_2 - \varepsilon_3 + e_1 \\ y_2 & = & m_3 - \varepsilon_3 + e_2 \\ y_3 & = & m_3 + e_3 \end{array}$$

where $\eta^T = \left[\begin{array}{ccccccc} \varepsilon_0 & \varepsilon_1 & \varepsilon_2 & \varepsilon_3 & e_1 & e_2 & e_3 \end{array}\right] \sim N\left(0, \Sigma\right)$, $\Sigma = \left[\begin{array}{cc} a\sigma^2 & 0 \\ 0 & \sigma^2 I_6 \end{array}\right]$. This can be compactly written

$$Y = m_3 \iota + A\eta$$

where $Y^T = \left[\begin{array}{cccc} \mu & y_1 & y_2 & y_3 \end{array}\right]$, ι is a vector of ones, and

$$A = \left[\begin{array}{ccccccc} -1 & -1 & -1 & -1 & 0 & 0 & 0 \\ 0 & 0 & -1 & -1 & 1 & 0 & 0 \\ 0 & 0 & 0 & -1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{array}\right]$$

Let $\sigma^2 = 1$. The sampling distribution for Y given m_3 is

$$\left(Y \mid m_3; \Sigma\right) \sim N\left(m_3 \iota, A\Sigma A^T\right)$$

and the likelihood function for m_3 given cash flows μ, y_1, y_2, y_3 is

$$\ell\left(m_3; Y, \Sigma\right) \propto \exp\left[-\frac{1}{2}\left(Y - m_3\iota\right)^T \left(A\Sigma A^T\right)^{-1} \left(Y - m_3\iota\right)\right]$$

Suggested:

1. Utilize the log-likelihood to find the modal value (or most likely value) for m_3 given Y.
2. Evaluate $Var\left[m_3 \mid Y; \Sigma\right] = -\left(\frac{\partial^2 \log -\ell}{\partial (m_3)^2}\right)^{-1}$.
3. Suppose m_0 is known, then $a = 0$. Evaluate your expression in 1 and 2.
4. Suppose m_0 is equally uncertain as m_t for $t > 0$, $a = 1$. Evaluate your expression in 1 and 2.
5. Suppose m_0 is extremely uncertain so as to be uninformative, $a \to \infty$. Evaluate your expression in 1 and 2.