Ralph's long-run incentives

Ralph manages Alice's operations. Ralph knows the following regarding asset returns. In initial state one state-transition returns are

$$A_1 = \begin{bmatrix} & state_{11} & state_{12} \\ asset_1 & 0.99 & 0.99 \\ asset_2 & 1.089 & 0.989 \end{bmatrix}, x = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

while initial state two state-transition returns are

$$A_2 = \begin{bmatrix} & state_{21} & state_{22} \\ asset_1 & 1.019 & 1.021 \\ asset_2 & 1.1231 & 1.0209 \end{bmatrix}, x = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

Arbitrage-free state (or Arrow-Debreu) prices are $y > 0$ such that $Ay = x$. State-transition probabilities are

$$\begin{aligned} F &= \frac{1}{\delta} DPD^{-1} \\ &= \begin{bmatrix} 0.0103128 & 0.9896872 \\ 0.00098999 & 0.99901001 \end{bmatrix} \end{aligned}$$

where initial states are referred to by the rows and transition states are reported in columns with steady-state probabilities

$$\begin{aligned} p^T F &= p^T \\ p^T &= \begin{bmatrix} 0.000999311 & 0.999000689 \end{bmatrix} \end{aligned}$$

Alice only knows the Kelly criterion will achieve her goal of long-run wealth maximization. Hence, Alice motivates Ralph to maximize long-run wealth by emphasizing compound growth. She implements this strategy by evaluating Ralph's performance based on logarithmic returns.

Suggested:

1. The risk-free return in initial state one produces shrinkage at rate 0.99, what is the risk-free return in initial state two?

2.. Determine state prices in each initial state $i = 1, 2$, y_i.

3. Determine Alice's returns and Ralph's log-returns per unit of total wealth invested from a Kelly investment strategy in each transition state, $r_{11}, r_{12}, r_{21}, r_{22}$.

4.If Ralph employs a short-run strategy expected returns are unbounded

(with unlimited borrowing/short-selling), what is the probability of bankruptcy, or equivalently, the probability Ralph is fired?

Suppose Ralph can acquire the following information where z_{ij} refers to information in initial state i regarding signal j.

	$state_{i1}$	$state_{i2}$	$\Pr(z_j)$
z_{11}	0.0000102026	9.89005×10^{-7}	0.0000111916
z_{12}	1.03057×10^{-7}	0.000988016	0.00098812
z_{21}	0.000979115	0.000998012	0.00197713
z_{22}	9.89005×10^{-6}	0.997013672	0.997023562
$\Pr(s)$	0.000999311	0.999000689	

5. Determine Alice's returns and Ralph's log-returns per unit of total wealth invested from a Kelly investment strategy in each transition state, $r_{11}, r_{12}, r_{21}, r_{22}$ for each signal z_{ij}.

6. What is the expected gain from the information if the initial state is unknown? Compare

$$E[gain] = E[r \mid z] - E[r^{pss}]$$

with mutual information

$$I(s; z) = H(s) + H(z) - H(s, z)$$

where H refers to entropy

$$H(\circ) = -\sum p_i \log p_i$$

and $E[r^{pss}]$ is the expected log-return when the fraction of wealth invested in each Arrow-Debreu portfolio equals p — the steady-state probability (in other words, Ralph has no information — not even initial state information) and $E[r \mid z]$ is the expected log-return when the fraction of wealth invested in each Arrow-Debreu portfolio equals the conditional state probability given information z_{ij}.

7. What is the expected gain from the information in each initial state i when the initial state is known? Compare

$$E[gain_i] = E[r_i \mid z_i] - E[r_i]$$

with mutual information

$$I(s_i; z_i) = H(s_i) + H(z_i) - H(s_i, z_i)$$