Ralph's long-run economy

Ralph is managing in an economy where every agent maximizes long-run wealth (employs a Kelly strategy) but each agent acquires their own information (information is heterogeneous). There are three equally endowed agents in the economy (with initial wealth normalized to one).

Returns on nominal assets are depicted in the rows and states in the columns of A with nominal prices x.

$$A = \begin{bmatrix} 1 & 1 \\ 0.9 & 1.1 \end{bmatrix}, x = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

Initial state prices satisfying $Ay = x$ are $y = \begin{bmatrix} 0.5 \\ 0.5 \end{bmatrix}$. In this setting, state prices, y, and probability beliefs over states, p, are a scalar multiple of one another

$$p = \frac{1}{\delta} y$$

where $\frac{1}{\delta}$ is the risk-free rate.

Each agent acquires information involving three possible signals where z_i refers to agent i's acquired information (Ralph is agent 1). On observing their own signal, the agents balance their portfolios. The positions taken by the agents reveal their private information and the agents rebalance to achieve equilibrium.

Suppose the agents' joint probability assignments regarding the information and states are symmetric (even though each agent acquires their own signal) and the conditional distributions for each agent based on their own information only is as below (the joint distribution is provided on the next page).

signal j	$\Pr(s_1 \mid z_1)$	$\Pr(z_1)$	$\Pr(s_1 \mid z_2)$	$\Pr(z_2)$	$\Pr(s_1 \mid z_3)$	$\Pr(z_3)$
$j = 1$	$\frac{3}{5}$	$\frac{1}{3}$	$\frac{3}{5}$	$\frac{1}{3}$	$\frac{3}{5}$	$\frac{1}{3}$
$j = 2$	$\frac{2}{5}$	$\frac{1}{3}$	$\frac{2}{5}$	$\frac{1}{3}$	$\frac{2}{5}$	$\frac{1}{3}$
$j = 3$	$\frac{1}{2}$	$\frac{1}{3}$	$\frac{1}{2}$	$\frac{1}{3}$	$\frac{1}{2}$	$\frac{1}{3}$
$\Pr(s)$	$\frac{1}{2}$		$\frac{1}{2}$		$\frac{1}{2}$	

Suppose the three agents acquire signals $z_1 = 1, z_2 = 2, z_3 = 3$.

Suggested:

1. Determine each agent's initial position in Arrow-Debreu assets as well as nominal assets. (hint: $w = \left(A^T\right)^{-1} \Omega k$ where w is a vector representing the fraction of wealth invested in the nominal assets, k is a vector representing the fraction of wealth invested in the Arrow-Debreu assets, and Ω is a diagonal matrix with elements $\frac{1}{y}$.)

2. Determine the implied values of the nominal assets based on each agent's private information z_j. Is the long-, short-position consistent with under-, over-pricing?

3. Determine each agent's rebalanced positions in Arrow-Debreu assets as well as nominal assets on observing all positions (and effectively other agent's probability beliefs based on their own private signal; hint: $p = k = \Omega^{-1} A^T w$). Is this an equilibrium? If not, continue rebalancing until equilibrium. What is the equilibrium price of the nominal assets?

4. Repeat 1-3, if the three agents acquire signals $z_1 = 1, z_2 = 1, z_3 = 1$
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The joint is distribution is below.

z_1, z_2, z_3	$\Pr(s_1, z)$	$\Pr(s_2, z)$	$\Pr(z)$	$\Pr(s_1 \mid z)$
111	$\frac{4}{135}$	$\frac{1}{135}$	$\frac{1}{27}$	$\frac{4}{5}$
112	$\frac{1}{45}$	$\frac{2}{135}$	$\frac{1}{27}$	$\frac{3}{5}$
113	$\frac{7}{270}$	$\frac{1}{90}$	$\frac{1}{27}$	$\frac{7}{10}$
121	$\frac{1}{45}$	$\frac{2}{135}$	$\frac{1}{27}$	$\frac{3}{5}$
122	$\frac{2}{135}$	$\frac{1}{45}$	$\frac{1}{27}$	$\frac{2}{5}$
123	$\frac{1}{54}$	$\frac{1}{54}$	$\frac{1}{27}$	$\frac{1}{2}$
131	$\frac{7}{270}$	$\frac{1}{90}$	$\frac{1}{27}$	$\frac{7}{10}$
132	$\frac{1}{54}$	$\frac{1}{54}$	$\frac{1}{27}$	$\frac{1}{2}$
133	$\frac{1}{45}$	$\frac{2}{135}$	$\frac{1}{27}$	$\frac{3}{5}$
211	$\frac{1}{45}$	$\frac{2}{135}$	$\frac{1}{27}$	$\frac{3}{5}$
212	$\frac{2}{135}$	$\frac{1}{45}$	$\frac{1}{27}$	$\frac{2}{5}$
213	$\frac{1}{54}$	$\frac{1}{54}$	$\frac{1}{27}$	$\frac{1}{2}$
221	$\frac{2}{135}$	$\frac{1}{45}$	$\frac{1}{27}$	$\frac{2}{5}$
222	$\frac{1}{135}$	$\frac{4}{135}$	$\frac{1}{27}$	$\frac{1}{5}$
223	$\frac{1}{90}$	$\frac{7}{270}$	$\frac{1}{27}$	$\frac{3}{10}$
231	$\frac{1}{54}$	$\frac{1}{54}$	$\frac{1}{27}$	$\frac{1}{2}$
232	$\frac{1}{90}$	$\frac{7}{270}$	$\frac{1}{27}$	$\frac{3}{10}$
233	$\frac{2}{135}$	$\frac{1}{45}$	$\frac{1}{27}$	$\frac{2}{5}$
311	$\frac{7}{270}$	$\frac{1}{90}$	$\frac{1}{27}$	$\frac{7}{10}$
312	$\frac{1}{54}$	$\frac{1}{54}$	$\frac{1}{27}$	$\frac{1}{2}$
313	$\frac{1}{45}$	$\frac{2}{135}$	$\frac{1}{27}$	$\frac{3}{5}$
321	$\frac{1}{54}$	$\frac{1}{54}$	$\frac{1}{27}$	$\frac{1}{2}$
322	$\frac{1}{90}$	$\frac{7}{270}$	$\frac{1}{27}$	$\frac{3}{10}$
323	$\frac{2}{135}$	$\frac{1}{45}$	$\frac{1}{27}$	$\frac{2}{5}$
331	$\frac{1}{45}$	$\frac{2}{135}$	$\frac{1}{27}$	$\frac{3}{5}$
332	$\frac{2}{135}$	$\frac{1}{45}$	$\frac{1}{27}$	$\frac{2}{5}$
333	$\frac{1}{54}$	$\frac{1}{54}$	$\frac{1}{27}$	$\frac{1}{2}$
$\Pr(s)$	$\frac{1}{2}$	$\frac{1}{2}$		

Alternatively, suppose the agents' joint probability assignments regarding the information and states are unique (or they have very different perceptions of other agents' information) and the conditional distributions for each agent based on their own information only is as below (the joint distribution is again provided on the next page).

signal j	$\Pr(s_1 \mid z_1)$	$\Pr(z_1)$	$\Pr(s_1 \mid z_2)$	$\Pr(z_2)$	$\Pr(s_1 \mid z_3)$	$\Pr(z_3)$
$j=1$	$\frac{7}{10}$	$\frac{1}{5}$	$\frac{297}{475}$	$\frac{19}{100}$	$\frac{79}{150}$	$\frac{27}{100}$
$j=2$	$\frac{59}{150}$	$\frac{3}{10}$	$\frac{2}{5}$	$\frac{51}{100}$	$\frac{137}{270}$	$\frac{27}{100}$
$j=3$	$\frac{121}{250}$	$\frac{1}{2}$	$\frac{443}{750}$	$\frac{3}{10}$	$\frac{12}{25}$	$\frac{23}{50}$
$\Pr(s)$	$\frac{1}{2}$		$\frac{1}{2}$		$\frac{1}{2}$	

Again, the three agents acquire signals $z_1 = 1, z_2 = 2, z_3 = 3$.

5. Repeat 1-4 for this alternative economy.

The joint is distribution is below.

z_1, z_2, z_3	$\Pr(s_1, z)$	$\Pr(s_2, z)$	$\Pr(z)$	$\Pr(s_1 \mid z)$
111	0.008	0.002	0.01	$\frac{4}{5}$
112	0.008	0.002	0.01	$\frac{4}{5}$
113	0.008	0.002	0.01	$\frac{4}{5}$
121	0.008	0.002	0.01	$\frac{4}{5}$
122	0.016	0.004	0.02	$\frac{4}{5}$
123	0.06	0.04	0.1	$\frac{3}{5}$
131	0.016	0.004	0.02	$\frac{4}{5}$
132	0.008	0.002	0.01	$\frac{4}{5}$
133	0.008	0.002	0.01	$\frac{4}{5}$
211	0.008	0.012	0.02	$\frac{2}{5}$
212	0.008	0.012	0.02	$\frac{2}{5}$
213	0.018	0.002	0.02	$\frac{9}{10}$
221	0.004	0.016	0.02	$\frac{1}{5}$
222	0.008	0.032	0.04	$\frac{1}{5}$
223	0.03	0.07	0.1	$\frac{3}{10}$
231	0.016	0.024	0.04	$\frac{2}{5}$
232	0.008	0.012	0.02	$\frac{2}{5}$
233	0.018	0.002	0.02	$\frac{9}{10}$
311	0.018	0.012	0.03	$\frac{3}{5}$
312	0.018	0.012	0.03	$\frac{3}{5}$
313	0.0248	0.0152	0.04	$\frac{31}{50}$
321	0.024	0.036	0.06	$\frac{2}{5}$
322	0.024	0.036	0.06	$\frac{2}{5}$
323	0.03	0.07	0.1	$\frac{3}{10}$
331	0.0402	0.0198	0.06	$\frac{67}{100}$
332	0.039	0.021	0.06	$\frac{13}{20}$
333	0.024	0.036	0.06	$\frac{2}{5}$
$\Pr(s)$	$\frac{1}{2}$	$\frac{1}{2}$		

Further, suppose the agents' joint probability assignments regarding the information and states are highly complementary (again the agents acquire their own signal) and the conditional distributions for each agent based on their own information only is as below (the joint distribution is provided on the next page).

signal j	$\Pr(s_1 \mid z_1)$	$\Pr(z_1)$	$\Pr(s_1 \mid z_2)$	$\Pr(z_2)$	$\Pr(s_1 \mid z_3)$	$\Pr(z_3)$
$j = 1$	$\frac{1}{2}$	$\frac{1}{3}$	$\frac{7}{15}$	$\frac{1}{3}$	$\frac{1}{2}$	$\frac{1}{3}$
$j = 2$	$\frac{1}{2}$	$\frac{1}{3}$	$\frac{1}{2}$	$\frac{1}{3}$	$\frac{1}{2}$	$\frac{1}{3}$
$j = 3$	$\frac{1}{2}$	$\frac{1}{3}$	$\frac{8}{15}$	$\frac{1}{3}$	$\frac{1}{2}$	$\frac{1}{3}$
$\Pr(s)$	$\frac{1}{2}$		$\frac{1}{2}$		$\frac{1}{2}$	

Again, the three agents acquire signals $z_1 = 1, z_2 = 2, z_3 = 3$.

6. Repeat 1-4 for this highly complementary economy.

The joint is distribution is below.

z_1, z_2, z_3	$\Pr(s_1, z)$	$\Pr(s_2, z)$	$\Pr(z)$	$\Pr(s_1 \mid z)$
111	$\frac{2}{135}$	$\frac{1}{45}$	$\frac{1}{27}$	$\frac{2}{5}$
112	$\frac{1}{45}$	$\frac{2}{135}$	$\frac{1}{27}$	$\frac{3}{5}$
113	$\frac{2}{135}$	$\frac{1}{45}$	$\frac{1}{27}$	$\frac{2}{5}$
121	$\frac{1}{45}$	$\frac{2}{135}$	$\frac{1}{27}$	$\frac{3}{5}$
122	$\frac{2}{135}$	$\frac{1}{45}$	$\frac{1}{27}$	$\frac{2}{5}$
123	$\frac{1}{54}$	$\frac{1}{54}$	$\frac{1}{27}$	$\frac{1}{2}$
131	$\frac{1}{54}$	$\frac{1}{54}$	$\frac{1}{27}$	$\frac{1}{2}$
132	$\frac{1}{54}$	$\frac{1}{54}$	$\frac{1}{27}$	$\frac{1}{2}$
133	$\frac{1}{45}$	$\frac{2}{135}$	$\frac{1}{27}$	$\frac{3}{5}$
211	$\frac{2}{135}$	$\frac{1}{45}$	$\frac{1}{27}$	$\frac{2}{5}$
212	$\frac{1}{45}$	$\frac{2}{135}$	$\frac{1}{27}$	$\frac{3}{5}$
213	$\frac{2}{135}$	$\frac{1}{45}$	$\frac{1}{27}$	$\frac{2}{5}$
221	$\frac{1}{45}$	$\frac{2}{135}$	$\frac{1}{27}$	$\frac{3}{5}$
222	$\frac{2}{135}$	$\frac{1}{45}$	$\frac{1}{27}$	$\frac{2}{5}$
223	$\frac{1}{54}$	$\frac{1}{54}$	$\frac{1}{27}$	$\frac{1}{2}$
231	$\frac{1}{54}$	$\frac{1}{54}$	$\frac{1}{27}$	$\frac{1}{2}$
232	$\frac{1}{54}$	$\frac{1}{54}$	$\frac{1}{27}$	$\frac{1}{2}$
233	$\frac{1}{45}$	$\frac{2}{135}$	$\frac{1}{27}$	$\frac{3}{5}$
311	$\frac{2}{135}$	$\frac{1}{45}$	$\frac{1}{27}$	$\frac{2}{5}$
312	$\frac{1}{45}$	$\frac{2}{135}$	$\frac{1}{27}$	$\frac{3}{5}$
313	$\frac{2}{135}$	$\frac{1}{45}$	$\frac{1}{27}$	$\frac{2}{5}$
321	$\frac{1}{45}$	$\frac{2}{135}$	$\frac{1}{27}$	$\frac{3}{5}$
322	$\frac{2}{135}$	$\frac{1}{45}$	$\frac{1}{27}$	$\frac{2}{5}$
323	$\frac{1}{54}$	$\frac{1}{54}$	$\frac{1}{27}$	$\frac{1}{2}$
331	$\frac{1}{54}$	$\frac{1}{54}$	$\frac{1}{27}$	$\frac{1}{2}$
332	$\frac{1}{54}$	$\frac{1}{54}$	$\frac{1}{27}$	$\frac{1}{2}$
333	$\frac{1}{45}$	$\frac{2}{135}$	$\frac{1}{27}$	$\frac{3}{5}$
$\Pr(s)$	$\frac{1}{2}$	$\frac{1}{2}$		