Ralph's long-run dynamic economy

Ralph is managing in an economy where every agent maximizes long-run wealth (employs a Kelly strategy) but each agent acquires their own information (information is heterogeneous). There are two equally endowed agents in the economy (with initial wealth normalized to one).

Returns on nominal assets in initial state i are depicted in the rows and states in the columns of A with nominal prices x.

$$A_1 = \begin{bmatrix} 1 & 1 \\ 0.9 & 1.1 \end{bmatrix}, A_2 = \begin{bmatrix} 1.1 & 1.1 \\ 0.9 & 1.2 \end{bmatrix}, x_i = \begin{bmatrix} 1 \\ 1 \end{bmatrix}, i = 1, 2$$

Initial state prices satisfying $A_i y_i = x_i$ are $y_1 = \begin{bmatrix} 0.5 \\ 0.5 \end{bmatrix}$ and $y_2 = \begin{bmatrix} \frac{1}{3.3} \\ \frac{2}{3.3} \end{bmatrix}$.

The recovery theorem indicates the matrix of state prices, P, and probability beliefs over states, F, are related by

$$\begin{aligned} F &= \frac{1}{\delta} D P D^{-1} \\ &= \frac{1}{0.945876} \begin{bmatrix} \frac{1}{0.746348} & 0 \\ 0 & \frac{1}{0.665556} \end{bmatrix} \begin{bmatrix} 0.5 & 0.5 \\ \frac{1}{3.3} & \frac{2}{3.3} \end{bmatrix} \begin{bmatrix} 0.746348 & 0 \\ 0 & 0.665556 \end{bmatrix} \\ &= \begin{bmatrix} 0..528611 & 0.471389 \\ 0.35926 & 0.64074 \end{bmatrix} \end{aligned}$$

where δ is the largest eigenvalue of P (the representative agent's time discount rate) and D^{-1} is the a diagonal matrix with elements the associated eigenvectors (the representative agent's relative marginal rates of utility for consumption between states). Since Ralph regards beliefs and preferences as independent, equilibrium state prices are implied by probability beliefs.

$$P = \delta D^{-1} F D$$

Each agent acquires information involving two possible signals where z_i refers to agent i's acquired information (Ralph is agent 1). On observing their own signal, the agents balance their portfolios. The positions taken by the agents reveal their private information and the agents rebalance to achieve equilibrium.

Suppose the agents' joint probability assignments regarding the information and states are symmetric (even though each agent k acquires their own signal j in initial state i, z_{ij}^k) and the conditional distributions for each agent based on their own information only is as below.

signal z_{1j}	$\Pr\left(s_1 \mid z_{1j}^1\right)$	$\Pr\left(z_{1j}^1\right)$	$\Pr\left(s_1 \mid z_{1j}^2\right)$	$\Pr\left(z_{1j}^2\right)$
$i, j = 1, 1$	0.561123	0.643054	0.561123	0.643054
$i, j = 1, 2$	0.470039	0.356946	0.470039	0.356946
$\Pr\left(s_{11}\right)$	0.528611		0.528611	

signal z_{2j}	$\Pr\left(s_1 \mid z_{2j}^1\right)$	$\Pr\left(z_{2j}^1\right)$	$\Pr\left(s_1 \mid z_{2j}^2\right)$	$\Pr\left(z_{2j}^2\right)$
$i,j = 2,1$	0.302101	0.734567	0.302101	0.734567
$i,j = 2,2$	0.517443	0.265433	0.517443	0.265433
$\Pr(s_{21})$	0.35926		0.35926	

The joint distribution is below.

z	$\Pr(s_{11}, z)$	$\Pr(s_{12}, z)$	$\Pr(z)$	$\Pr(s_{11} \mid z)$
z_{11}^1, z_{11}^2	0.235832	0.157222	0.393054	0.6
z_{11}^1, z_{12}^2	0.125	0.125	0.25	0.5
z_{12}^1, z_{11}^2	0.125	0.125	0.25	0.5
z_{12}^1, z_{12}^2	0.0427784	0.0641676	0.106946	0.4
$\Pr(s_{1j})$	0.528611	0.471389		

z	$\Pr(s_{21}, z)$	$\Pr(s_{22}, z)$	$\Pr(z)$	$\Pr(s_{21} \mid z)$
z_{21}^1, z_{21}^2	0.0969134	0.387654	0.484567	0.2
z_{21}^1, z_{22}^2	0.125	0.125	0.25	0.5
z_{22}^1, z_{21}^2	0.125	0.125	0.25	0.5
z_{22}^1, z_{22}^2	0.0123462	0.00308655	0.0154328	0.8
$\Pr(s_{2j})$	0.35926	0.64074		

Suppose the agents acquire signals $z_{11}^1, z_{11}^2, z_{21}^1, z_{21}^2$.

Suggested:

1. Determine each agent's initial position in Arrow-Debreu assets as well as nominal assets. (hint: $w_i = \left(A_i^T\right)^{-1} \Omega_i k_i$ where w_i is a vector representing the fraction of wealth invested in the nominal assets, k_i is a vector representing the fraction of wealth invested in the Arrow-Debreu assets, and Ω_i is a diagonal matrix with elements $\frac{1}{y_i}$ in initial state i.)

2. Determine the implied values of the nominal assets based on each agent's private information z_j. Is the long-, short-position consistent with under-, over-pricing?

3. Determine each agent's rebalanced positions in Arrow-Debreu assets as well as nominal assets on observing all positions (and effectively other agent's probability beliefs based on their own private signal; hint: $F_i = k_i = \Omega_i^{-1} A_i^T w_i$). Is this an equilibrium? If not, continue rebalancing until equilibrium. What is the equilibrium price of the nominal assets?