Ralph's inverse

Let I be the identity matrix, that is, the matrix when multiplied by another matrix or vector leaves that matrix or vector as the product, A I = I A = A. When a matrix is square and composed of linearly independent rows and columns (it is full rank), its inverse exists, $A A^{-1} = A^{-1} A = I$. Suppose $A = \begin{bmatrix} 1 & -1 & 0 \\ -1 & 0 & 0 \\ 0 & 1 & -1 \end{bmatrix}$ and $x = \begin{bmatrix} 5 \\ -2 \\ 2 \end{bmatrix}$.

Required:

1. Find a matrix A^{-1} such that $A A^{-1} = A^{-1} A = I$.

2. Find y such that Ay = x. Hint: multiply both sides of Ay = x by A^{-1}.

3. Use Gaussian elimination and back substitution to solve for y such that Ay = x. Notice, Gaussian elimination is effectively A = LU, the product of lower and upper triangular matrices, where $L^{-1}A = L^{-1}LU = U$ or $AU^{-1} = LUU^{-1} = L$ and $U^{-1}L^{-1}LU = A^{-1}A = I$.