Ralph's Information Partition

We now find Ralph thinking about multiple sources of information.

A. Consider the following matrix, concerning the cash flow random variable CF = 100 or CF = 200, along with two information sources. One source reports "g" or "b" while the other reports "s" or "u". Joint probabilities are given in the following table.

	CF = 100		CF = 200	
	g	b	g	b
s	.02	.16	.08	.04
u	.40	.02	.20	.08

Required:

Suppose Ralph installs both information sources. Determine the expected value of CF conditional on no information, on g only, on b only, on s only, on u only, and on g and s, g and u, b and s, and b and u.

Can you express these changes in CF expectation as $140 + \varepsilon_1 + \varepsilon_2$ (constant plus additive shocks), where the first "shock" term comes from the g/b report and the second shock comes from the s/u report? If not, explain.

B. Suppose cash flows are uniformly distributed between zero and 400, and Ralph installs an information system that reports two pairs of signals:

y_1 = high ($CF \in (200,400]$) or low ($CF \in [0,200]$) and

y_2 = "odd" ($CF \in [0,100] \cup (200,300]$) or "even" ($CF \in (100,200] \cup (300,400]$).

Hence, the joint density is

$$f(CF,y_1,y_2) = 1/400, \quad 0 \leq CF \leq 400,$$

with E[CF] = 200 and Var[CF] = 40,000/3.

Required:

1. Determine the conditional distribution, mean, and variance for CF given y_1 = high.
2. Determine the conditional distribution, mean, and variance for CF given y_1 = low.
3. Determine the conditional distribution, mean, and variance for CF given y_2 = "odd".
4. Determine the conditional distribution, mean, and variance for CF given y_2 = "even".
5. Determine the conditional distribution, mean, and variance for CF given y_1 = high and y_2 = "odd".
6. Determine the conditional distribution, mean, and variance for CF given y_1 = high and y_2 = "even".
7. Determine the conditional distribution, mean, and variance for CF given y_1 = low and y_2 = "odd".
8. Determine the conditional distribution, mean, and variance for CF given y_1 = low and y_2 = "even".
9. Determine $E_{y1}[E[CF \mid y_1]]$, $E_{y2}[E[CF \mid y_2]]$, and $E_{y1,y2}[E[CF \mid y_1,y_2]]$. How do these quantities compare with E[CF]?
10. Determine $E_{y1}[Var[CF \mid y_1]] + Var_{y1}[E[CF \mid y_1]]$, $E_{y2}[Var[CF \mid y_2]] + Var_{y2}[E[CF \mid y_2]]$, and $E_{y1,y2}[Var[CF \mid y_1,y_2]] + Var_{y1,y2}[E[CF \mid y_1,y_2]]$. How do these quantities compare with Var[CF]?