Ralph's Information Paradox[1]

Ralph, the manager of Paradox Corp., is puzzled by the relationship between accounting earnings/valuation and market valuation of his firm. Ralph is aware of a substantial body of academic literature which documents that, on average, when a firm's reported earnings are greater (lower) than expected that the firm's stock price increases (decreases). Ralph is puzzled because this (seemingly too) frequently fails to hold for Paradox Corp. Consequently, Ralph decides to return to fundamentals in order to try to understand these phenomena.

Deviations between accounting and market valuations of the firm are likely to occur between the inception and termination of the firm but do not occur at either end point. These deviations may occur even though accounting and market valuations are unbiased. **Unbiasedness** here refers to the property that the valuations reflect expected cash flows conditional on the information considered.

Definition 1: $A_t = (1+r)^{-T} E[d_T | y_t^a]$, where the firm pays only a single (liquidating) dividend at the termination point T, y_t^a is information used for accounting valuation through period t (that is, accounting employs restricted recognition or limited usage of information), $E[\cdot]$ is the expectation operator and r is the firm's discount rate.[2]

If **accounting valuation** is given by A_t, then accounting valuation is **unbiased**.

Definition 2: $V_t = (1+r)^{-T} E[d_T | y_t]$, where y_t is all available information through time t and other variables are defined as above.

If **market valuation** is given by V_t, then market valuation is also **unbiased**; however, A_t need not be equal to V_t, for $t \neq 0, T$.

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[1]The example is based on Antle, Demski and Ryan, "Accounting Earnings and Multiple Sources of Information," *Journal of Accounting, Auditing & Finance*, Fall 1994.

[2]We could also write a similar expression involving interim dividends but it would simply be more cumbersome without adding further insight.

Definition 3: $e_{t,t-2} = A_t - A_{t-2}$; the change in accounting firm value is **accounting earnings** (still assuming no interim dividends).

Definition 4: $U_{t,t-1} = e_{t,t-2} - E[e_{t,t-2}|y_{t-1}]$; the deviation between accounting earnings and expected accounting earnings conditional on information through period t-1 (y_{t-1}) is **unexpected accounting earnings**.

Consider the following example. T=4, r=0 (ignore discounting), $y_3 = 0$ (there is no information in period three). The period four outcomes are described in the table below where the first entry in each cell refers to the liquidating dividend and the second entry in each cell refers to the joint probability of the two information signals, y_1 and y_2^a.

		y_2^a	
		0	1
y_1	0	0, 0.005	1, 0.495
	1	99, 0.495	100, 0.005

y_1 and y_3 are available information to investors at times 1 and 3, respectively (but are not reflected in the information employed for accounting valuation); y_2^a and y_4^a are information used for accounting valuation at times 2 and 4, respectively. That is, accounting valuation disclosures occur only at the end of even periods.

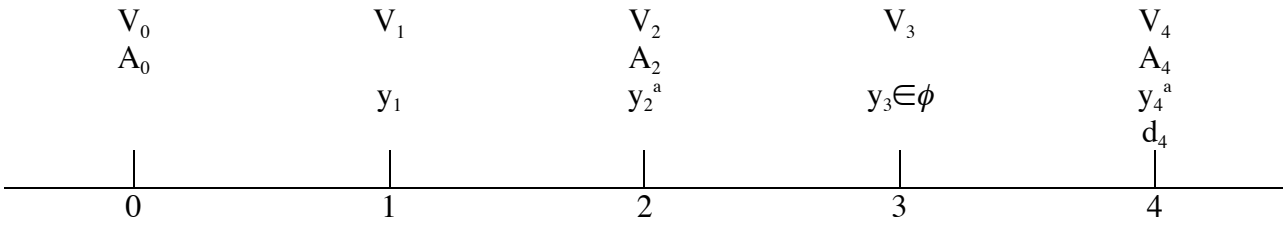

Figure. Timeline of information signals, valuations, and liquidating dividends.

Required :

(Hint: The key is to remember what information is used for accounting valuation, i.e., accounting ignores information from time 1.)

1. Derive the market value of the firm at time 0 (V_0).
   Derive the accounting value of the firm at time 0 (A_0).
   (Answer: $V_0 = A_0 = \$50$.)

2. If $y_1 = 0$ or $y_1 = 1$,
   (a) what is the firm's accounting value at time 2 (A_2) if $y_2^a = 0$?
   (b) what is the firm's accounting value at time 2 if $y_2^a = 1$?
   (Answer: $A_2(y_1 = 0 \text{ or } 1, y_2^a = 0) = 98.01$ and $A_2(y_1 = 0 \text{ or } 1, y_2^a = 1) = 1.99$)

3. What is the firm's **expected** accounting value at time 2 ($E[A_2|y_1]$) if
   (a) $y_1=0$,
   (b) $y_1=1$?
   (Answer: $E[A_2|y_1=0] = 2.9502$ and $E[A_2|y_1=1] = 97.0498$)

4. If $y_1 = 0$,
   (a) what is the firm's market value at time 1 (V_1)?
   (b) what is the firm's market value at time 2 (V_2) if $y_2^a = 0$?
   (c) what is the firm's market value at time 2 if $y_2^a = 1$?
   (Answer: $V_1(y_1 = 0) = .99$, $V_2(y_1 = 0, y_2^a = 0) = 0$ and $V_2(y_1 = 0, y_2^a = 1) = 1$.)

5. If $y_1 = 1$,
   (a) what is the firm's market value at time 1 (V_1)?
   (b) what is the firm's market value at time 2 (V_2) if $y_2^a = 0$?
   (c) what is the firm's market value at time 2 if $y_2^a = 1$?
   (Answer: $V_1(y_1 = 1) = 99.01$, $V_2(y_1 = 1, y_2^a = 0) = 99$ and $V_2(y_1 = 1, y_2^a = 1) = 100$.)

6. If $y_1 = 0$,

(a) what are the change in the firm's market value ($\Delta V_{2,1}$) and unexpected earnings ($U_{2,1}$) from time 1 to time 2 if $y_2^a = 0$?

(b) what are the change in the firm's market value and unexpected earnings from time 1 to time 2 if $y_2^a = 1$?

(Answer: $\Delta V_{2,1}(y_1 = 0, y_2^a = 0) = -0.99$, $U_{2,1}(y_1 = 0, y_2^a = 0) = 95.0598$, $\Delta V_{2,1}(y_1 = 0, y_2^a = 1) = 0.01$ and $U_{2,1}(y_1 = 0, y_2^a = 1) = -0.9602$.)

7. If $y_1 = 1$,

(a) what are the change in the firm's market value ($\Delta V_{2,1}$) and unexpected earnings ($U_{2,1}$) from time 1 to time 2 if $y_2^a = 0$?

(b) what are the change in the firm's market value and unexpected earnings from time 1 to time 2 if $y_2^a = 1$?

(Answer: $\Delta V_{2,1}(y_1 = 1, y_2^a = 0) = -0.01$, $U_{2,1}(y_1 = 1, y_2^a = 0) = 0.9602$, $\Delta V_{2,1}(y_1 = 1, y_2^a = 1) = 0.99$ and $U_{2,1}(y_1 = 1, y_2^a = 1) = -95.0598$.)

8. To help understand how such deviations between two unbiased valuation systems can occur determine accounting earnings in period four ($e_{4,2}$) for each pair of information signals in periods one and two

(a) $y_1 = 0$, $y_2^a = 0$,

(b) $y_1 = 0$, $y_2^a = 1$,

(c) $y_1 = 1$, $y_2^a = 0$, and

(d) $y_1 = 1$, $y_2^a = 1$.

(Answer: $e_{4,2}(y_1=0,y_2^a=0) = -98.01$, $e_{4,2}(y_1=0,y_2^a=1) = -0.99$, $e_{4,2}(y_1=1,y_2^a=0) = 0.99$ and $e_{4,2}(y_1=1,y_2^a=1) = 98.01$)