Ralph's inequality

Ralph believes Bell's inequality typifies quantum weirdness as well as the power of information (everything is fundamentally information). Suppose the entangled state of a system is $|\beta_{11}\rangle = \frac{1}{\sqrt{2}}(|01\rangle - |10\rangle)$ and an observable is

$$A = X_1 S_2 + X_1 T_2 + Z_1 S_2 - Z_1 T_2$$

where the subscript refers to the qubit on which the transformation operates, $S = -\frac{1}{\sqrt{2}}(X + Z)$ and $T = -\frac{1}{\sqrt{2}}(X - Z)$.

Required:

1. Find the expected value of A with respect to the state of the system $|\beta_{11}\rangle$. That is, evaluate $\langle A\rangle = \langle\beta_{11}| A |\beta_{11}\rangle$.

2. Each component of A (X, Z, S, and T) has eigenvalues ± 1, what is the maximum value of $A = X(S + T) + Z(S - T)$ from a classical perspective? (Hint: either $S + T$ or $S - T$ is 0.) How does this compare with your result in 1? What does this suggest about the power or synergy of entanglement?