Ralph's Income Benchmark[1]

Ralph is concerned with using income to evaluate performance. He believes that valuation of resources and claims will influence this evaluation (these are standard ideas in accounting). Ralph considers the following scenario. Suppose that there are two firms in the economy that produce two commodities from three resources.[2] The firms start with the following endowments:

	firm A	firm B
resource 1	2	2
resource 2	3	3
resource 3	3	7

The firms have the following production technology (the cell entries represent the amount of the resource required to produce one unit of the respective commodity; firm A produces commodity 1 and firm B produces commodity 2):

	commodity 1	commodity 2
resource 1	1	1
resource 2	0.5	2
resource 3	4	0.25

Demand (reflecting consumer preferences) for the two commodities is given by the following two inverse demand functions:
$p_1 = -x_1 + x_2$ and $p_2 = 10x_1 - x_2$, where p_i = unit price for commodity i and x_i = quantity of commodity i produced and sold. Quantities of commodities and resources are infinitely divisible so that fractional units can be produced and employed, respectively.

[1] This case is drawn from Arya, Fellingham, Glover, Schroeder, and Young, "Income and efficiency in incomplete markets," working paper, April 1996.
[2] The label "firms" really refers to industries occupied by many homogeneous producers.

Maximizing the 'value' of production subject to the above conditions yields 'Pareto optimal' (no consumer can be made better off without worsening the welfare of another consumer) or efficient production for the economy. If resources can be freely traded (markets are perfect and complete[3]), optimal production is (values are approximate) x_1 = 0.9836 and x_2 = 2.754. Importantly, the rates of exchange (market prices) for resources 1, 2, and 3, respectively, are 0, 3.5410, and 0.[4]

Required:

1. Verify the general equilibrium solution. What happens to entry and exit in these markets if input and/or output prices deviate? Why is this an equilibrium? (Hint: treat the firms as price takers in both input and output markets, can they alter production decisions to improve their own welfare?)

2. Determine the 'value' of production by computing total revenue for each firm and for the economy as a whole.

3. Determine the cost of production for each firm and the economy as a whole by applying the opportunity cost or market price to resources used.

4. Determine economic income for each firm and the economy as a whole by subtracting production costs from revenues including resource trades at equilibrium prices.

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[3] Note that the initial distribution of resources doesn't impact efficient production in this market setting.

[4] This is a mathematical program and the opportunity costs associated with the resources are their shadow prices (or Lagrange multipliers). The program below (written in matrix notation) solves the production problem. Since this is a competitive equilibrium the firms act as price takers.

$$\begin{aligned} \max_{p,x,v} \quad & p^T x - v^T r \\ \text{s.t.} \quad & Ax \leq r \\ & A^T v \geq p \\ & Dx = p \\ & x,v,p \geq 0 \end{aligned}$$

p = vector of commodity prices, v = vector of shadow prices for resources, x = vector of quantity of commodities produced, r = vector of total resource endowments, A = matrix representation of production

You should have determined that net income for each firm and the economy as a whole are zero. This demonstrates accounting in a general equilibrium setting. Note that since the current problem has no element of time, the interest rate is taken to be zero and excess inventory carries no value. If time were important and interest rates positive, then equilibrium income would be the 'normal' return on resources (i.e., the interest rate times the initial value of resources at their equilibrium prices).

5. What happens to the equilibrium (input and output prices and quantities) if firm B's initial endowment of resource 3 is one?

Now, consider an alternative scenario. Suppose that it is extremely costly to trade resource 3; consequently, no trade of resource 3 occurs (i.e., markets are incomplete). Even though the opportunity cost associated with resource 3 was zero above (with the initial set of resource endowments) this changes the firms' optimal production. The reason is that resource 3 was traded before, but there was a surplus after trading so the opportunity cost associated with an additional unit of resource 3 (its market price) was zero. In this setting, optimal production is $x_1 = .75$ and $x_2 = 2.8125$. The market prices on resources 1 and 2, respectively, are 0 and 2.34375. The opportunity costs associated with another unit of resource 3 for firm 1 and firm 2, respectively, are 0.2227 and 0. Notice that these are not market prices because resource 3 is untradable (prohibitively costly to trade) but they are opportunity costs to the firms. We'll refer to these as the firms' 'personal' prices since they are specific to the individual firm.[5]

6. For the alternative scenario, repeat questions 1 through 5.

Once again, you should determine that the firm's net income and net income for the economy is zero. Even with market incompleteness, there exists an (economic) income benchmark for optimal performance. Notice that the accounting here is much more

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technology, and D = matrix representation of inverse demand relating production quantity to prices. Since prices are endogenously determined, the program solution is sensitive to selection of starting values.

[5] Again, this is the solution to a mathematical program (the key is that production technology is changed to restrict trading of resource 3, effectively resource 3 becomes resource 3a and 3b).

subtle and requires an in-depth understanding of the equilibrium forces in the economy (e.g., one cannot look at the value of resource 3 for one firm to determine its value for another firm). What message for accounting can we draw from this example? Accounting is important (income and valuation provide performance benchmarks at the firm level and for the economy as a whole) and challenging (market imperfections and incompleteness require that accountants develop 'personal prices' for individual firm's assets). Is it any wonder that mark-to-market accounting is selectively applied in accounting practice? Resources 1 and 2 are valued at market (in the alternative scenario). However, notice their valuations are changed by market incompleteness (actually this only applies to resource 2 in the example, but the statement applies more generally).

7. Why do you suppose that we don't observe such simple benchmarks in practice? Is it because interest rates are positive? Possibly, but earlier we outlined the solution for that case. Is it because of uncertainty? Notice uncertainty affects both income and valuation (balance sheet) measurements here. If we wish to maintain the clean surplus relation in accounting (recall that it is the foundation for our valuation solution when accounting does not follow perfect and complete markets protocol), favorable (unfavorable) chance outcomes are going to yield greater (lower) than expected income and balance sheet valuations. In turn, if these chance outcomes are expected to balance out over time, then we expect that firms experiencing favorable (unfavorable) outcomes will generate negative (positive) abnormal earnings in the future – just as we argued earlier. Recall the importance of 'personal' prices or opportunity costs here.