Ralph's GLS

This is a continuation of Ralph's optimal accruals. Again, Ralph is only interested in the estimator for m_3 but recognizes that the relation between the cash flows is delicate. That is, the DGP is

$$Y = Xm_3 + \eta$$

where $Y = [y_1\ y_2\ y_3]^T$, $X = [1\ 1\ 1]^T$, and, importantly, η is a mean zero vector with variance $V = \begin{bmatrix} 3 & 1 & 0 \\ 1 & 2 & 0 \\ 0 & 0 & 1 \end{bmatrix}$. This DGP does not fit conditions suitable for OLS, so Ralph realizes he must generalize the solution for the estimator for m_3. His strategy involves two paths: GLS and Cholesky decomposition of V.

Required:

1. Determine the GLS estimator for m_3, $(X^TV^{-1}X)^{-1}\ X^TV^{-1}y$. Compare the results with those from Ralph's optimal accruals.

2. Determine the variance of the GLS estimator for m_3, $(X^TV^{-1}X)^{-1}$. Compare the results with those from Ralph's optimal accruals.

3. Find the Cholesky decomposition of $V = GG^T$ where G is lower triangular.
Rewrite the DGP by multiplying both sides by G^{-1}, $G^{-1}Y = G^{-1}Xm_3 + G^{-1}\eta$.
Now, $Var[G^{-1}\eta] = G^{-1}V(G^T)^{-1} = G^{-1}\ GG^T\ (G^T)^{-1} = I$. Hence, OLS conditions are satisfied for the transformed (by G^{-1}) DGP. Regress (via OLS) $G^{-1}Y$ onto $G^{-1}X$ to determine the estimator and its variance, $(X^T(G^T)^{-1}\ G^{-1}X)^{-1}$. Compare the results to those above.