Ralph's fundamental accounting

Ralph believes accounting takes a long-run view (e.g., going-concern) and log returns performance measurement reinforces the notion. In the long-run an organizations growth and its accounting converge to a steady-state. This fundamental relationship can be summarized by Fellingham's fundamental theorem of accounting.

$$\ln\left(1+\frac{income}{assets}\right)\to r_f + I\left(s;z\right)$$

where the left-hand side represents accounting results and the right-hand side is expected log returns without information (the optimal long-run or Kelly strategy yields the riskless rate of return) plus $I\left(s;z\right)$, which is mutual information or the expected (long-run) gain from information. Conditions supporting this relationship are (1) long-run preferences, (2) assets span the state space or, equivalently, the existence of Arrow-Debreu assets for each state, and (3) no scalable arbitrage opportunities exist.

A healthy, going-concern reinvests in many assets each period whose returns are stochastic but by the law of large numbers the stochastic (unobservable) components almost surely averages to zero. Let the convergent cost of these periodic investments be denoted C. Let convergent periodic cash flows generated by these assets be denoted $\sum cf_j$. The accounting for assets converges as

$$B + C - amort = B$$

where B is book value of assets and *amort* is amortization or accrual expense of assets and accounting income converges to economic income.

$$\begin{aligned} income &= \sum cf_j - amort \\ &= \sum cf_j - C \\ &= \left(e^r - 1\right)\left(B + C\right) \end{aligned}$$

This implies convergent $amort = C = \sum cf_j - \left(e^r - 1\right)\left(B + C\right)$. Putting this together yields

$$\begin{aligned} B + C - \left[\sum cf_j - \left(e^r - 1\right)\left(B + C\right)\right] &= B \\ \sum cf_j - \left(e^r - 1\right)\left(B + C\right) &= C \\ \frac{\sum cf_j - e^r C}{e^r - 1} &= B \end{aligned}$$

Now, accounting rate of return converges to

$$\frac{income}{B + C} = \frac{\sum cf_j - C}{\frac{\sum cf_j - e^r C}{e^r - 1} + C} = e^r - 1$$

and

$$\ln\left(1+\frac{income}{B+C}\right)=r=E\left[r\mid z\right]$$

where $r=E\left[r\mid z\right]=r_f+I\left(s;z\right)$ is the expected long-run rate of return based on whatever information the organization utilizes z to guide its long-run (Kelly) investment strategy.

In equilibrium residual income, $income-\left(e^r-1\right)\left(B+C\right)$, equals zero and the equilibrium value of the organization is $B+C$. If $B=C$, the convergent scale of operations involves $\sum cf_j=C\left(2e^r-1\right)$.

Suppose Ralph's assets involve convergent stochastic returns characterized as follows.

	s_1	s_2
probability	0.5	0.5
$asset_1$	1	1
$asset_2$	0.5	1.5

with convergent scale of operations $C=B=e^r$. Further, Ralph can potentially acquire information, z, as follows.

	s_1	s_2	$\Pr\left(z\right)$
z_1	3/8	1/8	1/2
z_2	1/8	3/8	1/2
$\Pr\left(s\right)$	1/2	1/2	

Suggested:

1. Determine state prices, y, such that $Ay=v$ where A is the returns matrix and v is a vector ones (normalized asset prices).

2. Let $\Omega=\begin{bmatrix}\frac{1}{y_1} & \cdots & 0\\ \vdots & \ddots & \vdots\\ 0 & \cdots & \frac{1}{y_n}\end{bmatrix}$. Determine

$$E\left[r\right]=\Pr\left(s\right)^T\ln\left(\Omega\Pr\left(s\right)\right),$$

$$E\left[r\mid z\right]=\sum_i\Pr\left(z_i\right)\Pr\left(s\mid z_i\right)^T\ln\left(\Omega\Pr\left(s\mid z_i\right)\right),$$

and

$$E\left[r\mid PI\right]=\sum_i\Pr\left(q_i\right)\Pr\left(s\mid q_i\right)^T\ln\left(\Omega\iota\right)$$

where PI (or q) is perfect information and ι is a vector of ones (the vector of ones is employed in place of $\Pr\left(s\mid q_i\right)$ to finesse/avoid $\ln 0$). hint: perfect information q is depicted as

	s_1	s_2	$\Pr\left(q\right)$
q_1	1/2	0	1/2
q_2	0	1/2	1/2
$\Pr\left(s\right)$	1/2	1/2	

3. Determine mutual information given imperfect information, z, and perfect information, PI. Compare mutual information with expected gains from information $E[r \mid z] - E[r]$ and $E[r \mid PI] - E[r]$.

Mutual information is

$$I(s; z) = h(s) + h(z) - h(s, z)$$

where

$$h(\cdot) = -\sum \Pr(\cdot) \ln \Pr(\cdot)$$

4. Determine convergent *income*, *amort*, B, C, and accounting rate of return, $\ln\left(1 + \frac{income}{B+C}\right)$, for no information, imperfect information, z, and perfect information, PI.

5. Compare expected returns with (or without) information to convergent log accounting returns, $\ln\left(1 + \frac{income}{assets}\right)$.