Ralph's Excess

Ralph owns a production facility comprised of two complementary business units, each with its own manager. Managerial specialization is important to organizational success as is coordination between business units to create and take advantage of synergies. Ralph and the managers realize that production resembles a Cobb-Douglas input-output relation. That is, many inputs are *necessary* to produce any outputs (for example, production halts if only managerial input is utilized) but there is sufficient flexibility to exploit economic efficiencies as they arise (production accommodates substitution of inputs at the margin). However, production technology is quite dynamic and inherently uncertain. Any measurements (say, of organizational or individual performance evaluation) are likely to interfere with production and potentially mute synergy. Incorporation of these features into the Cobb-Douglas frame is inelegant, so Ralph considers an alternative frame. Drawing on quantum information theory, Ralph imagines a quantum production technology frame for his organization.

First, some background on quantum information theory. Then, we consider Ralph's quantum production technology. There are four quantum axioms governing the behavior of quantum probabilities (see Nielsen and Chuang [2002]). The axioms are outlined in standard quantum bit (qubit) form.

1 Quantum information axioms

1.1 The superposition axiom:

A quantum unit (qubit) is specified by a two element vector, say $\begin{bmatrix} \alpha \\ \beta \end{bmatrix}$, with $|\alpha|^2 + |\beta|^2 = 1$.

Let $|\psi\rangle \equiv \begin{bmatrix} \alpha \\ \beta \end{bmatrix} = \alpha\,|0\rangle + \beta\,|1\rangle$,[1] $\langle\psi| = \begin{bmatrix} \alpha \\ \beta \end{bmatrix}^\dagger$ where $\dagger$ is the adjoint (conjugate transpose) operation.

[1]Dirac notation is a useful descriptor, as $|0\rangle = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$ and $|1\rangle = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$.

1.2 The transformation axiom:

A transformation of a quantum unit is accomplished by unitary (length-preserving) matrix multiplication. The Pauli matrices provide a basis of unitary operators.

$$I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \quad X = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$$
$$Y = \begin{bmatrix} 0 & -i \\ i & 0 \end{bmatrix} \quad Z = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$$

where $i = \sqrt{-1}$. The operations work as follows: $I \begin{bmatrix} \alpha \\ \beta \end{bmatrix} = \begin{bmatrix} \alpha \\ \beta \end{bmatrix}$, $X \begin{bmatrix} \alpha \\ \beta \end{bmatrix} = \begin{bmatrix} \beta \\ \alpha \end{bmatrix}$, $Y \begin{bmatrix} \alpha \\ \beta \end{bmatrix} = \begin{bmatrix} -\beta i \\ \alpha i \end{bmatrix}$, and $Z \begin{bmatrix} \alpha \\ \beta \end{bmatrix} = \begin{bmatrix} \alpha \\ -\beta \end{bmatrix}$. Other useful single qubit transformations are $H = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}$ and $\Theta = \begin{bmatrix} e^{i\theta} & 0 \\ 0 & 1 \end{bmatrix}$. Examples of these transformations, in Dirac notation are

$$H\left|0\right\rangle = \frac{\left|0\right\rangle + \left|1\right\rangle}{\sqrt{2}}; \, H\left|1\right\rangle = \frac{\left|0\right\rangle - \left|1\right\rangle}{\sqrt{2}}$$

$$\Theta\left|0\right\rangle = e^{i\theta}\left|0\right\rangle; \, \Theta\left|1\right\rangle = \left|1\right\rangle$$

1.3 The measurement axiom:

Measurement of a quantum state is accomplished by a linear projection from a set of projection matrices which add to the identity matrix.[2] The probability of a particular measurement occurring is the squared absolute value of the projection. (An implication of the axiom not explicitly used here is that the post-measurement state is the projection appropriately normalized; this effectively rules out multiple measurement.)

For example, let the projection matrices be $M_0 = \left|0\right\rangle\left\langle 0\right| = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}$ and $M_1 = \left|1\right\rangle\left\langle 1\right| = \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}$. Note that M_0 projects onto the $\left|0\right\rangle$ vector and M_1 projects onto the $\left|1\right\rangle$ vector. Also note that $M_0^\dagger M_0 + M_1^\dagger M_1 = M_0 + M_1 = I$. For $\left|\psi\right\rangle =$

[2]More precisely, the projection matrices satisfy the completeness condition, $\sum_m M_m^\dagger M_m = I$, where $M_m^\dagger$ is the adjoint (conjugate transpose) of projection matrix M_m.

$\alpha|0\rangle + \beta|1\rangle$, the projection of $|\psi\rangle$ onto $|0\rangle$ is $M_0|\psi\rangle$. The probability of $|0\rangle$ being the result of the measurement is $\langle\psi| M_0 |\psi\rangle = |\alpha|^2$.

1.4 The combination axiom:

Qubits are combined by tensor multiplication. For example, two $|0\rangle$ qubits are combined as $|0\rangle \otimes |0\rangle = \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}$ denoted $|00\rangle$. It is often useful to transform one qubit in a combination and leave another unchanged; this can also be accomplished by tensor multiplication. Let H_1 denote a Hadamard transformation on the first qubit. Then applied to a two qubit system, $H_1 = H \otimes I = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ 1 & 0 & -1 & 0 \\ 0 & 1 & 0 & -1 \end{bmatrix}$ and $H_1|00\rangle = \frac{|00\rangle+|10\rangle}{\sqrt{2}}$.

Another important two qubit transformation is the controlled not operator,

$$Cnot = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

Entangled two qubit states or Bell states are defined as follows,

$$|\beta_{00}\rangle = Cnot\ H_1|00\rangle = \frac{|00\rangle + |11\rangle}{\sqrt{2}}$$

and more generally,

$$\left|\beta_{ij}\right\rangle = Cnot\ H_1|ij\rangle \text{ for } i, j = 0, 1$$

The four two qubit Bell states form an orthonormal basis.

2 Quantum production technology

Next, Ralph describes his frame for quantum production technology.

2.1 Production technology

Productivity is represented by a two qubit interferometer (transformation function) and its application to a two qubit state: the pure synergy state is represented by $|\beta_{00}\rangle$ while the no synergy state is represented by $|00\rangle$. The transformation function is $F = H_2\Theta_2 H_2 H_1 \Theta_1 H_1$. where the managers each supply input $\theta_i \in \{\theta_l, \theta_h\}$.

Given the no synergy $|00\rangle$ setting, productivity is

$$
\begin{aligned}
F\,|00\rangle &= H_2\Theta_2H_2H_1\Theta_1\frac{|00\rangle + |10\rangle}{\sqrt{2}} = H_2\Theta_2H_2H_1\frac{e^{i\theta_1}\,|00\rangle + |10\rangle}{\sqrt{2}} \\
&= H_2\Theta_2H_2\frac{e^{i\theta_1}\,|00\rangle + e^{i\theta_1}\,|10\rangle + |00\rangle - |10\rangle}{2} \\
&= H_2\Theta_2\frac{[e^{i\theta_1}+1][|00\rangle + |01\rangle] + [e^{i\theta_1}-1][|10\rangle + |11\rangle]}{2\sqrt{2}} \\
&= H_2\frac{[e^{i\theta_1}+1][e^{i\theta_2}\,|00\rangle + |01\rangle] + [e^{i\theta_1}-1][e^{i\theta_2}\,|10\rangle + |11\rangle]}{2\sqrt{2}} \\
&= \frac{[e^{i\theta_1}+1][e^{i\theta_2}(|00\rangle + |01\rangle) + (|00\rangle - |01\rangle)]}{4} \\
&+ \frac{[e^{i\theta_1}-1][e^{i\theta_2}(|10\rangle + |11\rangle) + (|10\rangle - |11\rangle)]}{4} \\
&= \frac{[e^{i\theta_1}+1][(e^{i\theta_2}+1)\,|00\rangle + (e^{i\theta_2}-1)\,|01\rangle]}{4} \\
&+ \frac{[e^{i\theta_1}-1][(e^{i\theta_2}+1)\,|10\rangle + (e^{i\theta_2}-1)\,|11\rangle]}{4}.
\end{aligned}
\tag{A1}
$$

Similarly, given the pure synergy $|\beta_{00}\rangle$ setting, productivity is

$$
\begin{aligned}
F\,|\beta_{00}\rangle &= H_2\Theta_2H_2H_1\Theta_1\frac{|00\rangle + |10\rangle + |01\rangle - |11\rangle}{2} \\
&= H_2\Theta_2H_2H_1\frac{e^{i\theta_1}\,|00\rangle + |10\rangle + e^{i\theta_1}\,|01\rangle - |11\rangle}{2} \\
&= H_2\Theta_2H_2\frac{[e^{i\theta_1}+1][|00\rangle + |11\rangle] + [e^{i\theta_1}-1][|10\rangle + |01\rangle]}{2\sqrt{2}} \\
&= H_2\Theta_2\frac{e^{i\theta_1}[|00\rangle + |10\rangle] + |01\rangle - |11\rangle}{2} \\
&= H_2\frac{e^{i\theta_1}e^{i\theta_2}[|00\rangle + |10\rangle] + |01\rangle - |11\rangle}{2} \\
&= \frac{[e^{i\theta_1}e^{i\theta_2}+1]\,|\beta_{00}\rangle + [e^{i\theta_1}e^{i\theta_2}-1]\,|\beta_{01}\rangle}{2}.
\end{aligned}
\tag{A2}
$$

2.2 Projection matrices for measurement

The projection matrices are defined as the sum of the outer product of success vectors. Individual measurement reports a "success" or "failure" signal for each manager's production. For manager one, the success vectors are $|10\rangle$ and $|11\rangle$; and the failure vectors are $|00\rangle$ and $|01\rangle$. The projection matrices for manager one are,

$$M_{S1} = |10\rangle\langle 10| + |11\rangle\langle 11| = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}, \text{ and} \tag{A3}$$

$$M_{F1} = |00\rangle\langle 00| + |01\rangle\langle 01| = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}. \tag{A4}$$

Similarly, for manager two, the success vectors are $|01\rangle$ and $|11\rangle$; and the failure vectors are $|00\rangle$ and $|10\rangle$. The projection matrices for manager two are,

$$M_{S2} = |01\rangle\langle 01| + |11\rangle\langle 11| = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}, \text{ and} \tag{A5}$$

$$M_{F2} = |00\rangle\langle 00| + |10\rangle\langle 10| = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}. \tag{A6}$$

Note, $M_{S1}^{\dagger}M_{S1} + M_{F1}^{\dagger}M_{F1} = I$, and $M_{S2}^{\dagger}M_{S2} + M_{F2}^{\dagger}M_{F2} = I$.

Group measurement reports a "success" or "failure" signal for both managers. The success vectors are $|\beta_{01}\rangle$ and $|\beta_{11}\rangle$; and the failure vectors are $|\beta_{00}\rangle$ and $|\beta_{10}\rangle$. The projection matrices are,

$$M_{S} = |\beta_{01}\rangle\langle \beta_{01}| + |\beta_{11}\rangle\langle \beta_{11}| = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}, \text{ and} \tag{A7}$$

$$M_{F} = |\beta_{00}\rangle\langle \beta_{00}| + |\beta_{10}\rangle\langle \beta_{10}| = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}. \tag{A8}$$

Note, $M_S^\dagger M_S + M_F^\dagger M_F = I$.

2.3 Measure success probabilities

Project $\mathcal{F}\,|00\rangle$ or $\mathcal{F}\,|\beta_{00}\rangle$ onto success matrices (M_{S1} or M_S).[3] The probability of success is the squared length of the projection vectors. Under individual measurement with no synergy,

$$\begin{aligned} prob(success|IN, \theta_1, \theta_2) &= \langle 00|\,\mathcal{F}^\dagger M_{S1} \mathcal{F}\,|00\rangle = \|M_{S1}\mathcal{F}\,|00\rangle\|^2 \\ &= \left\| \frac{1}{4} \begin{bmatrix} 0 \\ 0 \\ (e^{i\theta_1}-1)(e^{i\theta_2}+1) \\ (e^{i\theta_1}-1)(e^{i\theta_2}-1) \end{bmatrix} \right\|^2 \\ &= \frac{1-\cos\theta_1}{2} = \sin^2\frac{\theta_1}{2}. \end{aligned} \tag{A9}$$

Similarly, under individual measurement with synergy,

$$\begin{aligned} prob(success|IS, \theta_1, \theta_2) &= \langle \beta_{00}|\,\mathcal{F}^\dagger M_{S1} \mathcal{F}\,|\beta_{00}\rangle = \|M_{S1}\mathcal{F}\,|\beta_{00}\rangle\|^2 \\ &= \left\| \frac{1}{2\sqrt{2}} \begin{bmatrix} 0 \\ 0 \\ e^{i\theta_1}e^{i\theta_2}-1 \\ e^{i\theta_1}e^{i\theta_2}+1 \end{bmatrix} \right\|^2 = \frac{1}{2}. \end{aligned} \tag{A10}$$

Under group measurement with no synergy,

$$\begin{aligned} prob(success|GN, \theta_1, \theta_2) &= \langle 00|\,\mathcal{F}^\dagger M_S \mathcal{F}\,|00\rangle = \|M_S\mathcal{F}\,|00\rangle\|^2 \\ &= \left\| \frac{1}{4} \begin{bmatrix} 0 \\ (e^{i\theta_1}+1)(e^{i\theta_2}-1) \\ (e^{i\theta_1}-1)(e^{i\theta_2}+1) \\ 0 \end{bmatrix} \right\|^2 \\ &= \sin^2\frac{\theta_1}{2}\cos^2\frac{\theta_2}{2} + \cos^2\frac{\theta_1}{2}\sin^2\frac{\theta_2}{2}. \end{aligned} \tag{A11}$$

[3] Manager 1's success probability is described here. Parallel results apply to manager 2 if M_{S1} is replaced by M_{S2}.

Under group measurement with synergy,

$$\begin{aligned}prob(success|GS,\theta_1,\theta_2) &= \langle\beta_{00}| F^{\dagger}M_S F |\beta_{00}\rangle = \|M_S F |\beta_{00}\rangle\|^2 \\ &= \left\| \frac{1}{2\sqrt{2}} \begin{bmatrix} 0 \\ e^{i\theta_1}e^{i\theta_2}-1 \\ e^{i\theta_1}e^{i\theta_2}-1 \\ 0 \end{bmatrix} \right\|^2 \\ &= \sin^2\frac{\theta_1+\theta_2}{2}. \end{aligned} \tag{A12}$$

where $\theta_i \in \{\theta_l = 0, \theta_h = \frac{\pi}{3}\}$.

2.4 Observables and expected payoffs

Measurements involve real values drawn from observables — in this production setting these values correspond to the payoffs. The individual measure observable for manager or process one is

$$\begin{aligned} P_{I_1} &= \begin{bmatrix} -10 & 0 & 0 & 0 \\ 0 & -10 & 0 & 0 \\ 0 & 0 & 35 & 0 \\ 0 & 0 & 0 & 35 \end{bmatrix} \\ &= -10\,|00\rangle\langle 00| - 10\,|01\rangle\langle 01| + 35\,|10\rangle\langle 10| + 35\,|11\rangle\langle 11| \end{aligned}$$

The expected payoff is

$$\begin{aligned} \langle P_{I_1}\rangle &= \langle 00| F^{\dagger}P_{I_1}F |00\rangle \\ &= -10\,\langle 00| F^{\dagger} |00\rangle\langle 00| F |00\rangle - 10\,\langle 00| F^{\dagger} |01\rangle\langle 01| F |00\rangle \\ &\quad +35\,\langle 00| F^{\dagger} |10\rangle\langle 10| F |00\rangle + 35\,\langle 00| F^{\dagger} |11\rangle\langle 11| F |00\rangle \\ &= -10\,\langle 00| F^{\dagger}M_{F_1}F |00\rangle + 35\,\langle 00| F M_{S_1} F |00\rangle \end{aligned}$$

In other words, -10 times the probability the first measure is a failure plus 35 times the probability measure one is a success.

The individual measure observable for manager or process two is

$$\begin{aligned} P_{I_2} &= \begin{bmatrix} -10 & 0 & 0 & 0 \\ 0 & 35 & 0 & 0 \\ 0 & 0 & -10 & 0 \\ 0 & 0 & 0 & 35 \end{bmatrix} \\ &= -10\,|00\rangle\langle 00| + 35\,|01\rangle\langle 01| - 10\,|10\rangle\langle 10| + 35\,|11\rangle\langle 11| \end{aligned}$$

The expected payoff is

$$
\begin{aligned}
\langle P_{I_2}\rangle &= \langle 00|\, F^{\dagger} P_{I_2} F\, |00\rangle \\
&= -10\,\langle 00|\, F^{\dagger}\, |00\rangle\, \langle 00|\, F\, |00\rangle + 35\,\langle 00|\, F^{\dagger}\, |01\rangle\, \langle 01|\, F\, |00\rangle \\
&\quad -10\,\langle 00|\, F^{\dagger}\, |10\rangle\, \langle 10|\, F\, |00\rangle + 35\,\langle 00|\, F^{\dagger}\, |11\rangle\, \langle 11|\, F\, |00\rangle \\
&= -10\,\langle 00|\, F^{\dagger} M_{F_2} F\, |00\rangle + 35\,\langle 00|\, F M_{S_2} F\, |00\rangle
\end{aligned}
$$

In other words, -10 times the probability the second measure is a failure plus 35 times the probability measure two is a success. The expected payoff for individual measures is $\langle P_{I_1}\rangle + \langle P_{I_2}\rangle$.

On the other hand, the group measure observable is

$$
\begin{aligned}
P_G &= \begin{bmatrix} -30 & 0 & 0 & 10 \\ 0 & 55 & 15 & 0 \\ 0 & 15 & 55 & 0 \\ 10 & 0 & 0 & -30 \end{bmatrix} \\
&= -20\,|\beta_{00}\rangle\, \langle \beta_{00}| + 70\,|\beta_{01}\rangle\, \langle \beta_{01}| - 40\,|\beta_{10}\rangle\, \langle \beta_{10}| + 40\,|\beta_{11}\rangle\, \langle \beta_{11}|
\end{aligned}
$$

The expected payoff is

$$
\begin{aligned}
\langle P_G\rangle &= \langle \beta_{00}|\, F^{\dagger} P_G F\, |\beta_{00}\rangle \\
&= -20\,\langle \beta_{00}|\, F^{\dagger}\, |\beta_{00}\rangle\, \langle \beta_{00}|\, F\, |\beta_{00}\rangle + 70\,\langle \beta_{00}|\, F^{\dagger}\, |\beta_{01}\rangle\, \langle \beta_{01}|\, F\, |\beta_{00}\rangle \\
&\quad -40\,\langle \beta_{00}|\, F^{\dagger}\, |\beta_{10}\rangle\, \langle \beta_{10}|\, F\, |\beta_{00}\rangle + 40\,\langle \beta_{00}|\, F^{\dagger}\, |\beta_{11}\rangle\, \langle \beta_{11}|\, F\, |\beta_{00}\rangle \\
&= -20\,\langle \beta_{00}|\, F^{\dagger} M_F F\, |\beta_{00}\rangle + 70\,\langle \beta_{00}|\, F^{\dagger} M_S F\, |\beta_{00}\rangle
\end{aligned}
$$

In other words, -20 times the probability the group measure is failure eigenstate $|\beta_{00}\rangle$ plus 70 times the probability the group measure is success eigenstate $|\beta_{01}\rangle$ minus 40 times the probability the group measure is failure eigenstate $|\beta_{10}\rangle$ plus 40 times the probability the group measure is success eigenstate $|\beta_{11}\rangle$. Since the latter two probabilities are equal to zero only values -20 and 70 are observed.

Required: Throughout the analysis below hold manager two's input at $\theta_h = \frac{\pi}{3}$.

Part A: Synergy

1. To gain some "comfort" with quantum information, verify expressions A9 through A12.

2. With a no synergy initial state $|00\rangle$ and individual measurement of each manager's productivity, what is the probability of productive success if manager one supplies input $\theta_l = 0$? $\theta_h = \frac{\pi}{3}$? Determine the expected payoff if the managers each supply $\theta_h = \frac{\pi}{3}$.

3. With a pure synergy initial state $|\beta_{00}\rangle$ and individual measurement of each manager's productivity, what is the probability of productive success if manager one supplies input $\theta_l = 0$? $\theta_h = \frac{\pi}{3}$? Determine the expected payoff if the managers each supply $\theta_h = \frac{\pi}{3}$.

4. With a no synergy initial state $|00\rangle$ and group measurement of productivity, what is the probability of productive success if manager one supplies input $\theta_l = 0$? $\theta_h = \frac{\pi}{3}$?

5. With a pure synergy initial state $|\beta_{00}\rangle$ and group measurement of productivity, what is the probability of productive success if manager one supplies input $\theta_l = 0$? $\theta_h = \frac{\pi}{3}$?

6. If Ralph and the managers can choose or design the initial state, synergy or no synergy, in combination with the production transformation function, F, which will they select?

7. In what sense does individual measurement destroy productive synergy? Can we have too many or excessive measures? What are the implications of influence activities which culminate in demand for more measures?

8. Productivity measurement is often treated as if it is benign. Comment.

Required: Throughout the analysis below hold manager two's input at $\theta_h = \frac{\pi}{3}$.

Part B: Performance evaluation

Suppose each of the managers has CARA utility,

$$U(S,a) = -\exp\left[-0.01\left(S - c(a)\right)\right]$$

reservation wage, $RW = 0$, and personal cost for input θ_l, $c(\theta_l = 0) = 0$, and for input θ_h, $c\left(\theta_h = \frac{\pi}{3}\right) = 10$. Both managers and Ralph recognize the information structure identified in A9 through A12. From the above it's clear Ralph desires both managers supply $\theta_h = \frac{\pi}{3}$ and Ralph's expected cost of supplying incentives to the managers involves a standard performance-based compensation contract for each manager i.

$$\min_{S_f, S_s} E\left[S_i \mid \theta_i = \theta_h\right]$$

$$\begin{array}{ll} s.t & CE(h) \geq RW = 0 \quad (IR) \\ & CE(h) \geq CE(l) \quad\;\; (IC) \end{array}$$

where S_f is the manager's compensation if measurement indicates a production failure, S_s is the manager's compensation if measurement indicates a production success, and $CE(\cdot)$ is the manager's certain equivalent. Incentives are supplied to each manager. That is, Ralph assumes when supplying incentives to one manager the other manager is motivated to supply the desired input, θ_h. This serves to stack the deck in favor of individual measures as group measures consider the input of both managers simultaneously (this treatment reduces the spread in likelihoods between θ_l and θ_h for group measures) while individual measures effectively ignore the other manager's input.

1. If it's possible for Ralph to supply incentives, find the optimal performance-based payments, S_f and S_s, and Ralph's expected compensation cost for the case of no synergy (initial state $|00\rangle$) and individual measurement of each manager's productivity. If not, indicate why it's not possible to supply incentives.

2. If it's possible for Ralph to supply incentives, find the optimal performance-based payments, S_f and S_s, and Ralph's expected compensation cost for the case of pure synergy (initial state $|\beta_{00}\rangle$) and individual measurement of each manager's productivity. If not, indicate why it's not possible to supply incentives.

3. If it's possible for Ralph to supply incentives, find the optimal performance-based payments, S_f and S_s, and Ralph's expected compensation cost for the case of no synergy (initial state $|00\rangle$) and group measurement of productivity. If not, indicate why it's not possible to supply incentives.

4. If it's possible for Ralph to supply incentives, find the optimal performance-based payments, S_f and S_s, and Ralph's expected compensation cost for the case of pure synergy (initial state $|\beta_{00}\rangle$) and group measurement of productivity. If not, indicate why it's not possible to supply incentives.

5. If Ralph and the managers can "design" the productivity setting, which would they select?

6. Often group measures and relative performance evaluation measures are plagued by the potential of manager collusion. Does this pose a concern for the case of pure synergy (initial state $|\beta_{00}\rangle$) and group measurement of productivity? How do the managers' certainty equivalents compare to their reservation wage if they conspire to both supply θ_l?

7. In a performance evaluation setting we don't think of measurement as benign. In what sense is it harmful to employ too many or excessive performance measures?