Ralph's Equilibrium

A. For a dynamic and uncertain setting the Markov transition probabilities are as follows.

$$F = \begin{bmatrix} 0.9 & 0.1 \\ 0.4 & 0.6 \end{bmatrix}$$

where 0.9 in the first row is the transition probability from state 1 to state 1 and the transition probability from state 1 to state 2 is 0.1. Likewise for row 2, 0.4 is the transition probability from state 2 to state 1 and 0.6 is the transition probability of remaining in state two. Ralph is attempting to learn if steady-state equilibrium state probabilities exist and, if so, what are the probabilities.

Required:

1. Solve $\begin{bmatrix} p & 1-p \end{bmatrix} F = \begin{bmatrix} p & 1-p \end{bmatrix}$ for p.

2. Find the eigenvector associated with the largest eigenvalue for F^T (the transpose of F). Rescale so that the eigenvector sums to one (hint: divide each element by the sum of the elements). Compare this vector to the solution in 1.

3. Suppose the system begins in state 2 (with probability 1; $\begin{bmatrix} 0 & 1 \end{bmatrix}$). What are the state probabilities after one transition? What are the state probabilities after ten transitions? Do the state probabilities appear to be converging? If so, toward what values are they converging?

B. A dynamic cash flow process evolves as follows.

$$\begin{bmatrix} y_t \\ y_{t-1} \end{bmatrix} = F \begin{bmatrix} y_{t-1} \\ y_{t-2} \end{bmatrix}$$

where

$$F = \begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix}$$

Suppose $y_0 = 0$ and $y_1 = 1$.

Required:

1. Determine $y_2, \ldots, y_{10}$. Are cash flows converging?

2. Determine the rate of growth in cash flow, $\frac{y_t}{y_{t-1}}$ for time 2 through 10. Does the growth rate seem to be converging?

3. Solve for the maximum eigenvalue for F. How does this compare with growth in cash flow?

C. Another dynamic cash flow process, Y_t, involving two components y_{1t} and y_{2t} evolves as follows.

$$Y_t = H\,Y_{t-1}$$
$$\begin{bmatrix} y_{1t} \\ y_{2t} \end{bmatrix} = H \begin{bmatrix} y_{1t-1} \\ y_{2t-1} \end{bmatrix}$$

where

$$H = \begin{bmatrix} 0.7 & 0.3 \\ 0.2 & 0.8 \end{bmatrix}$$

Required:

1. Suppose $y_{1,0} = 1$ and $y_{2,0} = 2$. Determine $Y_1, \ldots, Y_{20}$. Does the series converge?

2. Determine the maximum eigenvalue of H. How does this eigenvalue relate to convergence of the series? (Hint: write $H = ZMZ^{-1}$ where Z represents a matrix of eigenvectors of H and M is a diagonal matrix with the corresponding eigenvalues along the diagonal. Now, $Y_n = ZM^n Z^{-1} Y_0$.)