Ralph's Dual Accruals B

This is a continuation of Ralph's Dual Accruals A. Ralph's actions are unobservable, production is subject to moral hazard and a delicate coordination exercise arises (this was suppressed in part A). In other words, how can Alice supply motivate Ralph to perform as she desires? Alice offers Ralph a performance-based contract utilizing available contractible information (namely the cash flow history and possibly accruals). In addition to the cash flows, Alice and Ralph mutually observe other information y_t that is informative of Ralph's action (plus noise μ_t) and contractible.

$$y_t = a_t + \mu_t$$

The error terms (e_t, ε_t, and μ_t) are jointly normally distributed with mean zero, common variance $\sigma^2 = 1$ and are mutually stochastically independent.

Ralph has outside employment opportunities that pay reservation wage $RW = 0$. Ralph is risk averse with utility for payments s and actions a, $U(s,a) = -\exp(-r[s-c(a)])$ where $c(a)$ is Ralph's personal cost of action a and $r = 1$ is a measure of the degree of Ralph's risk aversion. Ralph supplies either $a_H = 1$ or $a_L = 0$ and Ralph's personal cost is $c(a) = 0.5\ a$. Ralph is compensated based on a linear function of the contractible variables, cf_t, y_t, and past cash flows with Ralph's contribution a_{t-k}^* removed $\{cf_1 – a_1^*, \ldots, cf_{t-1} – a_{t-1}^*\}$. As accruals (income excluding Ralph's compensation s and Ralph's equilibrium contribution to production a^*) can summarize past cash flows, the linear payment can be expressed as

$$s_t = \delta + \theta_1 y_t + \theta_2(cf_t - accrual_{t-1})$$

where δ is a fixed wage, the remainder represents incentive payments, and (from part A) $accruals_t = F_{2t}/F_{2t+1}\ (cf_t – a_t^*) + F_{2t-1}/F_{2t+1}\ accruals_{t-1}$ with $accruals_0 = m_0$.

Details can be reduced via two simplifications. First, Ralph's expected utility can be rewritten in certainty equivalent form as $E[s_t] – ½\ r\ Var[s_t] – c(a_t)$ where E[] is the expectation operator and Var[] is the variance operator. The fixed portion of wages is $\delta = RW – \{E[\theta_1 y_t + \theta_2(cf_t - accrual_{t-1}) \mid a_H] – r/2\ Var[s] – c(a_H)\}$. Second, the incentive payments (payments excluding the fixed wage δ) for period t are

$$s_t - \delta_t = 0.5\{F_{2t+1}/L_{2t}\ y_t + F_{2t-1}/L_{2t}\ (cf_t - accruals_{t-1})\},$$

where $L_n = L_{n-2} + L_{n-1}$, $L_0 = 2$, and $L_1 = 1$ (the Lucas series).[2]

Required:

1. Find δ and the θ's for Alice's first period employment contract with Ralph.

(Hint: $\delta = RW - \{E[\theta_1 y_t + \theta_2 (cf_t - accrual_{t-1}) \mid a_H] - ½\, r\, Var[s_t] - c(a_H)\}$. $Var[s_1] = 0.25\, Var[\hat{a}_1]$; $Var[\hat{a}_1]$ is to be identified in question 3 below.)

Verify that incentive compatibility is satisfied.

(Hint: $E[s_1 \mid a_H] - ½\, r\, Var[s_1] - c(a_H) \geq E[s_1 \mid a_L] - ½\, r\, Var[s_1] - c(a_L)$.)

2. Repeat question 1 for Alice's second period employment contract with Ralph.

3. Derive first period incentive payments via least squares and identify the variance of payments.

Are the payments equal to $s_t - \delta_t = 0.5\{F_{2t+1}/L_{2t}\, y_t + F_{2t-1}/L_{2t}\, (cf_t - accruals_{t-1})\}$ as claimed above (for t = 1)?

(Hint: Employ the least squares estimator for period one's act, $\hat{a}_1$ = the second element of $[(H_1^a)^T H_1^a]^{-1} (H_1^a)^T w_1$ where $H_1^a = \begin{bmatrix} -1 & 0 \\ 1 & 1 \\ 0 & 1 \end{bmatrix}$ and $w_1 = \begin{bmatrix} -m_0 \\ cf_1 \\ y_1 \end{bmatrix}$ (ignore the first element of the resultant vector). Scale $\hat{a}_1$ by 0.5. Also, $Var[\hat{a}_1]$ = the second row and column element of $\sigma^2[(H_1^a)^T H_1^a]^{-1}$ and $Var[s_1] = 0.25 Var[\hat{a}_1] = 0.25\, F_{2t+1}/L_{2t}\, \sigma^2$.)

4. Repeat question 3 for second period incentive payments.

Are the payments equal to $s_t - \delta_t = 0.5\{F_{2t+1}/L_{2t}\, y_t + F_{2t-1}/L_{2t}\, (cf_t - accruals_{t-1})\}$ as claimed above (for t = 2)?

[2] The Lucas series is very similar to the Fibonacci series but with a different starting point. Interestingly, their relation is summarized via $L_n = F_{n-1} + F_{n+1}$.

(Hint: Employ the least squares estimator for period two's act, $\hat{a}_2$ = the third element of $[(H_2^{\text{a}})^{\text{T}} H_2^{\text{a}}]^{-1} (H_2^{\text{a}})^{\text{T}} w_2$ where $H_2^{\text{a}} = \begin{bmatrix} -1 & 0 & 0 \\ 1 & 0 & 0 \\ 1 & -1 & 0 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix}$ and $w_2 = \begin{bmatrix} -m_0 \\ cf_1 - a_1^* \\ 0 \\ cf_2 \\ y_2 \end{bmatrix}$ (ignore the first two elements). Scale $\hat{a}_2$ by 0.5. Also, Var[$\hat{a}_2$] = the third row and column element of $\sigma^2[(H_2^{\text{a}})^{\text{T}} H_2^{\text{a}}]^{-1}$ and Var[s_2] = 0.25Var[$\hat{a}_2$] = 0.25 F_{2t+1}/L_{2t} σ^2.)

5. This example explores the statistical role of accruals for both valuation and evaluation. Accrual accounting competes with other information providers for resources. Briefly explain why/how accruals may help an organization manage uncertainty and private information.