Ralph's Dual Accruals

Production in Alice's firm combines physical assets (capital) with the manager's (Ralph) talent and effort. Since Ralph's actions are unobservable, production is subject to moral hazard and a delicate coordination exercise arises. Net operating cash flows (excluding Ralph's compensation) each period cf_t are comprised of a permanent component m_t from the physical assets, the manager's contribution a_t, and a random error e_t.

$$cf_t = m_t + a_t + e_t$$

The permanent component follows a Markov process subject to stochastic shocks ε_t.

$$m_t = m_{t-1} + \varepsilon_t$$

In addition to the cash flows Alice and Ralph mutually observe other information y_t that is informative of Ralph's action (plus noise μ_t) and contractible.

$$y_t = a_t + \mu_t$$

Alice and Ralph agree on the above stochastic relations. Other common knowledge conditions include the error terms (e_t, ε_t, and μ_t) are jointly normally distributed with mean zero, common variance $\sigma^2 = 1$ and are mutually stochastically independent. Prior beliefs are that the physical assets without the manager yield expected net cash flows of $m_0 = 1$. Ralph has outside employment opportunities that pay reservation wage $RW = 0$. Ralph is risk averse with utility for payments s and actions a, $U(s,a) = -\exp(-r[s-c(a)])$ where $c(a)$ is Ralph's personal cost of action a and $r = 1$ is a measure of the degree of Ralph's risk aversion. Ralph supplies either $a_H = 1$ or $a_L = 0$ and Ralph's personal cost is $c(a) = 0.5\ a$. Ralph is compensated based on a linear function of the contractible variables, cf_t, y_t, and past cash flows with Ralph's contribution a_{t-k}^* removed $\{cf_1 - a_1^*, \ldots, cf_{t-1} - a_{t-1}^*\}$. As accruals (income excluding Ralph's compensation s and Ralph's equilibrium contribution to production a^*) can summarize past cash flows, the linear payment can be expressed as

$$s_t = \delta + \theta_1 y_t + \theta_2(cf_t - accrual_{t-1})$$

where δ is a fixed wage and the remainder represents incentive payments.

Details can be reduced via three simplifications. First, Ralph's expected utility can be rewritten in certainty equivalent form as $E[s_t] - ½\ r\ Var[s_t] - c(a_t)$ where E[] is the expectation operator and Var[] is the variance operator. The fixed portion of wages is $\delta = RW - \{E[\theta_1 y_t + \theta_2(cf_t - accrual_{t-1}) \mid a_H] - r/2 \, \mathrm{Var}[s] - c(a_H)\}$. Second, as indicated above, accruals can be written as a linear combination of past cash flows (after removing the manager's equilibrium act a_{t-k}^{*}). That is, $y_{t-k} - a_{t-k}^{*} = m_{t-k} + e_{t-k}$ for k = {1, …, t-1}. Efficient usage of the cash flow history yields

$$\hat{m}_t = accruals_t = F_{2t}/F_{2t+1}\,(cf_t - a_t^{*}) + F_{2t-1}/F_{2t+1}\,accruals_{t-1},$$

where $F_n = F_{n-2} + F_{n-1}$, $F_0 = 0$, $F_1 = 1$ (the Fibonacci series), and the sequence is initialized by reference to common knowledge prior beliefs such that $accruals_0 = m_0$. Third, the incentive payments (payments excluding the fixed wage δ) for period t are

$$s_t - \delta_t = 0.5\{F_{2t+1}/L_{2t}\,y_t + F_{2t-1}/L_{2t}\,(cf_t - accruals_{t-1})\},$$

where $L_n = L_{n-2} + L_{n-1}$, $L_0 = 2$, and $L_1 = 1$ (the Lucas series).[1]

Suppose the interest rate is 10% per period and operating assets are acquired at the present value of their future (indefinite lived) expected cash flows. In addition, the firm's dividend policy involves paying out excess cash flows each period $cf_t - s_t$. Present value accounting is applied to the operating assets such that its value equals the present value of expected future cash flows at every report date and depreciation (appreciation) is the change in present value between two successive report dates ($dep_t = PV_{t-1} - PV_t$). Accounting for unearned income is determined by the recognition rules specified above for the estimator $\hat{m}_t$. Accounting for the first two periods is summarized via T accounts below (beginning with time 0 balances and reported balances in boxes).[2]

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[1] The Lucas series is very similar to the Fibonacci series but with a different starting point. Interestingly, their relation is summarized via $L_n = F_{n-1} + F_{n+1}$.

[2] Since $E[m_t] = E[cf_t - a^{*}] = accruals_{t-1}$, and $PV_t = \sum_{k=1}^{\infty} \frac{E[cf_{t+k} - a^{*}]}{(1+i)^k}$, $dep_t = PV_{t-1} - PV_t = \frac{accruals_{t-1} - accruals_t}{i}$. The key is that only expected cash flows (excluding compensation) are updated based on the cash flow realization.

Cash

t=0 0	
cf_1	s_1 cf_1 - s_1
t=1 0	
cf_2	s_2 $cf_2 - s_2$
t=2 0	
⋮	

Operating assets

t=0 PV_0	
	dep_1
t=1 $PV_1 = PV_0 - dep_1$	
	dep_2
t=2 $PV_2 = PV_1 - dep_2$	
⋮	

Liabilities (unearned income)

	t=0 0
$F_2/F_3(cf_1 - a_1^*) + F_1/F_3 m_0$	$cf_1 - a_1^*$
	t=1 $(1 - F_2/F_3)(cf_1 - a_1^*) -$ $F_1/F_3 m_0$
$F_2/F_5(cf_1 - a_1^* + m_0) +$ $F_4/F_5(cf_2 - a_2^*)$	$cf_2 - a_2^*$
	t=2 $(1 - F_2/F_3 - F_2/F_5)(cf_1 - a_1^*)$ $- (F_1/F_3 + F_2/F_5)m_0 +$ $(1 - F_4/F_5)(cf_2 - a_2^*)$
	⋮

Owners' equity

	t=0 PV_0
dep_1 s_1 $div_1 = cf_1 - s_1$	a_1^* $F_2/F_3(cf_1 - a_1^*) + F_1/F_3 m_0$
	t=1 $PV_0 - dep_1 -$ $(1 - F_2/F_3)(cf_1 - a_1^*) +$ $F_1/F_3 m_0$
dep_2 s_2 $div_2 = cf_2 - s_2$	a_2^* $F_2/F_5(cf_1 - a_1^* + m_0) +$ $F_4/F_5(cf_2 - a_2^*)$
	t=2 $PV_0 - (dep_1 + dep_2) -$ $(1 - F_2/F_3 - F_2/F_5)(cf_1 - a_1^*)$ $+ (F_1/F_3 + F_2/F_5)m_0 -$ $(1 - F_4/F_5)(cf_2 - a_2^*)$
	⋮

Note: income for period 1 is $F_2/F_3 cf_1 + (1 - F_2/F_3)a_1^* + F_1/F_3 m_0 - s_1 - dep_1$ and income for period 2 is $F_4/F_5 cf_2 + (1 - F_4/F_5)a_2^* + F_2/F_5(cf_1 - a_1^* + m_0) - s_2 - dep_2$.

Required:

Part A – valuation

1. Calculate economic value *for the operating assets* at time 0, time 1, and time 2 and economic income *from operating assets* for periods 1 and 2 based on time 0 expectations.

2. Complete the T accounts for the first two periods based on time 0 expectations and accrual accounting as described in the problem. Assume $s_1 = 0.58$ and $s_2 = 0.59$. Are accruals equal to expected economic income from operating assets for each period?

3. Derive first period accruals via least squares.

(Hint: Stack the equations that capture the available history and recognize that for purposes of inferring expected cash flows the manager's equilibrium act a^* is known. Now employ ordinary least squares to estimate the mean of cash flows $\hat{m}_1 = (H_1^T H_1)^{-1} H_1^T z_1 = accruals_1$ where $H_1 = \begin{bmatrix} -1 \\ 1 \end{bmatrix}$ and $z_1 = \begin{bmatrix} -m_0 \\ cf_1 - a_1^* \end{bmatrix}$. Verify that the weights on the variables are consistent with the weights reported in the problem.)

How does $\text{Var}[acc_1] = F_{2t}/F_{2t+1}\, \sigma^2$ compare with $\text{Var}[cf_1] = \sigma^2$?

4. Repeat question 1 for second period accruals.

(Hint: Employ the least squares estimator for period two

$$\hat{m}_2 = (H_2^T H_2)^{-1} H_2^T z_2.$$

The stacked equations now involve estimation of both m_1 (updated with second period cash flows) and m_2 (the estimator of prime interest), where $H_2 = \begin{bmatrix} -1 & 0 \\ 1 & 0 \\ 1 & -1 \\ 0 & 1 \end{bmatrix}$

and $z_2 = \begin{bmatrix} -m_0 \\ cf_1 - a_1^* \\ 0 \\ cf_2 - a_2^* \end{bmatrix}$. Verify that the weights on the variables are consistent with the weights reported in the problem.)

How does $\text{Var}[acc_2] = F_{2t}/F_{2t+1}\, \sigma^2$ compare with $\text{Var}[cf_2] = \sigma^2$?

5. Suppose $cf_1 = 0$. Determine economic value at time 1 and time 2 and economic income for period 1 and period 2 for operating assets.
Determine the T accounts at time 1 and time 2 using $(accruals_1 \mid cf_1 = 0)$ and $E[accruals_2 \mid cf_1 = 0]$. Assume $s_1 = 0.25$, $s_2 = 0.59$, and second period cash flows equal expected second period cash flows conditional on first period result.
Suppose Alice can costlessly dispose of the physical assets and dismiss Ralph. If $cf_1 = 0$, will Alice continue to produce?

6. Ignoring scale differences, compare the *information content* of $\{V_{t-1}, cf_t\}$ with $\{accrual_{t-1}, cf_t\}$ for m_t based on the history of cash flows. What advantages, if any, do the accruals described in the problem offer over fair value (economic value and economic income) accounting? Could accrual accounting as described be applied even though conditions favorable to "fair value" accounting do not exist?

<u>Part B – performance evaluation</u>

1. Find δ and the θ's for Alice's first period employment contract with Ralph.

(Hint: $\delta = RW - \{E[\theta_1 y_t + \theta_2 (cf_t - accrual_{t-1}) \mid a_H] - \frac{1}{2} r \text{Var}[s_t] - c(a_H)\}$. $\text{Var}[s_1] = 0.25 \text{Var}[\hat{a}_1]$; $\text{Var}[\hat{a}_1]$ is to be identified in question 3 below.)

Verify that incentive compatibility is satisfied.

(Hint: $E[s_1 \mid a_H] - \frac{1}{2} r \text{Var}[s_1] - c(a_H) \geq E[s_1 \mid a_L] - \frac{1}{2} r \text{Var}[s_1] - c(a_L)$.)

2. Repeat question 1 for Alice's second period employment contract with Ralph.

3. Derive first period incentive payments via least squares and identify the variance of payments.

Are the payments equal to $s_t - \delta_t = 0.5\{F_{2t+1}/L_{2t}\, y_t + F_{2t-1}/L_{2t}\,(cf_t - accruals_{t-1})\}$ as claimed above (for t = 1)?

(Hint: Employ the least squares estimator for period one's act, $\hat{a}_1$ = the second element of $[(H_1^a)^T H_1^a]^{-1} (H_1^a)^T w_1$ where $H_1^a = \begin{bmatrix} -1 & 0 \\ 1 & 1 \\ 0 & 1 \end{bmatrix}$ and $w_1 = \begin{bmatrix} -m_0 \\ cf_1 \\ y_1 \end{bmatrix}$ (ignore the first element of the resultant vector). Scale $\hat{a}_1$ by 0.5. Also, Var[$\hat{a}_1$] = the second row and column element of $\sigma^2[(H_1^a)^T H_1^a]^{-1}$ and Var[s_1] = 0.25Var[$\hat{a}_1$] = 0.25 $F_{2t+1}/L_{2t}\,\sigma^2$.)

4. Repeat question 3 for second period incentive payments.

Are the payments equal to $s_t - \delta_t = 0.5\{F_{2t+1}/L_{2t}\, y_t + F_{2t-1}/L_{2t}\,(cf_t - accruals_{t-1})\}$ as claimed above (for t = 2)?

(Hint: Employ the least squares estimator for period two's act, $\hat{a}_2$ = the third element of $[(H_2^a)^T H_2^a]^{-1} (H_2^a)^T w_2$ where $H_2^a = \begin{bmatrix} -1 & 0 & 0 \\ 1 & 0 & 0 \\ 1 & -1 & 0 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix}$ and $w_2 = \begin{bmatrix} -m_0 \\ cf_1 - a_1^* \\ 0 \\ cf_2 \\ y_2 \end{bmatrix}$ (ignore the first two elements). Scale $\hat{a}_2$ by 0.5. Also, Var[$\hat{a}_2$] = the third row and column element of $\sigma^2[(H_2^a)^T H_2^a]^{-1}$ and Var[s_2] = 0.25Var[$\hat{a}_2$] = 0.25 F_{2t+1}/L_{2t} σ^2.)

5. This example explores the statistical role of accruals for both valuation and evaluation. Accrual accounting competes with other information providers for resources. Briefly explain why/how accruals may help an organization manage uncertainty and private information.