Ralph's discrete choice

Ralph observes a binary choice, $D$, and wishes to assess the choice probability conditional on background conditions described by $X$. It is clear to Ralph the unconditional $Pr(D = 1) = 0.5$ as he believes his sample is representative of the unknown population. Ralph believes the conditional probability follows a logistic distribution such that $Pr(D_j = 1 \mid X) = 1/(1 + exp[-X_j^T g])$ where $X$ is a matrix with two columns (one and $x$) and $g$ is a two element vector of parameters ($g_0$ and $g_1$). Ralph's sample from the data generating process (*DGP*) is tabulated below.

| *D* | one | *x* |
|---|---|---|
| 1 | 1 | 2 |
| 1 | 1 | 2 |
| 1 | 1 | 0 |
| 0 | 1 | -1 |
| 0 | 1 | 0 |
| 0 | 1 | 1 |

After some consideration, Ralph determines the conditional probability assessment is a maximum likelihood problem. The log-likelihood function is

$$\text{log-}L = \sum_{j=1}^{n} D_j * log\{1/(1+exp[-X_j^T g])\} + (1 - D_j) * log\{1/(1+exp[X_j^T g])\}$$

Required:

1. Find $g = [g_0 \;\; g_1]^T$ that maximizes the log-likelihood or minimizes the negative log-likelihood.

2. Find $Pr(D_j = 1 \mid X_j)$ for j = 1, …, 6. (Hint: repeated values of $X_j$ have the same conditional probabilities.)