Ralph's derivatives

Ralph trades derivatives in a two-state economy with a riskless asset and a risky asset whose state-contingent payoffs and current prices are as follows.

asset	state 1	state 2	current price (p)
riskless	1	1	0.9
risky	9	12	10
derivative	0	1	v

Ralph recognizes that arbitrage opportunities are eliminated in well functioning markets and this setting admits no arbitrage equilibrium state pricing which can be utilized to value the derivative. That is, in equilibrium the state prices, $y \geq 0$, are the solution to

$$Ay = p$$

where A is a matrix of state-contingent payoffs on assets with observable equilibrium prices (state prices are also called Arrow-Debreu prices as they are the prices for portfolios of assets that pay 1 in a single state and zero otherwise).

Required:

1. Solve for $y = [y_1\ y_2]^T$ where $y \geq 0$.

2. Find the equilibrium price for the derivative, v, based on the solution in 1.

3. Suppose the state prices, y, involved some negative element (for example, suppose the risky asset payoff in state 2 is 11), why would this violate the no arbitrage condition? (Hint: the theorem of the separating hyperplane indicates either there exists $y \geq 0$ such that $Ay = p$ or there exists a λ such that $A^T\lambda \geq 0$ and $\lambda^T p < 0$.)
Can you find a λ such that the state-contingent portfolio payoff, $A^T\lambda$, is nonnegative but the investment cost for the portfolio, $\lambda^T p$, is negative?