

## Ralph's Debt Securitization<sup>1</sup>

Ralph is contemplating derivatives. In particular, creating derivatives (frequently referred to as credit default swaps – CDSs) that effectively insure tranche default on securitized debt (collateralized debt obligations – CDOs). Private debt is securitized when lenders bundle and sell private debt agreements. Tranches are created when these bundled instruments are split up into default portfolios. For example, suppose we bundle three loans (for simplicity) and sell one tranche, call it tranche A, such that the tranche owner suffers no default loss unless all three loans default, another tranche, tranche B, for which the tranche owner suffers default loss only if at least two loans default, and a third tranche, tranche C, for which the tranche owner suffers default loss if any loans default. Absent private information concerns, securitization produces more efficient risk sharing amongst investors. If the probability of default is independent for the loans, then tranche A would command a higher price than tranche B and tranche B a higher price than tranche C. For simplicity, each security (bundled loan) is normalized to \$1 and default results in loss of \$1 (interest rates and recovery rates on defaulted loans are normalized to zero).

Suppose 10,000 tranche A, 360 tranche B, and 40 tranche C insurance derivatives (CDSs) are sold at a rate of \$1.10 per expected dollar of defaults (that is, \$1.10 times probability of default on the tranche times the dollar amount of the loan).<sup>2</sup>

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<sup>1</sup> Adapted from Meder, Schwartz, Spires, and Young, 2010.

<sup>2</sup> At the height of the derivatives market (before its collapse), several sources claim the notational amount of such derivatives totaled several times annual GDP (gross domestic product).

Required:

A. Default independent case. Suppose we have the following probability assessments regarding the state of the economy and loan defaults conditional on the state of the economy.

| state             | Pr(Default   state) | Pr(no default   state) |
|-------------------|---------------------|------------------------|
| good; Pr(G) = 0.5 | 0.1                 | 0.9                    |
| bad; Pr(B) = 0.5  | 0.1                 | 0.9                    |

1. Determine the likelihood of default for each tranche.
2. Determine the expected net payoff to the issuer of the CDS (the insurer).

B. Default dependent case. Suppose we have the following probability assessments regarding the state of the economy and loan defaults conditional on the state of the economy.

| state             | Pr(Default   state) | Pr(no default   state) |
|-------------------|---------------------|------------------------|
| good; Pr(G) = 0.5 | 0.0                 | 1.0                    |
| bad; Pr(B) = 0.5  | 0.2                 | 0.8                    |

1. Determine the likelihood of default for each tranche.
2. Determine the expected net payoff to the issuer of the CDS (the insurer) if the issuer sets the price of the derivatives based on default independence (as in case A).

C. Default dependent case. Suppose we have the following probability assessments regarding the state of the economy and loan defaults conditional on the state of the economy.

| state             | Pr(Default   state) | Pr(no default   state) |
|-------------------|---------------------|------------------------|
| good; Pr(G) = 0.8 | 0.0                 | 1.0                    |
| bad; Pr(B) = 0.2  | 0.5                 | 0.5                    |

1. Determine the likelihood of default for each tranche.
2. Determine the expected net payoff to the issuer of the CDS (the insurer) if the issuer sets the price of the derivatives based on default independence (as in case A).

D. Default dependent case. Suppose we have the following probability assessments regarding the state of the economy and loan defaults conditional on the state of the economy.

| state             | Pr(Default   state) | Pr(no default   state) |
|-------------------|---------------------|------------------------|
| good; Pr(G) = 0.9 | 0.0                 | 1.0                    |
| bad; Pr(B) = 0.1  | 1.0                 | 0.0                    |

1. Determine the likelihood of default for each tranche.
2. Determine the expected net payoff to the issuer of the CDS (the insurer) if the issuer sets the price of the derivatives based on default independence (as in case A).

- E. 1. How important is default correlation to the pricing structure of bundled debt securities and related insurance derivatives?
2. If these securities and their derivatives are “fair valued” for financial statement purposes, are they likely to employ mark-to-market (level one or two) evidence or mark-to-model evidence (level 3) evidence? Do you expect the markets for these instruments to have a large number of buyers and sellers?