Ralph's Beauty Contest

Ralph, an accounting policy maker, is contemplating an appropriate regulatory level of public accounting report precision. Conventional wisdom suggests greater report precision of public information is welfare enhancing. However, existence of a "beauty contest" (discussed below) causes Ralph to pause and ponder. Each investor j can acquire private information, x_j, as well as public accounting information, y. All signals are mutually independent, noisy indicators of fundamental value, θ.[1]

$$y = \theta + \varepsilon_y, \quad \varepsilon_y \sim N\left(0, \sigma_y^2 = \tfrac{1}{\alpha}\right)$$

$$x_j = \theta + \varepsilon_x, \quad \varepsilon_x \sim N\left(0, \sigma_x^2 = \tfrac{1}{\beta}\right), \text{ for all } j = 1, \ldots, n$$

Since investors care not only about a firm's fundamental value but also about other investors' perceptions of other investors' perceptions (and so on) of value. Keynes' General Theory (1936) referred to equity valuation as akin to a beauty contest in which the contest seeks not to select the contestant an individual finds most attractive but rather a contest to identify the contestant who more individuals find most attractive. Accordingly, investors choose a utility maximizing (or quadratic loss minimizing) strategy a_j.

$$U_j = -(1-r)(a_j - \theta)^2 - r(a_j - \overline{a})^2$$

where $\overline{a} = \frac{1}{n-1}\sum_{j \neq i} a_i$ is the average of investors' strategies and the beauty contest component involves common knowledge weight on other investors' perceptions, r. The fundamental value component favors investing and utilizing the assets while the beauty contest component favors trading the assets. Hence, expected utility for investor j conditional on available information is

$$E_j[U_j \mid x_j, y] = -(1-r)E_j\left[(a_j - \theta)^2 \mid x_j, y\right] - rE_j\left[(a_j - \overline{a})^2 \mid x_j, y\right]$$

An investor's optimal strategy solves the first order condition (foc)

$$\frac{\partial E_j[U_j \mid x_j, y]}{\partial a_j} = -2(1-r)(a_j - E_j[\theta \mid x_j, y]) - 2r(a_j - E_j[\overline{a} \mid x_j, y]) = 0$$

The foc yields an optimal strategy for investor j

$$a_j = (1-r)E_j[\theta \mid x_j, y] + rE_j[\overline{a} \mid x_j, y]$$

The remaining challenge involves identification of $E_j[\overline{a} \mid x_j, y]$. Suppose investors conjecture linear strategies by all investors (remarkably, this is a unique equilibrium).

$$a_i = kx_i + (1-k)y$$

[1] All parties perceive θ has unbounded uncertainty or zero precision.

Since y is public (common) information but x_j is private information, an investor's best guess of others' information is $E_j[\theta \mid x_j, y] = \frac{\alpha y + \beta x_j}{\alpha+\beta}$. Then, on average

$$\begin{aligned}
E_j[\overline{a} \mid x_j, y] &= kE_j[\theta \mid x_j, y] + (1-k)\,y \\
&= k\frac{\alpha y + \beta x_j}{\alpha+\beta} + (1-k)\,y \\
&= \frac{\alpha + (1-k)\,\beta}{\alpha+\beta} y + \frac{k\beta}{\alpha+\beta} x_j
\end{aligned}$$

This leads to an optimal strategy

$$\begin{aligned}
a_j &= (1-r)\,E_j[\theta \mid x_j, y] + rE_j[\overline{a} \mid x_j, y] \\
&= (1-r)\,E_j[\theta \mid x_j, y] + r\,(kE_j[\theta \mid x_j, y] + (1-k)\,y) \\
&= (1-r+rk)\,E_j[\theta \mid x_j, y] + r\,(1-k)\,y \\
&= (1-r+rk)\,\frac{\alpha y + \beta x_j}{\alpha+\beta} + \frac{r\,(1-k)\,(\alpha+\beta)\,y}{\alpha+\beta} \\
a_j &= \frac{\alpha + \beta r\,(1-k)}{\alpha+\beta} y + (1-r+rk)\,\frac{\beta}{\alpha+\beta} x_j
\end{aligned}$$

Since $a_j = (1-k)\,y + kx_j$, some algebra shows $k = \frac{\beta(1-r)}{\alpha+\beta(1-r)}$ satisfies the two equations. Hence, an optimal strategy is

$$a_j^* = \frac{\alpha}{\alpha + (1-r)\,\beta} y + \frac{(1-r)\,\beta}{\alpha + (1-r)\,\beta} x_j$$

Ralph's welfare measure only values the fundamental component.

$$\begin{aligned}
W(a, \theta) &= -\frac{1}{n}\sum_{j=1}^{n} (a_j - \theta)^2 \\
&= -\frac{1}{n}\sum_{j=1}^{n} \left(k\varepsilon_{x_j} + (1-k)\,\varepsilon_y\right)^2
\end{aligned}$$

In other words, expected welfare is inversely related to the variance for the strategies a around θ.

$$E[W(a, \theta) \mid \theta] = -E\left[(a-\theta)^2 \mid \theta\right]$$

With perfect information, $a_j = \theta$ for all investors and $E[W(a, \theta) \mid \theta] = 0$. With noisy public information only, $a_j = y$ for all investors and $E[W(a, \theta) \mid \theta] = -\sigma_y^2 = -\frac{1}{\alpha}$.

But with noisy public and private information, things get more interesting. Because y is common information it tends to be overweighted (relative to the social optimum) as a result of chasing the beauty contest. That is, based on the equilibrium strategy identified above

$$
\begin{aligned}
E\left[W\left(a,\theta\right)\mid\theta\right] &= -E\left[\left(\frac{\alpha}{\alpha+\left(1-r\right)\beta}y+\frac{\left(1-r\right)\beta}{\alpha+\left(1-r\right)\beta}x_j-\theta\right)^2\mid\theta\right] \\
&= -\left[\left(\frac{\alpha}{\alpha+\left(1-r\right)\beta}\right)^2\sigma_y^2+\left(\frac{\left(1-r\right)\beta}{\alpha+\left(1-r\right)\beta}\right)^2\sigma_x^2\right] \\
&= -\left[\left(\frac{\alpha}{\alpha+\left(1-r\right)\beta}\right)^2\frac{1}{\alpha}+\left(\frac{\left(1-r\right)\beta}{\alpha+\left(1-r\right)\beta}\right)^2\frac{1}{\beta}\right] \\
&= -\left[\frac{\alpha}{\left(\alpha+\left(1-r\right)\beta\right)^2}+\frac{\left(1-r\right)^2\beta}{\left(\alpha+\left(1-r\right)\beta\right)^2}\right] \\
E\left[W\left(a,\theta\right)\mid\theta\right] &= -\frac{\alpha+\beta\left(1-r\right)^2}{\left(\alpha+\beta\left(1-r\right)\right)^2}
\end{aligned}
$$

Comparative statics indicate social welfare is increasing in the precision of private information.

$$
\frac{\partial E\left[W\left(a,\theta\right)\mid\theta\right]}{\partial\beta}=\frac{\left(1-r\right)\left[\left(1+r\right)\alpha+\left(1-r\right)^2\beta\right]}{\left(\alpha+\beta\left(1-r\right)\right)^3}>0
$$

However, the implication for social welfare of increasing precision of public information is ambiguous.

$$
\frac{\partial E\left[W\left(a,\theta\right)\mid\theta\right]}{\partial\alpha}=\frac{\alpha-\left(1-r\right)\left(2r-1\right)\beta}{\left(\alpha+\beta\left(1-r\right)\right)^3}
$$

This implies social welfare is increasing in public information precision if

$$
\frac{\alpha}{\beta}\geq\left(1-r\right)\left(2r-1\right)
$$

In other words, increasing the precision of public information is beneficial when the private information is noisy relative to public information and investors heavily weight the beauty contest or when investors place little weight on the beauty contest.

Required: Suppose $\alpha = 1$ and $\beta = 1$.

1. Suppose $r = 0$. What is a representative investor's optimal strategy (that is, the usage of or weights on public and private information in forming expectations)? Is expected social welfare increasing or decreasing in public report precision?

2. Suppose $r = \frac{1}{4}$. What is a representative investor's optimal strategy? Is expected social welfare increasing or decreasing in public report precision? How does this compare with $r = 0$?

3. Suppose $r = \frac{3}{4}$. What is a representative investor's optimal strategy? Is expected social welfare increasing or decreasing in public report precision? How does this compare with $r = 0$?

4. Suppose $r = 1$. What is a representative investor's optimal strategy? Is expected social welfare increasing or decreasing in public report precision? How does this compare with $r = 0$?

5. Repeat 1-4 for $\alpha = 1$ and $\beta = 10$. What does the analysis suggest about the role of the beauty contest? Does the beauty contest enhance social welfare or is it a dead weight loss?