Ralph's Accounting Reserves

Ralph, an owner-manager who is forced to liquidate part of his holdings at the end of each period, finds it difficult to resist upward misrepresentation of his private information (often referred to as earnings management). Ralph is curious about the resultant relation between change in firm value ΔP_t and reported accruals z_t. To keep things simple, firm value equals the expected future dividends for ten period's beyond the current period (at which time the firm is dissolved), the market interest rate is zero, current period cash flows are fully paid out in dividends, and dividends $\widetilde{d}_t$ are uniform iid $\{0, 1, 2\}$. Ralph has private information $\widetilde{y}_t^p$ about next period's dividend $\widetilde{y}_t^p = \widetilde{d}_{t+1}$ (for simplicity, there is no other information). If the private information is revealed, change in ex-dividend firm value at time t is

$$\begin{aligned}(\Delta P_t \mid y_t^p) &\equiv E\left[\widetilde{d}_{t+1} \mid \widetilde{y}_t^p = y_t^p\right] - E\left[\widetilde{d}_{t+1}\right] \\ &= y_t^p - E\left[y_t^p\right]\end{aligned}$$

Suppose Ralph faithfully reveals his private information through reported accruals z_t

$$z_t = y_t^p = E\left[\widetilde{d}_{t+1} \mid \widetilde{y}_t^p = y_t^p\right]$$

Then,

$$\begin{aligned}(\Delta P_t \mid z_t) &\equiv E\left[\widetilde{d}_{t+1} \mid \widetilde{z}_t = y_t^p\right] - E\left[\widetilde{d}_{t+1}\right] \\ &= z_t - E\left[y_t^p\right] \\ &= y_t^p - E\left[y_t^p\right]\end{aligned}$$

Upward misrepresentation as an equilibrium reporting strategy However, Ralph is able to misrepresent his private information by reporting accruals manipulated via θ_t as auditors are unable to detect any accrual misstatements below a materiality threshold equal to $\delta = 1$. Hence, reported accruals are $z_t = y_t^p + \theta_t$. Traders anticipate that Ralph reports $z_t = y_t^p + \theta_t$, consequently the change in market price is

$$\begin{aligned}(\Delta P_t \mid z_t) &= z_t - \theta_t - E\left[y_t^p\right] \\ &= y_t^p - E\left[y_t^p\right]\end{aligned}$$

Given this anticipated behavior, Ralph's equilibrium behavior is to report with upward misrepresentation as conjectured by investors.

Accounting reserves and multi-period reporting restrictions The auditor as well as investors naturally focus on upward misstatement and θ is limited to $\delta = 1$. However, Ralph may not always be able to upwardly misrepresent his private information due accrual ("cookie jar") reserve limitations (there is

only so much shifting of accruals across time periods that can be disguised as faithful reporting). In such circumstances, he may choose to increase reserves for future periods (think "big bath").[1] For simplicity, the auditor also limits downward misreports to $-\delta$. Finally, there is an intermediate region in which the auditor effectively eliminates misreporting opportunities. Hence, reported accruals as limited by auditor-disciplined reserves are

$$z_t = y_t^p + \theta_t$$

where $\theta_t \in \{1, 0, -1\}$.

Opportunities to misreport are impacted by current reserves equal to beginning reserves plus current performance. Favorable (unfavorable) performance, as indicated by y_t^p, increases (decreases) reserves. Upward (downward) misrepresentations deplete (boost) accrual reserves

$$R_t = R_{t-1} + y_t^p - \theta_t$$

Accruals may not be upwardly manipulated so that end of period reserves fall below some auditor detectable limit $R_t > \underline{r} = 0$. Putting this together, the resultant auditor-disciplined reporting strategies (and shorthand notation θ_t^k, $k \in \{u, 0, d\}$) are

$$\theta_t = \delta = 1, \quad \text{if } 0 < R_{t-1} + y_t^p - \delta \qquad (\theta_t^u)$$

$$\theta_t = -\delta = -1, \quad \text{if } 0 \geq R_{t-1} + y_t^p \qquad (\theta_t^d)$$

$$\theta_t = 0, \quad \text{if } 0 - R_{t-1} < y_t^p \leq \underline{r} - R_{t-1} + \delta \qquad (\theta_t^0)$$

Investors process Ralph's report with misreporting in mind. Given investors' expectations and Ralph's inability to credibly signal otherwise, Ralph's equilibrium reporting strategy is to upwardly misreport whenever audit tests permit, downwardly misreport when reserves are exhausted and downward misrepresentation passes audit tests, and faithfully report his private information when the auditor's test demand it. The likelihood of misreporting depends on the distribution of y_t^p (uniformly distributed as $\{0, 1, 2\}$), the distribution of perceived beginning reserves R_{t-1}, and $\delta = 1$. For period $t = 1$, investors' perceive the distribution of beginning reserves $R_{t-1} = R_0$ is uniformly distributed as $\{0, 1, 2, 3\}$.[2]

The change in equilibrium price for the firm following a report of z_t, evaluated over the support for beginning reserves R_{t-1}, is

$$\begin{aligned} \Delta P_t &= E\left[\widetilde{d}_{t+1} \mid \widetilde{z}_t = z_t, \underline{R}_{t-1} \leq R_{t-1} \leq \overline{R}_{t-1}\right] - E\left[d_{t+1}\right] \\ &= p_t^u\left(z_t - \delta\right) + p_t^0 z_t + p_t^d\left(z_t + \delta\right) - E\left[y_t^p\right] \end{aligned}$$

[1] Even though our analysis focuses on the first period, bear in mind that this is a multi-period reporting issue .

[2] If investors know R_{t-1} there is no uncertainty as Ralph's strategy in combination with the accruals report and the stock of reserves perfectly identifies Ralph's private information. As is often the case, combining stock and flow data is powerfully informative.

where p_t^k is the probability of θ_t^k $(k \in \{u, 0, d\})$ given the period t report z_t.

Ralph's equilibrium (auditor-disciplined) reporting strategy is

$$z_t = I_t^u \left(y_t^p + \delta\right) + I_t^d \left(y_t^p - \delta\right) + I_t^0 y_t^p$$

$$\begin{array}{lll} I_t^u = 1 & \text{if } \theta_t = \delta = 1 & \left(\theta_t = \theta_t^u\right) \\ \quad = 0 & \text{otherwise} & \\ I_t^d = 1 & \text{if } \theta_t = -\delta = -1 & \left(\theta_t = \theta_t^d\right) \\ \quad = 0 & \text{otherwise} & \\ I_t^0 = 1 & \text{if } \theta_t = 0 & \left(\theta_t = \theta_t^0\right) \\ \quad = 0 & \text{otherwise} & \end{array}$$

Suggested:

1. For potential reports $z_1 = -1, 0, 1, 2, 3$ and each level of possible beginning reserves $R_0 \in \{0, 1, 2, 3\}$, determine investors' perceptions of Ralph's equilibrium report strategy. (Hint: for each level of initial reserves R_0, each possible inferred value $y_1^p = \{0, 1, 2\}$ relates to only one equilibrium report value z_1).

2. Determine the equilibrium change in price $\Delta P\left(z_t\right)$ for each potential equilibrium report $z_1 = -1, 0, 1, 2, 3$.

3. For which, if any, potential equilibrium reports $z_1 = -1, 0, 1, 2, 3$ are there winners and losers (i.e., ambiguity in the signal)?

4. How does auditing (monitoring) impact earnings management as an equilibrium reporting strategy?