Ralph's Accounting Reserves

Ralph finds it difficult to resist upward mis-reporting. As a result, Ralph is interested in the relation between change in firm value ΔP_t and reported accruals z_t. To keep things simple, firm value equals the present value of expected future dividends for ten period's beyond the current period (at which time the firm is dissolved), the market interest rate is zero, current period cash flows are fully paid out in dividends, and dividends $\widetilde{d}$ are uniform *iid* $\{0, 1, 2\}$. Firm managers have private information $\widetilde{y}_t^p$ about next period's dividend $\widetilde{y}_t^p = \widetilde{d}_{t+1}$.[1] If the private information is revealed, change in ex dividend firm value at time t is

$$\begin{aligned} \Delta P_t &\equiv E\left[\widetilde{d}_{t+1} \mid \widetilde{y}_t^p = y_t^p\right] \\ &= y_t^p \end{aligned}$$

Suppose management reveals its private information through income I_t (cash flows plus change in accruals) where (fair value) accruals z_t

$$z_t = E\left[\widetilde{d}_{t+1} \mid \widetilde{y}_t^p = y_t^p\right] = y_t^p$$

are reported. Then, income is

$$\begin{aligned} I_t &= d_t + (z_t - z_{t-1}) \\ &= d_t + \left(y_t^p - y_{t-1}^p\right) \end{aligned}$$

and

$$\begin{aligned} \Delta P_t &\equiv E\left[\widetilde{d}_{t+1} \mid \widetilde{d}_t = d_t, I_t = d_t + \left(y_t^p - y_{t-1}^p\right)\right] \\ &= E\left[\widetilde{d}_{t+1} \mid \widetilde{z}_t = y_t^p\right] \\ &= z_t \end{aligned}$$

Ralph is the firm's owner-manager who, for liquidity reasons, is forced to liquidate part of his holdings at the end of each period. Ralph is able to misrepresent the fair value estimate by reporting accruals $z_t = y_t^p + \theta$. Auditors are unable to detect any accrual overstatements below a threshold equal to δ. Traders anticipate that Ralph reports $z_t = y_t^p + \delta$, consequently the change in market price is

$$\Delta P_t = z_t - E[\theta] = z_t - \delta$$

Given this anticipated behavior, Ralph's equilibrium behavior is to report as conjectured.

The auditor naturally focuses on upward mis-statement and θ is limited to δ. However, Ralph may choose to increase reserves for future periods when he is unable to upward mis-report (think "big bath"). As investors naturally

[1] For simplicity, there is no other information.

price protect against upward mis-reporting, occasional downward mis-reporting increases the opportunity for Ralph to upwardly report as anticipated in future periods. The auditor limits downward mis-reports to $-\delta$. Finally, there is an intermediate region in which the auditor effectively eliminates mis-reporting. Hence, reported accruals are

$$z_t = y_t^p + \theta_t$$

Accrual manipulation is limited by auditor-disciplined reserves. Opportunities to mis-report are impacted by current performance as favorable (unfavorable) performance (reflected in y_t^p) increases (decreases) reserves . Also, upward (downward) mis-reports decrease (increase) reserves

$$R_t = R_{t-1} + y_t^p - \theta_t$$

Accruals may not be upwardly manipulated so that reserves fall below some auditor detectable limit $R_t \geq \underline{r} = 0$. Putting this together the resultant mis-reporting (and shorthand notation θ_t^k, $k \in \{u, 0, d\}$) is

$$\begin{aligned}
\theta_t \leq \quad \delta \quad & \text{if } \underline{r} < R_{t-1} + y_t^p - \delta & (\theta_t^u) \\
\theta_t \geq -\delta \quad & \text{if } \underline{r} \geq R_{t-1} + y_t^p & (\theta_t^d) \\
\theta_t = \quad 0 \quad & \text{if } \underline{r} - R_{t-1} < y_t^p \leq \underline{r} - R_{t-1} + \delta & (\theta_t^0)
\end{aligned}$$

Investors process Ralph's report with mis-reporting in mind. Given investors' expectations and Ralph's inability to credibly signal otherwise, Ralph's equilibrium reporting strategy is to upwardly mis-report the maximum whenever possible, downwardly mis-report the maximum when reserves are exhausted, and not mis-report when results are closely scrutinized in the band between. The likelihood of mis-reporting depends on the distribution of y_t^p (uniformly distributed as $\{0, 1, 2\}$), the distribution of perceived reserves R_{t-1}, and $\delta = 1$. Assume investors' perceive the distribution of reserves R_0 is uniformly distributed as $\{0, 1, 2, 3\}$.[2]

The change in equilibrium price for the firm following a report of z_t,evaluated over the support for beginning reserves R_{t-1}, is

$$\begin{aligned}
\Delta P_t &= E\left[\widetilde{d}_{t+1} \mid \widetilde{z}_t = z_t, \underline{R}_{t-1} \leq R_{t-1} \leq \overline{R}_{t-1}\right] \\
&= p_t^u (z_t - \delta) + p_t^0 z_t + p_t^d (z_t + \delta)
\end{aligned}$$

where p_t^k is the probability of θ_t^k $(k \in \{u, 0, d\})$ given the period t report.

Ralph knows beginning reserves $R_0 = 2$. Recall investors' perceive R_0 to be uniformly distributed over $\{0, 1, 2, 3\}$, and Ralph's equilibrium reporting strategy is

$$z_t = I_t^u (y_t^p + \delta) + I_t^d (y_t^p - \delta) + I_t^0 y_t^p$$

[2] If investors know R_{t-1} there is no uncertainty as the entrepreneur's strategy in combination with the accruals report and the stock of reserves perfectly identifies the entrepreneur's private information. As is often the case, combining stock and flow data is powerfully informative. We focus on the case with residual uncertainty.

$$
\begin{array}{lll}
I_t^u = 1 & \text{if } \theta_t = \delta & (\theta_t = \theta_t^u) \\
= 0 & \text{otherwise} & \\
I_t^d = 1 & \text{if } \theta_t = -\delta & \left(\theta_t = \theta_t^d\right) \\
= 0 & \text{otherwise} & \\
I_t^0 = 1 & \text{if } \theta_t = 0 & \left(\theta_t = \theta_t^0\right) \\
= 0 & \text{otherwise} &
\end{array}
$$

where again θ_t^k is defined by

$$
\begin{aligned}
\theta_t &= \delta \text{ if } \underline{r} < R_{t-1} + y_t^p - \delta && (\theta_t^u) \\
\theta_t &= -\delta \text{ if } \underline{r} \geq R_{t-1} + y_t^p && (\theta_t^d) \\
\theta_t &= 0 \quad \text{if } \underline{r} - R_{t-1} < y_t^p \leq \underline{r} - R_{t-1} + \delta && (\theta_t^0)
\end{aligned}
$$

Required:

1. Use the following normal-form game to explain why upward mis-reporting is equilibrium reporting behavior when investors perceive the opportunity is available. Seller is the row player with reporting strategies θ^k. Buyer is the column player who bids according to perceived report θ^k. Your explanation should include the basis for the outcomes in the matrix (the upper left entry is the seller's payoff and the lower right entry in each cell is the buyer's payoff).

buyer

seller	θ^u	θ^0	θ^d
θ^u	0, 0	1, −1	2, −2
θ^0	−1, 1	0, 0	1, −1
θ^d	−2, 2	−1, 1	0, 0

2. Explain why it's difficult for Ralph to convey information about reserves R_{t-1}. Does Ralph have incentives to report $R_0 = 2$? What if Ralph knew $R_0 = 1$? Would investors find such reports credible (are they self-enforcing)? (Hint: consider the consequences to the buyer if Ralph reported $R_0 = 1$ when $R_0 = 2$; it may be helpful to refer to the above matrix.)

3. For each possible private signal $y_1^p \in \{0, 1, 2\}$, identify Ralph's equilibrium report z_1.

4. For each possible private signal $y_1^p \in \{0, 1, 2\}$ and each level of possible reserves $R_0 \in \{0, 1, 2, 3\}$, determine investors' perceptions of Ralph's report strategy.

5. Determine the equilibrium change in price $\Delta P(z_t)$ for each potential report $z_1 = -1, 0, 1, 2, 3$.

6. For which, if any, potential reports $z_1 = -1, 0, 1, 2, 3$ are there winners and losers? (You might find it helpful to revisit question 2.) How does auditing (monitoring) impact the winners/losers problem?