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Interaction between Productivity and Measurement

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INTRODUCTION

Measurement often appears far from benign. Close to home we have the system-wide, at times corrosive effects of student evaluations and popular press rankings. Modern medicine exhibits cycles of advances in measurement, e.g., blood chemistry, followed by development of pharmaceuticals designed to improve those measures. Many are the stories of the athlete more interested in personal than team glory. And then there is the unfolding case of earnings, where we encounter developments in financial engineering and organizational arrangements that seem to have no purpose beyond improving the earnings measure.

Economically, we are accustomed to measures being simply a source of information, where Bayesian revision in decision making or valuation and optimal contracting in trading arrangements determine the information's effect on organizational and individual behavior. Although subtleties surface in a multitask setting, and we know ''bad'' information can drive out ''good'' information (e.g., Holmstrom and Milgrom 1991), it seems the story is deeper. A commitment to produce a particular set of measures has the potential to affect the organization's behavior, to affect productivity, in ways that go beyond traditional information effects.

Here we put forward and analyze one such setting. Exactly how the act of measurement produces effects beyond mere information effects is an open question. Our approach is to look to the physical sciences, where measurement is far from benign, and to import the natural effect of measurement in that setting to a human organization. We do not claim this

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is the only way to proceed, or even the most enlightening way. But it does produce some startling, counter-intuitive results.[1]

The idea, then, is to posit and explore a modest structural model in which measurement's effects resonate through the organization. We use a single-task setting, so that we are not confronted by task balance issues and the possibility that turning off explicit incentives on one task may improve the supply of another task. Into this setting we import measurement effects cloned from the way Nature deals with interactions and measurement. In particular, quantum probabilities form the basis for our structural model.

There are some advantages to this approach. Interaction is an important physical phenomenon, and Nature exploits interactions in a remarkably efficient manner. The analysis is also subject to some transparency as it is axiomatic; four quantum axioms are reviewed in Appendix A and form the basis for interaction effects in our model. This offers the advantage of being able to trace the presumed structural features on which our analysis is based to its foundational assumptions. But as we stressed at the outset, we do not contend this analysis is the best way to exhibit the corrosive effects of information. Rather, its advantage is its background and documentation in the physical sciences.

There are also some disadvantages to this approach. Quantum probabilities take no prisoners in the sense that interaction between measurement and the (physical) system is inevitable, and occurs in particular albeit stochastic fashion. We do not claim this transcends the behavior of social organizations. But we do claim information can have a corrosive side in such organizations, manifesting itself in undocumented fashion.[2]

To reiterate, we claim information can be corrosive. We also claim we do not know how to model this phenomenon. So we resort to a hypothetical exploration, taking license from the physical sciences.

Quantum probabilities yield two intertwined lessons. First, existence of a "true" state is fiction. Rather, a superposition of all possibilities exists. Second, measurement is not benign. Measurement unavoidably involves interaction with the system and any such interaction changes the system. It is this systemic effect of measurement that is the highlight of our exploration.

On the surface, the exploration centers on whether to measure individual or group performance. Absent any interesting interactions among the members of the group, the answer is perhaps clear. Obtaining only a group measure cannot increase the "amount" of usable information. On the other hand, when subtle and complex (and hopefully productive) interactions exist, the problem is more difficult.

An individual measure may not capture that individual's contribution to group productivity. Individually the measure may not look impressive, but taking into account the abilities and contributions of the other members may yield an entirely different assessment. When intricate teamwork is a key force, whether the team won or lost is quite informative about the contributions of team members. Furthermore, productive interactions are the hallmark of successful organizations. It seems sensible to analyze accounting for organizations in a setting in which interaction is the key compelling reason giving rise to the organization in the first place.

[1] For related discussions, see Demski et al. (2006, 2009).

[2] The large supply of earnings management documentations is a case in point. Precisely how does repetitive earnings measurement lead to the observed statistical pattern between earnings management proxies and *a priori* earnings management temptations? For that matter, might random as opposed to programmed reckoning periods for earnings measurement be less corrosive? Likewise, might widespread adoption of XBRL have a corrosive downside as disingenuous benchmarks proliferate?

Notice, however, we have not included analysis of simultaneously producing individual and group measures. Importing Nature's processing of productivity removes that option from the table. Intuitively, the mere assembling of individual measures in our setting has the power to affect if not destroy synergy between the agents.[3] For that matter, we further caution the reader. We do not offer new, startling insights into the (well-researched) world of, say, team production. Rather, the focus is on corrosive possibilities. Think Barry Bonds in a world where individual statistics are unavailable or a dean deprived of citation counts.

We begin with a standard, reduced form agency problem in the second section. Cost of control, meaning the risk premia burned in an effort to acquire the exogenously desired agent action, is the metric used to evaluate alternative information structures.

The third section considers two extreme cases. The first case is a setting with no interactions. Accordingly, individual measures yield a lower cost of control than does a group measure. The next case is a setting with maximum coordination. (Synergy is the term we use for positive interaction effects.) Given maximum synergy, a group measure is more efficient. This result holds even though there is only one signal or measure available when the group measure is used, as opposed to multiple individual signals under individual measurement. With maximum synergy the coordination effect is stronger than the effect associated with increasing the number of signals.[4]

The underlying structure allows confronting other questions as well. Of particular interest is the behavior of control costs when synergy is neither at its maximum level nor absent.[5] In a setting with partial synergy, the inherent tension between assessing individual accomplishment and promoting synergistic behavior surfaces. In the fourth section, we exhibit regions where group measurement is more or less efficient than individual measurement. More broadly, any such control problem may become more severe due to changes in the information environment, technology, or preferences. As the severity of the control problem increases, the region of group measurement superiority also increases. Group measurement actually becomes relatively more efficient (relative to individual measures) as the agency problem worsens. In other words, more measures actually decrease efficiency. Indeed, as the analysis shows, when the inclination to acquire more measures is greatest, the effect of many individual measures is most corrosive.

PRODUCTIVITY AND MEASUREMENT

In most analyses, productivity is a black box. Our paper differs only in that we employ a state of superposition to convey a setting with no underlying truth. Productivity is represented by transformation of an initial state of the system. The initial state indicates whether the agents are independent, entangled, or partially entangled. How the agents interact is modeled as an inherent feature of the system. Measurement of the transformed

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[3] These points raise questions, for example, about the prudence of adopting fair value accounting wherever remotely feasible (level three evidence as defined by the FASB comes to mind). Accounting policy makers and others seem to presume the existence of "true" value and the task is simply to figure out how to measure it. However, it is well known that existence of value is delicate (see, for example, Debreu 1959) even if one ignores inherent uncertainty. Also, the act of attempting to measure changes the productivity landscape. Enron Corp. is a ready example (see Haldeman 2006), as is the contentious debate in the U.S. Congress surrounding the Patriot Act.

[4] The Rolling Stones is always recognized for the collective performance of its members. Each member's contribution is hardly available.

[5] It also turns out Blackwell type comparisons between individual and aggregate measurement are possible at times, but impossible at other times.

system produces information about the agents' productivity. The same productive transformation applies to all states. The agents' inputs are binary for simplicity and conveyed via an angular representation well suited to the transformation process.[6] Agent input is endogenous. Measurement is the experimental manipulation and is either individual or group.

Measurements are binary and indicate whether the agents' inputs have a favorable impact on productivity; we denote these signals as "success" or "failure." The angular representation of agents' inputs leads to a natural monotone increasing mapping between the angle (agent's input) and success probability (favorable productivity).[7] With this setup we next outline our focal agency problem.

We begin with a simple agency problem constructed around the following information structure:

	Success (s)	Failure (f)
Work (h)	p	$1 - p$
Shirk (l)	0	1

(1)

Many of the comparisons in what follows can be made directly from the information structure. When additional context is necessary, a standard control problem will be used. The principal is risk-neutral and the risk-averse agent exhibits multiplicative separable CARA expected utility representation:

$$U(I - c) = -e^{-r(I-c)},$$

where I is the dollar payment, c the personal cost of the unobservable act, and r the Arrow-Pratt risk aversion parameter. Assuming the principal always wishes to induce the high act (work or h), the design program takes on a familiar appearance:

$$\text{Minimize } E(pmt) = pI_s + (1 - p)I_f$$

$$\text{Subject to } pU(I_s - c_h) + (1 - p)U(I_f - c_h) \geq U(RW)$$

$$pU(I_s - c_h) + (1 - p)U(I_f - c_h) \geq U(I_f - c_l)$$

RW is the reservation wage, and the payments and personal costs are appropriately subscripted. Since with multiplicative CARA the structure of the solution is invariant with respect to reservation wage and the minimum personal cost, we set $RW = c_l = 0$. The solution is thus parameterized by p, r, and c_h. Exploiting the multiplicative CARA preference representation and the fact both constraints are binding, the problem can be written in matrix form, where the information structure is highlighted:

$$\begin{bmatrix} p & 1 - p \\ 0 & 1 \end{bmatrix} \begin{bmatrix} U(I_s) \\ U(I_f) \end{bmatrix} = \begin{bmatrix} U(c_h) \\ U(0) \end{bmatrix}.$$

[6] Our black box transformation is represented by an interferometer, an instrument employed in physics to explore quantum processes.
[7] Details are provided in the appendices.

To illustrate, set $p = \frac{1}{4}$, $r = .01$, and $c_h = 10$. We then have:

$$I_f = 0,$$

$$I_s = 47.91,$$

$$E(pmt) = 11.98.$$

In order to bring synergy into view, the model must contain another agent, who, to keep things simple, is identical to the first agent. The intriguing aspect of this model is it projects to the noted benchmark in (1), but also allows, when synergy is present, for too much information to be corrosive. This occurs not because the agents collude or even engage in sabotage. Rather, it occurs under the radar so to speak, just as we suspect is the case in a real organization.

The notion of synergy or interaction is, of course, an important part of the story. With no synergy the agents' acts are independent and with synergy the agents' acts are entangled. As noted, the agents' productivity is described as "success" or "failure." Measurement can be individual or group. Individual measurement provides a "success" or "failure" signal for each agent. Group measurement provides a single "success" or "failure" signal. There are four cases: individual measurement with no synergy (*IN*), individual measurement with synergy (*IS*), group measurement with no synergy (*GN*), and group measurement with synergy (*GS*).

We first characterize the success probabilities for each case. To better describe how Nature exploits interaction, it is convenient to represent the agent's act in polar form. So we use angle θ_j to represent agent j's act. Recall the agents are identical, have but two possible acts from which to choose, and will be motivated to supply the "high" act. We then have $\theta_j \in \{\theta_l,\theta_h\}$ for each agent. This angular representation device for capturing the agents' productivity leads to the success probabilities reported in the following Lemma. All proofs are contained in Appendix B.

Lemma 1: The success probabilities for the four cases are as follows:

$$\text{prob}(\text{success}|IN, \theta_1, \theta_2) = \sin^2 \frac{\theta_j}{2}, j = 1 \text{ or } 2, \tag{2}$$

$$\text{prob}(\text{success}|IS, \theta_1, \theta_2) = \frac{1}{2}, \tag{3}$$

$$\text{prob}(\text{success}|GN, \theta_1, \theta_2) = \sin^2 \frac{\theta_1}{2} \cos^2 \frac{\theta_2}{2} + \cos^2 \frac{\theta_1}{2} \sin^2 \frac{\theta_2}{2}, \tag{4}$$

$$\text{prob}(\text{success}|GS, \theta_1, \theta_2) = \sin^2 \frac{\theta_1 + \theta_2}{2}. \tag{5}$$

Lemma 1 implies that individual versus group measurement is, as usual, not a benign choice and also that the mere act of measurement affects productivity. Moreover, in the benchmark single-agent story in (1), the success probability given high act (working) turns out to be $p = \sin^2 \frac{\theta_h}{2}$, implying $\theta_h > 0$; and the success probability given low act (shirking) is $0 = \sin^2 \frac{\theta_l}{2}$, implying $\theta_l = 0$.[8] It will also turn out that we require $\theta_h \leq 90°$. We stress this specific characterization follows from the assumed axiomatic structure.

We next analyze the extreme cases of no synergy and pure synergy. These play the role of benchmarks in documenting how the model behaves when confronted with familiar territory.

BENCHMARK CASES

No Synergy Case

For the case of individual measurement with no synergy, the probabilities are straightforward, as the probability of success for one agent is independent of the effort level of the other agent. For example, for the first agent we have:

$$\text{prob}(\text{success}|\text{work}_1, \text{work}_2) = \text{prob}(\text{success}|\text{work}_1, \text{shirk}_2) = p. \quad (6)$$

On the other hand, group measurement produces one, and only one, signal for the joint output: whether the output of their joint efforts was measured as a success or a failure. The success probabilities in the group measurement case are reported in Proposition 1.

Proposition 1: Consider the case of group measurement with no synergy (*GN*). The success probabilities are summarized as follows:

$$\text{prob}(\text{success}|\text{work}_1, \text{work}_2) = 2p(1 - p),$$
$$\text{prob}(\text{success}|\text{work}_1, \text{shirk}_2) = \text{prob}(\text{success}|\text{shirk}_1, \text{work}_2) = p,$$
$$\text{prob}(\text{success}|\text{shirk}_1, \text{shirk}_2) = 0.$$

In the no synergy setting, the probability of success given both agents work is smaller for a group measure than for individual measures ($2p(1 - p) < 2p$). If but one or neither agent works hard, then the success probability collapses to the single agent case. We reiterate this specific characterization follows from the assumed axiomatic structure.

From here we write the group measurement control problem in best response form. That is, we assume the other agent will work hard. Collusion, the shirk/shirk strategy, is not a pressing concern as will be discussed shortly. The control problem based on group measurement is as follows:

$$\text{Minimize } E(pmt) = 2p(1 - p)I_s + [1 - 2p(1 - p)]I_f$$
$$\text{Subject to } 2p(1 - p)U(I_s - c_h) + [1 - 2p(1 - p)]U(I_f - c_h) \geq U(RW)$$
$$2p(1 - p)U(I_s - c_h) + [1 - 2p(1 - p)]U(I_f - c_h) \geq pU(I_s - c_l) + (1 - p)U(I_f - c_l).$$

The two constraints, as readily verified, are binding. This implies the program can also be written as a matrix equation highlighting the group information structure:

[8] In polar coordinates the projection of an angle, θ, onto the vertical (horizontal) axis is $\sin\theta$ ($\cos\theta$). To change into probabilities, the projection is squared; since by Pythagoras $\sin^2 \theta + \cos^2 \theta = 1$, a probability measure is thereby created. Notice that if θ is viewed as effort level, it is sensible to think of $\sin^2 \frac{\theta_h}{2}$ as the probability of success—increasing in effort θ as long as θ does not exceed π. When Nature deals with multiple angles, the same "probability as squared projection" idea is maintained.

$$\begin{bmatrix} 2p(1-p) & 1-2p(1-p) \\ p & 1-p \end{bmatrix} \begin{bmatrix} U(I_s) \\ U(I_f) \end{bmatrix} = \begin{bmatrix} U(c_h) \\ U(0) \end{bmatrix}$$

The information structure representation allows the comparison in Corollary 1.

Corollary 1: Assume no synergy. Group measurement matrix:

$$\begin{bmatrix} 2p(1-p) & 1-2p(1-p) \\ p & 1-p \end{bmatrix}$$

is a garbling of the individual measurement matrix $\begin{bmatrix} p & 1-p \\ 0 & 1 \end{bmatrix}$, with garbling matrix given by $\begin{bmatrix} 1-p & p \\ p & 1-p \end{bmatrix}$.

Since group measurement is a garbling of individual measurement,[9] and since we are dealing with an agency problem, we know from Grossman and Hart (1983) that individual measurement is superior for all parametric versions of our setting with no synergy. (Again, the shirk/shirk combination is not of concern here.)

To illustrate this for the earlier introduced numerical example parameters, we have a group solution of:

$$I_f = -17.42,$$

$$I_s = 84.62,$$

$$E(pmt) = 20.84$$

The control cost, the expected payment, increases from 11.98 to 20.84 for each agent when group measurement is used.

Further observe that shirk/shirk is not an equilibrium. Since the probability of success if both agents shirk is zero, the certainty equivalent is negative for both agents, not even achieving the reservation wage.

There are, of course, no interactions among agents in the no synergy case. Not surprisingly, individual measures are more informative than group measures. In fact, group measures are a garbling of individual measures. Next we explore a pure synergy setting where measurement is potentially quite corrosive.

Pure Synergy Case

Synergy arises from the productive interaction among agents. It is defined here in terms of the probabilities.

Definition 1: Synergy is present when the success probability for the group is greater than the sum of the success probabilities when measured individually.

[9] All that is claimed here is the algebraic fact that:

$$\begin{bmatrix} 2p(1-p) & 1-2p(1-p) \\ p & 1-p \end{bmatrix} = \begin{bmatrix} p & 1-p \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1-p & p \\ p & 1-p \end{bmatrix}.$$

See Marschak and Miyasawa (1968), Blackwell (1953), or Blackwell and Girschick (1954).

Definition 2: Pure synergy is present when synergy is maximally present, where the success probability for the group is given by $\sin^2 \frac{\theta_1 + \theta_2}{2}$, (5) in Lemma 1.

In order for synergy to be defined in this manner, the individual success probability must be less than $\frac{1}{2}$. Otherwise we face the possibility the group success probability might exceed unity. $p \leq \frac{1}{2}$ will be imposed throughout the rest of the analysis.[10]

An important benefit of synergy, of course, is the benefit associated with added productivity. The focus in this paper, however, is solely on the cost of control. Basically the question is whether fewer measurements increase the control costs. The previous section verified that, not surprisingly, in a setting without synergy group measurement results in higher control costs than individual measurements.

The benchmark synergy setting is presented in Proposition 2, for the case of group measurement.

Proposition 2: Consider the case of group measurement with pure synergy (*GS*). The success probabilities are summarized as follows:

$$\text{prob}(\text{success}|\text{work}_1, \text{work}_2) = 4p(1 - p),$$

$$\text{prob}(\text{success}|\text{work}_1, \text{shirk}_2) = \text{prob}(\text{success}|\text{shirk}_1, \text{work}_2) = p,$$

$$\text{prob}(\text{success}|\text{shirk}_1, \text{shirk}_2) = 0.$$

The alternative measurement regime is an individual measurement for each agent. When Nature attempts an individual measurement in a pure synergy context, no information is forthcoming. It is as if the attempt to measure individual contribution in a synergistic setting leads to pure noise.[11] One interpretation is it is impossible to identify individual contribution; another interpretation is the attempt to highlight individual contribution invites a concern for that measure which in and of itself destroys the synergy. Course evaluations arguably have this effect when we focus on the larger curriculum. Regardless, the presumed productivity function and axioms lead to the conclusion that measuring one individual's contribution without considering joint effects is worthless. The act of measurement is introducing a corrosive "extra-information" effect.

Proposition 3: Consider the case of individual measurement with pure synergy (*IS*). The success probabilities are summarized as follows:

$$\text{prob}(\text{success}|\text{work}_1, \text{work}_2) = \frac{1}{2},$$

$$\text{prob}(\text{success}|\text{work}_1, \text{shirk}_2) = \text{prob}(\text{success}|\text{shirk}_1, \text{work}_2) = \frac{1}{2},$$

$$\text{prob}(\text{success}|\text{shirk}_1, \text{shirk}_2) = \frac{1}{2}.$$

Obviously, it is not possible to induce a high effort level in a pure synergy environment using individual measurement.

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[10] This is the earlier noted restriction that $\theta_h \leq 90°$.

[11] Consider Abbott and Costello's "Who's on First" routine. If you read but one actor's half of the skit, it is utterly meaningless. Together, however, it is indescribably amusing.

Corollary 2: Assume pure synergy. The individual measurement matrix is a garbling of the group measurement matrix, with garbling matrix given by:

$$\begin{bmatrix} \frac{1}{2} & \frac{1}{2} \\ \frac{1}{2} & \frac{1}{2} \end{bmatrix}.$$

Turning to group measurement, reconsider the numerical example. The best response program is:

$$\text{Minimize } E(pmt) = 4p(1 - p)I_s + [1 - 4p(1 - p)]I_f$$

$$\text{Subject to } 4p(1 - p)U(I_s - c_h) + [1 - 4p(1 - p)]U(I_f - c_h) \geq U(RW)$$

$$4p(1 - p)U(I_s - c_h) + [1 - 4p(1 - p)]U(I_f - c_h) \\ \geq pU(I_s - c_l) + (1 - p)U(I_f - c_l)$$

For $p = \frac{1}{4}$ (as $\theta_h = 60°$), the solution is:

$$I_f = -4.65,$$

$$I_s = 15.40,$$

$$E(pmt) = 10.39.$$

Here, as well, collusion is not a problem, as the certainty equivalent for shirk/shirk is less than the reservation wage.

It is suggestive in the example that the group measurement with pure synergy case has a lower control cost than individual measurement with no synergy. But as stated in the following corollary, the comparison is context specific.

Corollary 3: The group measurement with pure synergy and individual measurement with no synergy cases are not Blackwell comparable, as in general neither one is a garbling of the other.

Even though they are not Blackwell comparable, they are comparable when embedded in the CARA multiplicative agency problem.[12] For this particular setting the cost of control for individual/no synergy always exceeds the cost of control for group/synergy as illustrated in the numerical example (see Fellingham and Schroeder 2007).

The results of this section are that when synergy is at its maximum (pure synergy), group measurement is better than individual measurement in the sense that control costs are less. Because individual measures fail to capture the effects of interactions, they destroy incentives to interact. In this way the extreme cases give comfort to the exercise, as the model is behaving in fairly intuitive fashion.

[12] This is suggestive of Kim's (1995) linkage between reduced agency cost and a mean preserving spread of the likelihood ratio, a broader condition than that of the Blackwell Theorem. Keep in mind, however, that Kim (1995) deals with a single agent setting and a local expression for incentive compatibility.

The previous two sections analyze no synergy and pure synergy cases. Presumably, there are cases in which there is some, but not pure, synergy. Partial synergy is the subject of the next section.

PARTIAL SYNERGY

It turns out that Nature is also quite clever at processing information when there is some, but not pure, synergy. This leads to our definition of partial synergy.

Definition 3: Partial synergy is present when pure synergy is present with probability α and no synergy is present with probability $1 - \alpha$, $0 < \alpha < 1$.

Under partial synergy, now, the conditional success probabilities are those derived in the preceding section. By the laws of probability, the unconditional probabilities are linear combinations of the benchmark cases.[13] This leads to the success probabilities reported in Proposition 4, for individual and group measurement systems, given a best-response contracting problem with the other agent choosing to work.

Proposition 4: The success probabilities in the partial synergy case are summarized as follows:

		Individual Measurement	
		Success	**Failure**
No Synergy	Work	p	$1 - p$
	Shirk	0	1
Pure Synergy	Work	1/2	1/2
	Shirk	1/2	1/2
Partial Synergy	Work	$(1 - \alpha)p + \frac{1}{2}\alpha$	$1 - (1 - \alpha)p - \frac{1}{2}\alpha$
	Shirk	$\frac{1}{2}\alpha$	$1 - \frac{1}{2}\alpha$

and

		Group Measurement	
		Success	**Failure**
No Synergy	Work, work	$2p(1 - p)$	$1 - 2p(1 - p)$
	Shirk, work	p	$1 - p$
Pure Synergy	Work, work	$4p(1 - p)$	$1 - 4p(1 - p)$
	Shirk, work	p	$1 - p$
Partial Synergy	Work, work	$2p(1 - p)(1 + \alpha)$	$1 - 2p(1 - p)(1 + \alpha)$
	Shirk, work	p	$1 - p$

Not surprisingly, for extreme values of α we return to a garbling characterization. For example, for "little" synergy (α close to zero), group measurement is a garbling of individual measurement. On the other hand, for large values of α, just the opposite holds. The bounds defining the regions of garbling dominance are provided in Corollary 4.

[13] Quantum information theory represents this in density operator form (see the discussion of the quantum axioms for density operators in Appendix A). Application of density operators confirms these claims on success probabilities in the partial synergy setting.

Corollary 4: Group measurement is a garbling of individual measurement when $2p - \alpha(5 - 4p) + 2\alpha^2(1 - p) \geq 0$. Individual measurement is a garbling of group measurement when $\alpha^2(1 - p) - p + \frac{\alpha}{2} \geq 0$.

Return to the numerical example with $p = \frac{1}{4}$. Group measurement is a garbling of individual measurement when $\frac{1}{2} - 4\alpha + \frac{3}{2}\alpha^2 \geq 0$, which is satisfied for $0 < \alpha \leq \frac{4 - \sqrt{13}}{3} \simeq .1315$, that is, for "small" values of α. At the other end of the α continuum, individual measurement is a garbling of group measurement when $\frac{3}{2}\alpha^2 + \alpha - \frac{1}{2} \geq 0$, that is for $\frac{1}{3} \leq \alpha < 1$. It is also interesting to note that, regardless of the value of p, individual measurement is a garbling of group measurement whenever $\alpha \geq \frac{1}{\lambda}$ where λ is the "golden ratio," an important number in the history of mathematics. This follows from the fact that the greatest lower bound on α occurs for the upper bound on p: $p = \frac{1}{2}$; substitute into the condition in the Corollary. $\frac{1}{\lambda}$ is approximately 0.61803.

Continuing with $p = \frac{1}{4}$, for values of the synergy parameter, α, between $\frac{4 - \sqrt{13}}{3}$ and $\frac{1}{3}$, the measurement systems are not Blackwell comparable. In order to establish a ranking, it is necessary to embed the problem in more structure. As before, let $RW = c_l = 0$, $c_h = 10$, and $r = 0.01$. We find the control costs are equal for $\alpha = 0.2064$. That is, for α levels above the cut-off, group measurement results in a lower cost of control for the specific control problem. At the cut-off the expected payment is 13.5868 for both measurement systems. The signal-act probabilities at the cutoff are displayed as follows:

Individual Measurement	**Success**	**Failure**
Work	0.3016	0.6984
Shirk	0.1032	0.8968

Group Measurement	**Success**	**Failure**
Work, Work	0.4524	0.5476
Shirk, Work	0.25	0.75

$RW = c_l = 0$;
$c_h = 10$;
$r = 0.01$;
$p = 0.25$; and
$\alpha = 0.2064$.

This suggests comparative statics. Denote α^* as the cut-off synergy level where the control costs are equal for the two different measurement systems. Intuitively, it follows from the garbling relations that for synergy values exceeding α^*, group measurement has lower control cost than individual measurement. In other words, α^* bounds the region of group measurement superiority.

The example below suggests when the control problem becomes more pressing, group measurement becomes relatively more efficient. To illustrate, let the private cost c_h vary and track α^*.

FIGURE 1
Group Measurement More Efficient as Private Cost Increases

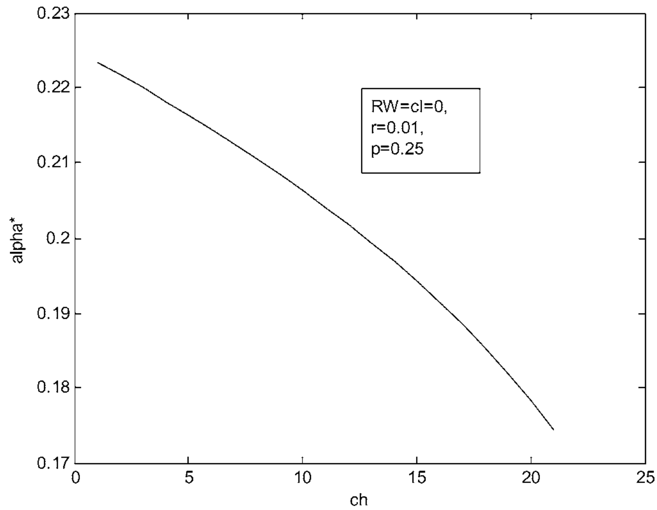
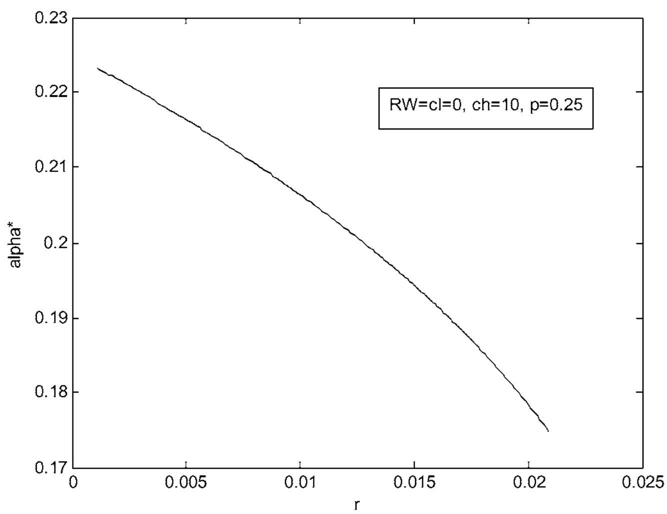


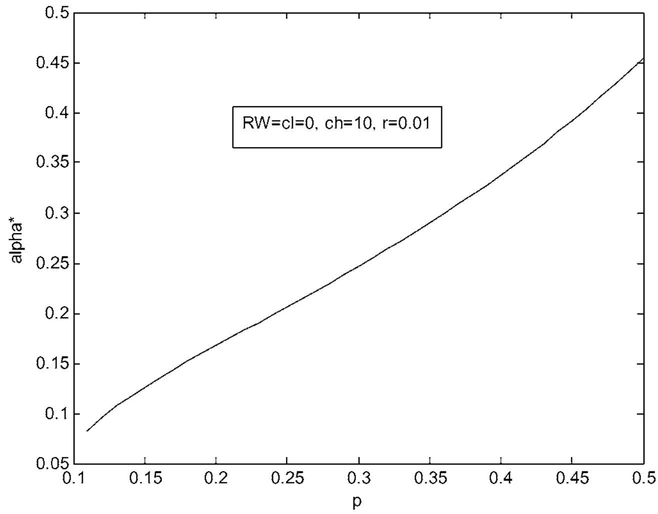
FIGURE 2
Group Measurement More Efficient as Private Risk Aversion Increases



c_h	$E[pmt]$	α^*
5	5.7848	0.2164
10	13.5868	0.2064
15	24.6725	0.1943

$RW = c_l = 0$;
$r = 0.01$; and
$p = 0.25$.

FIGURE 3
Group Measurement More Efficient as Success Probability Declines



As the private cost, c_h, increases so does the severity of the control problem—more risk premium is burned. The change in α^* indicates the greater the severity of the control problem, the larger the region over which group measurement is superior. Likewise, control problem severity increases with the risk parameter, r, and severity decreases as success probability, p, increases. The figures illustrate when control severity increases, group measures become relatively more efficient.

CONCLUDING REMARKS

The point to our odyssey is to raise the issue of potentially corrosive effects of information. Familiar anecdotes are the manager who manages to the numbers, the baseball player who plays for his individual measures, the industry that lives with citation counts, and the college managed by course evaluations. These all have not only an admittedly apocryphal side, but they are also suggestive of a hole in our understanding.

Turning to the physical sciences, it has long been understood that the mere act of measurement can impact the physical system. Is the same true of human systems? We suspect so, but are perplexed by how this might transpire and survive.

Borrowing from the physical sciences, we construct a structural model in which the act of measurement mimics its counterpart at the subatomic level. The resulting model exhibits the usual presumption that independent activities are best monitored with independent measures and group activities with a group measure. Mismatching, so to speak, is corrosive; and also has ties to garbling.

Moving away from the extreme cases to what we term partial synergy, we naturally encounter a more mixed message. Surprisingly, however, increasing the strength of the underlying control problem does not move us in the direction of more measures.

As a control problem increases in severity, the natural inclination is to collect more measures of behavior. For example, an increase in lucrative outside opportunities is often met with tighter monitors inside the organization, supposedly inducing the agent to be conscientious in his organizational duties.[14] The example illustrates that, in the presence of nontrivial synergies, the inclination to increase monitoring may be misguided. The control cost actually decreases with a group measure. The reason is that the salient behavior characteristic is coordination, which becomes more important when the control problem is aggravated. A group information structure more effectively illuminates agent coordination, and does not invite sabotaging that coordination by keeping an eye, so to speak, on one's individual performance measure.

We stress, however, that the exercise is aimed at the corrosive side of information. Precisely how this comes about is an open question. But our anecdotes resonate in a productivity-measurement function borrowed from the physical sciences.

So, can accounting be corrosive?

APPENDIX A

There are four quantum axioms governing the behavior of quantum probabilities (see Nielsen and Chuang 2002). We first outline the axioms in standard (qubit) form. Then we express each in terms of von Neumann's density operator representation as it is more amenable to mixed states encountered with our partial synergy setting.

1. The superposition axiom:

   A quantum unit (qubit) is specified by a two-element vector, say $\begin{bmatrix}\alpha\\ \beta\end{bmatrix}$, with $|\alpha|^2 + |\beta|^2 = 1$.

   Let $|\psi\rangle \equiv \begin{bmatrix}\alpha\\ \beta\end{bmatrix} = \alpha|0\rangle + \beta|1\rangle$,[15] $\langle\psi| = \begin{bmatrix}\alpha\\ \beta\end{bmatrix}^{\dagger}$ where $\dagger$ is the adjoint (conjugate transpose) operation.

   A density operator is the sum of the probabability-weighted, p_j, mixture of states expressed in outer product form:

$$\rho = \sum_{j=1}^{n} p_j|\psi_j\rangle\langle\psi_j|$$

   as the outer product is a unitary matrix and $\Sigma_{j=1}^{n} p_j = 1$, $\mathrm{Tr}[\rho] = 1$ where $\mathrm{Tr}[\cdot]$ is the trace operator.

2. The transformation axiom:

   A transformation of a quantum unit is accomplished by matrix multiplication.

   Useful single qubit transformations are $H = \frac{1}{\sqrt{2}}\begin{bmatrix}1 & 1\\ 1 & -1\end{bmatrix}$ and $\Theta = \begin{bmatrix}e^{i\theta} & 0\\ 0 & 1\end{bmatrix}$. Examples of the transformations in Dirac notation:

[14] The latest round of so-called congressional reforms aimed at documenting and controlling interactions with lobbyists is a case in point.

[15] Dirac notation is a useful descriptor, as $|0\rangle = \begin{bmatrix}1\\ 0\end{bmatrix}$ and $|1\rangle = \begin{bmatrix}0\\ 1\end{bmatrix}$.

$$H|0\rangle = \frac{|0\rangle + |1\rangle}{\sqrt{2}};\ H|1\rangle = \frac{|0\rangle - |1\rangle}{\sqrt{2}}$$
$$\Theta|0\rangle = e^{i\theta}|0\rangle;\ \Theta|1\rangle = |0\rangle$$

Transformation of a density operator is also a unitary operation.

$$U\rho U^{\dagger} = \sum_{j=1}^{n} p_j U|\psi_j\rangle\langle\psi_j|U^{\dagger}$$

3. The measurement axiom:

   Measurement of a quantum state is accomplished by a linear projection from a set of projection matrices that add to the identity matrix.[16] The probability of a particular measurement occurring is the squared absolute value of the projection. (A part of the axiom not explicitly used in the paper is that the post-measurement state is the projection appropriately normalized; this effectively rules out multiple measurement.)

   For example, let the projection matrices be $M_0 = |0\rangle\langle 0| = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}$ and $M_1 = |1\rangle\langle 1| = \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}$. Note that M_0 projects onto the $|0\rangle$ vector and M_1 projects onto the $|1\rangle$ vector. Also note that $M_0^{\dagger}M_0 + M_1^{\dagger}M_1 = M_0 + M_1 = I$. For $|\psi\rangle = \alpha|0\rangle + \beta|1\rangle$, the projection of $|\psi\rangle$ onto $|0\rangle$ is $M_0|\psi\rangle$. The probability of $|0\rangle$ being the result of the measurement is $\langle\psi|M_0|\psi\rangle = |\alpha|^2$.

   The probability of $|0\rangle$ being the result of the measurement in density operator representation is $\mathrm{Tr}[M_0^{\dagger}M_0\rho]$. Notice $\mathrm{Tr}[M_0^{\dagger}M_0\rho] = \Sigma_{j=1}^{n} p_j \mathrm{Tr}[M_0\rho_j]$ where $\rho_j = |\psi_j\rangle\langle\psi_j|$. We exploit this property in the analysis of partial synergy.

4. The combination axiom:

   Qubits are combined by tensor multiplication. For example, two $|0\rangle$ qubits are combined as $|0\rangle \otimes |0\rangle = \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}$ denoted $|00\rangle$. It is often useful to transform one qubit in a combination and leave another unchanged; this can also be accomplished by tensor multiplication. Let H_1 denote a Hadamard transformation on the first qubit. Then applied to a two qubit system, $H_1 = H \otimes I = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ 1 & 0 & -1 & 0 \\ 0 & 1 & 0 & -1 \end{bmatrix}$ and $H_1|00\rangle = \frac{|00\rangle + |10\rangle}{\sqrt{2}}$.

   Another important two qubit transformation is the controlled not:

[16] More precisely, the projection matrices satisfy the completeness condition, $\Sigma_m M_m^{\dagger} M_m = I$, where $M_m^{\dagger}$ is the adjoint (conjugate transpose) of projection matrix M_m.

$$Cnot = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

Entangled two qubit states or Bell states are defined as follows:

$$|\beta_{00}\rangle = Cnot\ H_1|00\rangle = \frac{|00\rangle + |11\rangle}{\sqrt{2}}$$

and more generally:

$$|\beta_{ij}\rangle = Cnot\ H_1|ij\rangle \text{ for } i, j = 0,1.$$

The four two qubit Bell states form an orthonormal basis.

Combinations of density operators are analogous tensor operations.

$$\rho_1 \otimes \rho_2 = (|\psi_1\rangle \otimes |\psi_2\rangle)(\langle\psi_1| \otimes \langle\psi_2|) = |\psi_{12}\rangle\langle\psi_{12}|$$

APPENDIX B

Proof of Lemma 1

We prove the lemma in the following three steps.

Step 1: Productivity.

Productivity is represented by a two qubit interferometer (transformation function) and its application to a two qubit state (synergy or no synergy). The transformation function is $F = H_2\Theta_2H_2 \otimes H_1\Theta_1H_1$. Given the no synergy $|00\rangle$ setting, productivity is:

$$\begin{aligned} F|00\rangle &= H_2\Theta_2H_2 \otimes H_1\Theta_1 \frac{|00\rangle + |10\rangle}{\sqrt{2}} = H_2\Theta_2H_2 \otimes H_1 \frac{e^{i\theta_1}|00\rangle + |10\rangle}{\sqrt{2}} \\ &= H_2\Theta_2H_2 \otimes \frac{e^{i\theta_1}|00\rangle + e^{i\theta_1}|10\rangle + |00\rangle - |10\rangle}{2} \\ &= H_2\Theta_2 \frac{[e^{i\theta_1} + 1][|00\rangle + |01\rangle] + [e^{i\theta_1} - 1][|10\rangle + |11\rangle]}{2\sqrt{2}} \\ &= H_2 \frac{[e^{i\theta_1} + 1][e^{i\theta_2}|00\rangle + |01\rangle] + [e^{i\theta_1} - 1][e^{i\theta_2}|10\rangle + |11\rangle]}{2\sqrt{2}} \\ &= \frac{[e^{i\theta_1} + 1][e^{i\theta_2}(|00\rangle + |01\rangle) + (|00\rangle - |01\rangle)]}{4} \\ &\quad + \frac{[e^{i\theta_1} - 1][e^{i\theta_2}(|10\rangle + |11\rangle) + (|10\rangle - |11\rangle)]}{4} \\ &= \frac{[e^{i\theta_1} + 1][(e^{i\theta_2} + 1)|00\rangle + (e^{i\theta_2} - 1)|01\rangle]}{4} \end{aligned}$$

$$+\frac{[e^{i\theta_1}-1][(e^{i\theta_2}+1)|10\rangle+(e^{i\theta_2}-1)|11\rangle]}{4}. \tag{A1}$$

Similarly, given the pure synergy $|\beta_{00}\rangle$ setting, productivity is:

$$\begin{aligned}
F|\beta_{00}\rangle &= H_2\Theta_2H_2 \otimes H_1\Theta_1\,\frac{|00\rangle+|10\rangle+|01\rangle-|11\rangle}{2}\\
&= H_2\Theta_2H_2 \otimes H_1\,\frac{e^{i\theta_1}|00\rangle+|10\rangle+e^{i\theta_1}|01\rangle-|11\rangle}{2}\\
&= H_2\Theta_2H_2 \otimes \frac{[e^{i\theta_1}+1][|00\rangle+|11\rangle]+[e^{i\theta_1}-1][|10\rangle+|01\rangle]}{2\sqrt{2}}\\
&= H_2\Theta_2\,\frac{e^{i\theta_1}[|00\rangle+|10\rangle]+|01\rangle-|11\rangle}{2}\\
&= H_2\,\frac{e^{i\theta_1}e^{i\theta_2}[|00\rangle+|10\rangle]+|01\rangle-|11\rangle}{2}\\
&= \frac{[e^{i\theta_1}e^{i\theta_2}+1]|\beta_{00}\rangle+[e^{i\theta_1}e^{i\theta_2}-1]|\beta_{01}\rangle}{2}.
\end{aligned} \tag{A2}$$

Step 2: Define projection matrices.

The projection matrices are defined as the sum of the outer product of success vectors. Individual measurement reports a "success" or "failure" signal for each agent. For agent one, the success vectors are $|10\rangle$ and $|11\rangle$; and the failure vectors are $|00\rangle$ and $|01\rangle$. The projection matrices for agent one are:

$$M_{S1} = |10\rangle\langle 10| + |11\rangle\langle 11| = \begin{bmatrix}0&0&0&0\\0&0&0&0\\0&0&1&0\\0&0&0&1\end{bmatrix}, \text{ and} \tag{A3}$$

$$M_{F1} = |00\rangle\langle 00| + |01\rangle\langle 01| = \begin{bmatrix}1&0&0&0\\0&1&0&0\\0&0&0&0\\0&0&0&0\end{bmatrix}. \tag{A4}$$

Similarly, for agent two, the success vectors are $|01\rangle$ and $|11\rangle$; and the failure vectors are $|00\rangle$ and $|10\rangle$. The projection matrices for agent two are:

$$M_{S2} = |01\rangle\langle 01| + |11\rangle\langle 11| = \begin{bmatrix}0&0&0&0\\0&1&0&0\\0&0&0&0\\0&0&0&1\end{bmatrix}, \text{ and} \tag{A5}$$

$$M_{F2} = |00\rangle\langle 00| + |10\rangle\langle 10| = \begin{bmatrix}1&0&0&0\\0&0&0&0\\0&0&1&0\\0&0&0&0\end{bmatrix}. \tag{A6}$$

To check, $M_{S1}^{\dagger}M_{S1} + M_{F1}^{\dagger}M_{F1} = I$, and $M_{S2}^{\dagger}M_{S2} + M_{F2}^{\dagger}M_{F2} = I$.

Group measurement reports a "success" or "failure" signal for both agents. The success vectors are $|\beta_{01}\rangle$ and $|\beta_{11}\rangle$; and the failure vectors are $|\beta_{00}\rangle$ and $|\beta_{10}\rangle$. The projection matrices are:

$$M_S = |\beta_{01}\rangle\langle\beta_{01}| + |\beta_{11}\rangle\langle\beta_{11}| = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}, \text{ and} \tag{A7}$$

$$M_F = |\beta_{00}\rangle\langle\beta_{00}| + |\beta_{10}\rangle\langle\beta_{10}| = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}. \tag{A8}$$

To check, $M_S^\dagger M_S + M_F^\dagger M_F = I$.

Step 3: Measure success probabilities.

Project $F|00\rangle$ or $F|\beta_{00}\rangle$ onto success matrices (M_{S1} or M_S). The probability of success is the square length of the projection vectors. Under individual measurement with no synergy:

$$\begin{aligned} \text{prob}(\text{success}|\text{IN}, \theta_1, \theta_2) &= \langle 00|F^\dagger M_{S1} F|00\rangle = \|M_{S1}F|00\rangle\|^2 \\ &= \left\| \frac{1}{4} \begin{bmatrix} 0 \\ 0 \\ (e^{i\theta_1} - 1)(e^{i\theta_2} + 1) \\ (e^{i\theta_1} - 1)(e^{i\theta_2} - 1) \end{bmatrix} \right\|^2 \\ &= \frac{1 - \cos\theta_1}{2} = \sin^2 \frac{\theta_1}{2}. \end{aligned} \tag{A9}$$

Similarly, under individual measurement with synergy:

$$\begin{aligned} \text{prob}(\text{success}|\text{IS}, \theta_1, \theta_2) &= \langle \beta_{00}|F^\dagger M_{S1} F|\beta_{00}\rangle = \|M_{S1}F|\beta_{00}\rangle\|^2 \\ &= \left\| \frac{1}{2\sqrt{2}} \begin{bmatrix} 0 \\ 0 \\ e^{i\theta_1}e^{i\theta_2} - 1 \\ e^{i\theta_1}e^{i\theta_2} + 1 \end{bmatrix} \right\|^2 = \frac{1}{2}. \end{aligned} \tag{A10}$$

Under group measurement with no synergy:

$$\begin{aligned} \text{prob}(\text{success}|\text{GN}, \theta_1, \theta_2) &= \langle 00|F^\dagger M_S F|00\rangle = \|M_S F|00\rangle\|^2 \\ &= \left\| \frac{1}{4} \begin{bmatrix} 0 \\ (e^{i\theta_1} + 1)(e^{i\theta_2} - 1) \\ (e^{i\theta_1} - 1)(e^{i\theta_2} + 1) \\ 0 \end{bmatrix} \right\|^2 \\ &= \sin^2 \frac{\theta_1}{2} \cos^2 \frac{\theta_2}{2} + \cos^2 \frac{\theta_1}{2} \sin^2 \frac{\theta_2}{2}. \end{aligned} \tag{A11}$$

Under group measurement with synergy:

$$\begin{aligned}
\text{prob}(\text{success}|\text{GS}, \theta_1, \theta_2) &= \langle \beta_{00}| F^{\dagger} M_S F |\beta_{00}\rangle = \| M_S F |\beta_{00}\rangle \|^2 \\
&= \left\| \frac{1}{2\sqrt{2}} \begin{bmatrix} 0 \\ e^{i\theta_1} e^{i\theta_2} - 1 \\ e^{i\theta_1} e^{i\theta_2} - 1 \\ 0 \end{bmatrix} \right\|^2 \\
&= \sin^2 \frac{\theta_1 + \theta_2}{2},
\end{aligned} \tag{A12}$$

where $\theta_i \in \{\theta_l, \theta_h\}$. ■

Proof of Proposition 1

By Lemma 1:

$$\begin{aligned}
\text{prob}(\text{success}|\text{GN}, \theta_h, \theta_h) &= \text{prob}(\text{success}|\text{GN}, \text{work}_1, \text{work}_2) \\
&= 2\sin^2 \frac{\theta_h}{2} \cos^2 \frac{\theta_h}{2} \\
&= 2\sin^2 \frac{\theta_h}{2} \left[1 - \sin^2 \frac{\theta_h}{2} \right] \\
&= 2p(1 - p).
\end{aligned} \tag{A13}$$

The last equality is given by $p = \sin^2 \frac{\theta_h}{2}$. Also:

$$\begin{aligned}
\text{prob}(\text{success}|\text{GN}, \theta_h, 0) &= \text{prob}(\text{success}|\text{GN}, \text{work}_1, \text{shirk}_2) \\
&= \text{prob}(\text{success}|\text{GN}, \text{shirk}_1, \text{work}_2) \\
&= \sin^2 \frac{\theta_h}{2} = p.
\end{aligned} \tag{A14}$$

And:

$$\text{prob}(\text{success}|\text{GN}, 0,0) = \text{prob}(\text{success}|\text{GN}, \text{shirk}_1, \text{shirk}_2) = 0. \tag{A15}$$ ■

Proof of Corollary 1

We have:

$$\begin{bmatrix} 2p(1 - p) & 1 - 2p(1 - p) \\ p & 1 - p \end{bmatrix} = \begin{bmatrix} p & 1 - p \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 - p & p \\ p & 1 - p \end{bmatrix}.$$ ■

Proof of Proposition 2

By Lemma 1:

$$\begin{aligned}
\text{prob}(\text{success}|\text{GS}, \theta_h, \theta_h) &= \text{prob}(\text{success}|\text{GS}, \text{work}_1, \text{work}_2) \\
&= \sin^2\theta = \left[2\sin\frac{\theta_h}{2}\cos\frac{\theta_h}{2}\right]^2 \\
&= 4\sin^2\frac{\theta_h}{2}\left[1 - \sin^2\frac{\theta_h}{2}\right] \\
&= 4p(1-p). \qquad \text{(A16)}
\end{aligned}$$

And:

$$\begin{aligned}
\text{prob}(\text{success}|\text{GS}, \theta_h, 0) &= \text{prob}(\text{success}|\text{GS}, 0, \theta) \\
&= \text{prob}(\text{success}|\text{GS}, \text{work}_1, \text{shirk}_2) \\
&= \text{prob}(\text{success}|\text{GS}, \text{shirk}_1, \text{work}_2) \\
&= \sin^2\frac{\theta_h}{2} = p, \qquad \text{(A17)}
\end{aligned}$$

$$\text{prob}(\text{success}|\text{GS}, 0, 0) = \text{prob}(\text{success}|\text{GS}, \text{shirk}_1, \text{shirk}_2) = 0. \qquad \text{(A18)} \ ■$$

Proof of Proposition 3

It is directly implied by Lemma 1. ■

Proof of Corollary 2

The garbling matrix is $\begin{bmatrix} \frac{1}{2} & \frac{1}{2} \\ \frac{1}{2} & \frac{1}{2} \end{bmatrix}$. ■

Proof of Corollary 3

The garbling matrix, were it to exist, would be $\begin{bmatrix} 3(1-p) & 1-3(1-p) \\ p & 1-p \end{bmatrix}$ or its inverse, as $\begin{bmatrix} 4p(1-p) & 1-4p(1-p) \\ p & 1-p \end{bmatrix} = \begin{bmatrix} p & 1-p \\ 0 & 1 \end{bmatrix}\begin{bmatrix} 3(1-p) & 1-3(1-p) \\ p & 1-p \end{bmatrix}$ or the reverse. Since $0 < p \leq \frac{1}{2}$ yields negative entries, neither the above nor its inverse is a garbling matrix. ■

Proof of Proposition 4

prob(success|partial synergy, ·) is defined to be:

$$(1-\alpha)\text{prob}(\text{success}|\text{no synergy}, \cdot) + \alpha\text{prob}(\text{success}|\text{pure synergy}, \cdot), \qquad \text{(A19)}$$

This follows directly from representing ensembles of states via density operators (see Nielsen and Chuang 2002, 98–102). ■

Proof of Corollary 4

Group measurement with partial synergy is a garbling of individual measurement with partial synergy if there exists a garbling matrix G such that:

$$\begin{bmatrix} (1-\alpha)p + \frac{\alpha}{2} & 1-(1-\alpha)p - \frac{\alpha}{2} \\ \frac{\alpha}{2} & 1-\frac{\alpha}{2} \end{bmatrix} G = \begin{bmatrix} 2p(1+\alpha)(1-p) & 1-2p(1+\alpha)(1-p) \\ p & 1-p \end{bmatrix}. \tag{A20}$$

Solving for G:

$$G = \frac{1}{1-\alpha}\begin{bmatrix} [1+2\alpha-\alpha^2](1-p) - \frac{1}{2}\alpha & (\alpha^2-2\alpha)(1-p)+p-\frac{1}{2}\alpha \\ -\frac{1}{2}\alpha + p - \alpha^2(1-p) & (1-p)(1+\alpha^2)-\frac{1}{2}\alpha \end{bmatrix}. \tag{A21}$$

$\alpha \leq 1$ implies:

$$[1+2\alpha-\alpha^2](1-p) - \frac{1}{2}\alpha \geq (1-p)(1+\alpha^2) - \frac{1}{2}\alpha, \text{ and} \tag{A22}$$

$$-\frac{1}{2}\alpha + p - \alpha^2(1-p) \geq (\alpha^2-2\alpha)(1-p) + p - \frac{1}{2}\alpha. \tag{A23}$$

$\alpha \geq 0$ implies:

$$(1-p)(1+\alpha^2) - \frac{1}{2}\alpha \geq (\alpha^2-2\alpha)(1-p) + p - \frac{1}{2}\alpha, \tag{A24}$$

therefore, G is a garbling matrix if $(\alpha^2 - 2\alpha)(1-p) + p - \frac{1}{2}\alpha \geq 0$.

Individual measurement with partial synergy is a garbling of group measurement with partial synergy if there exists a garbling matrix G such that:

$$\begin{bmatrix} 2p(1+\alpha)(1-p) & 1-2p(1+\alpha)(1-p) \\ p & 1-p \end{bmatrix} G = \begin{bmatrix} \frac{\alpha}{2} + (1-\alpha)p & \frac{\alpha}{2} + (1-\alpha)(1-p) \\ \frac{\alpha}{2} & 1-\frac{\alpha}{2} \end{bmatrix}. \tag{A25}$$

Solving,

$$G = \frac{1}{1 - 2p + 2\alpha(1-p)} \times \begin{bmatrix} (1-p)(1+\alpha^2) - \frac{\alpha}{2} & [1 + 2\alpha - \alpha^2](1-p) - 1 + \frac{\alpha}{2} \\ (1-p)\alpha^2 - p + \frac{\alpha}{2} & [1 + 2\alpha - \alpha^2](1-p) - \frac{\alpha}{2} \end{bmatrix}. \quad \text{(A26)}$$

Since $\alpha \leq 1$ and G is a garbling matrix if $(1-p)\alpha^2 - p + \frac{\alpha}{2} \geq 0$. ■

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