Asset Revaluation Regulations

by

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We gratefully acknowldge the many helpful comments and suggestions we have received from the editor, anonymous referees, Ron Dye, Hans Frimor, Bob Magee, Sri Sridhar, and seminar participants at Stanford, Columbia, Florida State, Duke and Northwestern Universities and the EIASM Workshop on Accounting and Economics.

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Abstract

GAAP mandates a variety of departures from historical cost valuation. We consider a simple model that produces corresponding variety, depending on prevailing regulatory objectives and economic conditions. The model entails entrepreneurial investment in an asset followed by private information about asset value that cannot be communicated. A lemons problem arises in the asset resale market, creating a role for mandated disclosure in the form of audited asset revaluation.

1 Introduction

GAAP provides a bewildering and seemingly inconsistent array of revaluation requirements. These requirements include lower of cost or market for inventory, net realizable value for receivables, a less restrictive variant of lower of cost or market for long-lived assets, no revaluation for R&D (or most liabilities for that matter), and fair value for a variety of financial instruments. We examine a model in which the prevailing valuation policy can affect the scale of investment and asset prices. We find that all of the aforementioned policies can emerge as optimal policies, depending on regulatory objectives and model parameters. This finding suggests standard economic considerations may instill a GAAP with a wide variety of revaluation requirements.

The central feature of the model is a simple lemons problem (Akerlof [1970]) where an investor or firm acquires an asset and then privately observes value-relevant information before the asset resale market opens. Some potential sellers are forced by liquidity concerns to liquidate their holdings while other investors may opportunistically impersonate those who are liquidity constrained. This potential impersonation creates a lemons problem in the resale market. The prospect of this lemons problem can affect investment incentives, as entrepreneurs anticipate mis-pricing of assets and recognize that if they ultimately become liquidity constrained they may be unable to earn a reasonable return on their investment.[1]

In principle, mandated revaluation of low-valued assets (in effect, movement away from historical cost) can lead to more accurate pricing, protect distressed investors, and enhance incentives for investment. However, the revaluation regulations also can impose costs on

[1] Cross and Prentice [2006] cite the lemons problem as an important determinant of U. S. securities regulation and the recent Sarbanes Oxley Act.

those required to undertake the costly revaluation. These costs can distort investment decisions and influence incentives for voluntary revaluation. The optimal design of revaluation regulations requires a careful balancing of these benefits and costs.

We examine the optimal such balancing as follows. Section 2 provides an overview of our analysis and discusses related literature. Section 3 describes the key elements of the basic setting that we analyze. Section 4 presents key findings in this setting. Section 5 adds additional structure that facilitates a more complete characterization of equilibrium prices and optimal asset revaluation policies. Section 6 considers extensions of the basic setting. Section 7 offers concluding thoughts.[2]

2 Overview and Literature Review

Departures from historical cost measurement are numerous, varied, and seemingly ever increasing. Fair value reporting is applied to numerous assets (but has less force in liability measurement). Truncation approaches, such as lower of cost or market or asset impairment reporting, are also prevalent. R&D reporting is a *de facto* commitment to no revaluation. Numerous explanations for this heterogeneous mixture are possible, including irrationality, lack of political will, clever sequential response to ever more sophisticated transaction technologies, regulatory capture, and so on. We develop a simple model in which rational regulation can lead to virtually any of these measurement approaches, depending on the relative importance of the economic forces at play and the prevailing regulatory objective.

The model encompasses rational pricing, private information, costly verification, and (possibly distorted) investment choices. Initially, before they acquire any private information, entrepreneurs decide on the scale of a risky investment (e.g., Brown, Izan and Loh [1992] and Whittred and Chan [1992]). Subsequently they learn privately whether they will be forced to liquidate their investment (due to unmodeled liquidity issues) and also receive an updated (private) assessment of the true value of the investment. At this point, a resale market with a classic lemons problem opens. The lemons problem arises because investors cannot prove whether they are, in fact, distressed, i.e., must liquidate their investments immediately. Consequently, non-distressed investors can masquerade as distressed investors in an attempt to sell their low value assets at the prevailing market price for assets that have not been revalued. Rational expectations prevail in the resale market,[3] and these expectations can be

[2] The proofs of all formal conclusions are provided in the Appendix.

[3] In particular, the equilibrium price incorporates the impact of opportunistic behavior by non-distressed entrepreneurs. The presence of indistinguishable distressed and non-distressed traders permits opportunistic

influenced by the requirements imposed on market participants to report truthfully expected asset values.

Our analysis has ties to different strands of the literature. One strand examines different frictions that financial measurement might alleviate. Managerial myopia (e.g., Bachar, Melumad and Weyns [1997] or Liang and Wen [2006]), informed and uninformed traders (e.g., Grossman and Stiglitz [1980]), capital structure tensions (e.g., Liang and Zhang [2006]), managerial opportunism (e.g., Dye [1988] or Newman, Patterson and Smith [2005]), spillovers that affect participants in the product market (e.g., Feltham and Xie [1992] or Verrecchia and Weber [2006]), market inefficiency (e.g., Aboody, Hughes and Liu [2002]), forced liquidation (e.g., Dye and Verrecchia [1995] or Kanodia, Sapra and Venugopalan [2004]), and risk sharing (e.g., Leland and Pyle [1977] or Datar, Feltham and Hughes [1991]) are familiar sources of friction. Our model rests on risk neutrality and private information that cannot be communicated, coupled with probabilistic liquidation. As noted, investors in our model cannot credibly communicate their motivation for trading, just as in (for example) Dye's [1985] disclosure model where an agent cannot credibly communicate whether he has received private information. We also focus on settings where investors cannot credibly communicate their updated assessments of asset value other than through the mandated asset revaluation requirements.[4]

A second related strand of the literature concerns the underlying accounting structure. Our model begins with an investment choice and historical cost recognition of the resulting asset. When (and if) the asset is subsequently offered for sale, the prevailing accounting regulations dictate the extent to which the historical cost valuation must be revised to better reflect updated beliefs about asset value. The regulations might entail fair value (e.g., Christensen and Frimor [2006] or Bachar, Melumad and Weyns [1997]) or asset write-downs, as in impairment type reporting.[5] Asset write-downs have an extensive history, especially in the long-standing use of lower of cost or market measurement and, more recently, the use of restructuring charges. FAS 121 and FAS 144 brought the latter (largely unregulated) activity under the regulatory umbrella.[6] Research on write-downs has been extensive, especially

behavior to persist in equilibrium, performing much the same function noise traders play in Grossman and Stiglitz's [1980] analysis.

[4] Section 6 considers a setting where investors can voluntarily disclose audited asset values.

[5] We do not exclude asset write-ups (which might be viewed as "negative impairments"). Walker [1992] provides an historical review of the SEC's various rulings on asset write-ups. The current expansion into fair value reporting requirements also merits mention in this regard.

[6] Loan impairments and impairment testing of goodwill provide additional illustrations.

during the pre-FAS 121 period. This research has been largely empirical, with an emphasis on documenting valuation effects and possible opportunism. (See Alciatore *et al.* [1998] for an extensive review.) More recently, Riedl [2004] examines statistical properties of asset write-offs before and after the introduction of regulation (specifically, FAS 121). He finds that macro and industry economic factors play relatively minor roles, while opportunistic considerations play more central roles, following the introduction of the regulation. The efficacy of such regulation and its effects on investment, however, remain open questions.

More broadly, the asymmetry of asset write-downs can be viewed as a form of conservatism, where bad news is recognized more aggressively than good news. (See, for example, Watts [2003a, 2003b], Basu [1997], Gigler and Hemmer [2001], Bachar, Melumad and Weyns [1997], Christensen and Demski [2004], and Lin [2006].) Likewise, asset write-downs can be viewed as a type of disclosure. (See Healy and Palepu [2001], Core [2001], Verrecchia [2001] and Dye [2001] for recent reviews of this extensive literature.)[7] Our model is similar to Verrecchia's [1983] model of discretionary disclosure in that disclosure is costly but may nevertheless be pursued in both models. In contrast to Verrecchia, we emphasize investment effects, welfare-maximizing regulation, and the combined effects of mandated and discretionary disclosure.

In parallel fashion, Magee [2006] considers recognition per se as opposed to recognition followed by possible revaluation. In Magee's model, an entrepreneur faces an investment option with generally unknown but privately observed risk that cannot be communicated. If investment takes place, the entrepreneur is subsequently forced to liquidate before the project has run its course. Recognition enters as a means to distinguish more risky from less risky projects, potentially to the entrepreneur's advantage. In contrast, we stress the revaluation option.

Bachar, Melumad and Weyns' [1997] analysis is closest to ours in terms of model formulation and accounting structure. In their model, a firm's manager seeks to maximize the recorded value of the firm at the end of an initial time period. The manager employs private information to determine when to either undertake costly selective auditing or sell assets in order to certify their value. The authors compare lower of cost or market, fair value, and historical cost methods, emphasizing the magnitude of the deadweight loss caused by the manager's myopic behavior. They also examine how certification costs affect policy performance, as do we.

[7] Bergemann and Valimaki [2006] survey the role of information and disclosure in mechanism design settings.

Our analysis differs in three important respects, though. First, we consider the design of an optimal asset revaluation policy, within specified policy classes, rather than comparing the performance of selected policies. Second, we focus on the lemons problem in the market for assets rather than on problems caused by managerial myopia. Third, we consider the impact of revaluation requirements on investment behavior, and allow for differential concern with the welfare of distressed and non-distressed investors.

Our model provides two primary qualitative conclusions in the basic setting where all risk-neutral investors are identical and where their investments are observable. First, the revaluation policy that maximizes aggregate expected surplus imposes no revaluation requirement. This is the case because the losses the lemons problem imposes on distressed entrepreneurs are offset by the opportunistic gains it affords to non-distressed entrepreneurs in the absence of any revaluation requirement. Consequently, the lemons problem does not affect the expected payoff from investment and so the surplus-maximizing (first-best) level of investment arises in the absence of revaluation. Second, a non-trivial asset revaluation requirement is optimal when revaluation costs are sufficiently small and the welfare of distressed investors is valued more highly than the welfare of non-distressed investors. We also find that different asset revaluation policies can be preferable in this setting. Under some circumstances, the preferred policy is a *targeted* policy in which all assets with values below a specified threshold must be revalued. In other circumstances, the preferred policy can be a *proportional* policy in which the auditor enforces revaluation only when the asset is below a specified percentage of historical cost (e.g., lower of cost or market). This finding suggests the variety of policies observed in GAAP could conceivably reflect a combination of distinct policies, each of which performs particularly well under a distinct set of circumstances.

Different findings emerge in different settings. For example, non-trivial revaluation requirements can increase total expected surplus when investors differ *ex ante* and when their capabilities or their investment levels are not observed publicly. Thus, although revaluation mandates do not increase surplus when market participants are only uncertain about investment outcomes and trading motives, revaluation mandates can increase surplus when market participants also are imperfectly informed about key elements of the investment process. Similarly, if the original investment projects also vary in quality, non-trivial revaluation mandates can be optimal even in the absence of any differential concern with the welfare of distressed investors.

We also find that the opportunity to voluntarily revalue high-value assets can reduce aggregate expected surplus. When the cost of voluntary revaluation is sufficiently low, a distressed investor with a high asset value will find it profitable to revalue his asset. The associated revaluation cost, coupled with the corresponding reduction in the equilibrium price of non-revalued assets, serves to reduce the aggregate expected surplus of entrepreneurs. Mandatory revaluation does not reduce the surplus reduction introduced by the possibility of voluntary revaluation. However, a prohibition on voluntary revaluation would increase aggregate expected surplus.[8]

In summary, we find that even in the context of a highly structured model with a simple lemons problem, a revaluation mandate may (e.g., FAS 144) or may not (e.g., FAS 2) be optimal. When such a mandate is optimal, it may be preferable to link the revaluation requirement explicitly to historical cost (e.g., FAS 121 or 144) or to implement no such direct link (e.g., FAS 5 or 115). Absent an explicit link to historical cost, the optimal policy can require revaluation at a variety of thresholds below (or even above) historical cost.

3 The Basic Setting

We consider settings in which entrepreneurs decide initially how much to invest in a risky asset. In the basic setting on which we focus initially, the expected present value of the future cash flows associated with investment I by any risk-neutral entrepreneur is $\hat{x}(I)$, which is an increasing, concave function. The first best investment, denoted I^{FB}, is the solution to $\max_I \{ \hat{x}(I) - I \}$. We assume $\hat{x}\left(I^{FB}\right) - I^{FB} > 0$.

The present value of the realized cash flows from investment I is modeled as $\widetilde{x} = \hat{x}(I) + \widetilde{\mu} + \widetilde{\varepsilon}$, where $\widetilde{\mu}$ and $\widetilde{\varepsilon}$ are independent, mean-zero random variables with respective non-degenerate densities $h(\mu)$ and $g(\varepsilon)$. For simplicity, the scale of investment does not affect either $\widetilde{\mu}$ or $\widetilde{\varepsilon}$.

After spending I to purchase and develop an asset (which might be a firm, for example), each entrepreneur learns privately the realized value of $\widetilde{\mu}$ for his investment. Denote this realized value by μ. After observing μ, the expected value of the cash flows is revised to $E[\widetilde{x}|I, \mu] = \hat{x}(I) + \mu$. At the same time, each entrepreneur discovers privately whether exogenous financial considerations compel him to sell his asset. Each entrepreneur becomes so distressed with probability $\pi \in (0, 1)$. An entrepreneur is non-distressed (and so is not compelled to sell his asset) with probability $1 - \pi$.[9]

[8]FAS 2 effectively precludes revaluation for a class of assets. We also find that when a non-trivial asset revaluation regulation is imposed, the expected net payoff is always higher when the revaluation mandate is imposed only on asset sellers than when it is imposed on all asset owners.

[9]Random distress admits opportunism in the resale market while facilitating a tractable analysis of the effects

A resale market for assets opens after each investor learns his $\widetilde{\mu} = \mu$ realization and whether he must sell his asset. Any asset offered for sale must be accompanied by an audited set of financial statements. The audited statements reveal the size of the investment (via the cash flow statement or, in our simple setting, the reported book value of the new asset). Consequently, all market participants know the initial investment associated with any asset that is offered for sale.[10] Absent revaluation, the accounting book value, v, of any asset offered for sale reflects its historical cost, i.e., $v = I$.

If an asset revaluation regulation is in effect, it is assumed to take the familiar form of fair value reporting or truncation (by requiring documentation of low asset values). Formally, the financial statements will report accounting book value

$$v = \begin{cases} \hat{x}(I) + \mu & \text{if } \hat{x}(I) + \mu \leq x_c(I) \\ I & \text{otherwise} \end{cases}, \tag{1}$$

where $x_c(I)$ is a bound on asset expected value below which any asset with book value I that is offered for sale must be revalued accurately for financial reporting purposes.[11] ($x_c(I)$ is specified before any investment is undertaken.) Revaluation policies of this sort can take on many familiar forms. For example, the policy resembles a fair value requirement when $x_c(I)$ is sufficiently large, a lower of cost or market requirement when $x_c(I) = I$, and the impairment test in FAS 144 when $x_c(I)$ is somewhat below I. In addition, the policy is informationally equivalent to R&D reporting when $x_c(I)$ is sufficiently low that no revaluation is ever required.

The audited financial statements, then, reveal the initial investment I for all assets. In addition, they reveal the expected present value of cash flows given μ if the asset's value is below $x_c(I)$. Market participants receive only the information in the financial statements,

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of revaluation regulations on investment behavior. Noise traders serve a parallel purpose in other studies. Given our focus on optimal revaluation policies and the associated welfare analysis, the introduction of unmodeled noise traders would be problematic. See Dye [2001] for additional discussion of this issue.

[10] Because all entrepreneurs initially are identical in this basic setting, they all choose to undertake the same level of investment, I. Alternative settings are considered below.

[11] As noted above, it is optimal in the present setting to impose asset revaluation requirements only on asset sellers, rather than on all asset owners. When revaluation requirements are imposed on all asset owners, non-distressed entrepreneurs are forced to incur revaluation costs. In our model, where the requirements are imposed only on asset sellers, non-distressed entrepreneurs with particularly low asset values avoid revaluation costs without aggravating the lemons problem. The non-distressed entrepreneurs recognize they can only sell their low-value assets if they have their assets revalued, and so rationally choose to retain, rather than sell, their low-valued assets.

so accounting is the only source of information in the resale market in this basic model.[12]

The cost of auditing the financial statements is normalized to zero if the audit simply verifies the historical cost of the asset ($v = I$). For simplicity, we initially assume it is prohibitively costly to verify claims of asset values in excess of $x_c(\cdot)$. This simplifying assumption reflects the observation that, in practice, it is often more difficult to provide conclusive evidence that an asset has a particularly high value than it is to prove the asset has a particularly low value.[13] Auditors can verify asset values below $x_c(\cdot)$, but such verification necessitates nontrivial incremental work with associated resource cost k. Compliance with the identified lower-tail revaluation policy is exogenously enforced (by SEC enforcement or legal liability, for example).[14] For simplicity, we also assume that competition among audit firms eliminates rent for auditors.[15]

If an asset is offered for sale in the resale market, then, the market participants know: (1) the entrepreneur chose to sell the asset; (2) the investment size is I; and (3) the expected value of the asset is $E[\widetilde{x}|I,\mu] = \hat{x}(I) + \mu = v$ if $v < x_c(I)$ whereas the expected value is $E[\widetilde{x}|I,\mu] \geq x_c(I)$ if $v = I$. The market prices for assets reflect the rational expectations of risk neutral traders. Let $P(v, I)$ denote the equilibrium price of an asset with underlying investment I and accounting book value v. An asset with accounting value $v \neq I$ that

[12] It is common to treat accounting as the sole information source for market participants (e.g., Dye and Sridhar [2004], Liang and Zhang [2006], and most earnings response studies), although multiple sources of information are central in some studies (e.g., Demski and Feltham [1994] and Christensen and Frimor [2006]). Our analysis in Section 6 allows investors to voluntarily certify (i.e., disclose) the value of the assets they offer for sale.

[13] High value often stems from such ethereal considerations as goodwill, while low value often results from such readily observed, concrete considerations as physical damage or product market collapse. FAS 144 and the predecessor FAS 121 reflect claims that the costs of certifying the fair value of a well-performing asset outweigh the corresponding benefits. For example, paragraph 141 of FAS 121 states "... Comment letters and ... testimony ... clearly indicated that a requirement to specifically test each asset or group of assets for impairment each period would not be cost-effective." Of course, ascertaining realized losses from physical damage (e.g., from hurricanes, oil spills, or other environmental accidents) or other sources is far from trivial. However, such valuations are not uncommon in practice, and so are assumed to be feasible at cost k. Alternative audit technologies are considered below.

[14] This regulation requires the entrepreneur to reveal what he knows about the asset's value if his assessed value of the asset is below $x_c(I)$. This requirement is intended to mirror institutional practice. If sufficient penalties were available and enforceable, if discovery were generally reliable, and if the legal infrastructure could commit itself to randomized discovery, full disclosure with random auditing would be an appealing policy. However, these assumptions, like the policy they could underlie, seem unrealistic.

[15] The presumed audit cost structure reflects the idea that once the regulation is announced, the audit industry efficiently adapts its technology, as in the classical theory of cost (e.g., Chambers [1988]). This presumes the audit technology is unaffected by the scale of investment and audit cost reflects the audit technology, not the entrepreneur's technology.

is offered for sale will sell at price $P(v, I) = v$ because its value is revealed accurately by the mandated audit. If an asset with reported book value $v = I$ is offered for sale, the lemons problem introduces uncertainty about the true prevailing asset value. All distressed entrepreneurs offer their assets for sale.

After observing $\widetilde{\mu} = \mu$, a non-distressed entrepreneur has the option of selling the asset. If $E[\widetilde{x}|I, \mu] = \hat{x}(I) + \mu \leq x_c(I)$, selling the asset would secure a net payoff of $\hat{x}(I) + \mu - k = v - k$ for the entrepreneur while retaining the asset would net $\hat{x}(I) + \mu = v > v - k$. This is the case because a revalued low-value asset sells for its true value, and retaining the asset eliminates the audit cost k. If $\hat{x}(I) + \mu \in (x_c(I), P(I, I))$, where $P(I, I)$ denotes the equilibrium price of assets with book value I , the non-distressed entrepreneur will sell the asset at price $P(I, I)$. If $\hat{x}(I) + \mu \geq P(I, I)$, the non-distressed entrepreneur will retain his asset because its value exceeds the equilibrium price of assets with book value I.

The price of an asset with accounting book value $v = I$ that is offered for sale is simply the expected present value of the cash flows from the asset conditional on: (1) $\hat{x}(I) + \mu \geq x_c(I)$; (2) all distressed entrepreneurs with such assets have offered them for sale; and (3) all non-distressed entrepreneurs with such assets and private values of $\hat{x}(I) + \mu \in (x_c(I), P(I, I))$ also have offered their assets for sale. Let S denote the event of offering the asset for sale with a book value of $v = I$, which implies $\hat{x}(I) + \mu \geq x_c(I)$. Then the expected value of an asset with book value I offered for sale is:

$$E[\widetilde{x}|S, I] = \hat{x}(I) + \int_{\varepsilon} \int_{\mu} [\mu + \varepsilon] h(\mu|S) g(\varepsilon) d\mu d\varepsilon = \hat{x}(I) + \int_{\mu} \mu h(\mu|S) d\mu, \tag{2}$$

where $h(\mu|S) = h(\mu, S) / \int_{\mu} h(\mu, S) d\mu$ is the probability that $\widetilde{\mu} = \mu$, given the asset is offered for sale. This probability is explicitly specified as:

$$h(\mu|S) = \begin{cases} h(\mu) & \text{if } P(I, I) - \hat{x}(I) \geq \mu \geq x_c(I) - \hat{x}(I) \\ \pi h(\mu) & \text{if } P(I, I) - \hat{x}(I) \leq \mu \\ 0 & \text{otherwise} \end{cases}. \tag{3}$$

The equilibrium prices in the resale market will be:

$$P(v, I) = \begin{cases} v & \text{if } v \neq I \\ E[\widetilde{x}|S, I] & \text{otherwise} \end{cases}. \tag{4}$$

An entrepreneur anticipates these equilibrium prices when choosing the investment level that maximizes his expected net payoff. Formally, after observing $x_c(\cdot)$, the entrepreneur chooses I to maximize:[16]

$$\begin{aligned} V(I) &= \pi\{ \int_{\mu \leq x_c - \hat{x}(I)} [P(v,I)-k]h(\mu)d\mu + \int_{\mu \geq x_c - \hat{x}(I)} P(I,I)h(\mu)d\mu\} \\ &+[1-\pi]\{\int_{\varepsilon} \int_{\mu \leq x_c - \hat{x}(I)} [\hat{x}\,(I)+\mu+\varepsilon]h(\mu)g(\varepsilon)d\mu d\varepsilon + \int_{P(I,I)-\hat{x}(I) \geq \mu \geq x_c - \hat{x}(I)} P(I,I)h(\mu)d\mu \\ &+\int_{\varepsilon} \int_{\mu \geq P(I,I)-\hat{x}(I)} [\hat{x}\,(I)+\mu]h(\mu)g(\varepsilon)d\mu d\varepsilon\} - I \ . \end{aligned} \quad (5)$$

Before proceeding to characterize equilibrium outcomes in the basic setting, we review the timing of activities in this setting. The relevant time line is the following:

Regulator sets revaluation policy, x_c.	Entrepreneur invests I.	Entrepreneur learns privately whether he is distressed.	μ is realized and privately observed.	Distressed entrepreneur sells asset; non-distressed decides whether to sell.

Notice the revaluation policy is designed to increase the information available to market participants, to disgorge private information from the seller. Absent revaluation, the balance sheet may overstate or understate the prevailing expected value of an asset. The additional information provided by the revaluation reduces balance sheet error and may affect initial investment through its anticipated impact on equilibrium asset prices.

4 Findings in the Basic Setting

We now describe some key outcomes in the basic setting. Proposition 1 confirms the presence of a non-trivial lemons problem – assets that trade with historical cost balance sheets trade at a discount relative to *a priori* value – and thus a potential role for asset revaluation policy in this setting.

[16] For expositional simplicity, the dependence of $x_c(\cdot)$ on I is not stated explicitly in the following expression (and several subsequent expressions).

Proposition 1. *$P(I,I) < \hat{x}(I)$ in the basic setting for any investment I and revaluation policy $x_c(\cdot)$ such that revaluation is not certain (i.e., such that trade at price $P(I,I)$ occurs with positive probability).*

Central questions now are whether this lemons problem affects investment and welfare and, if so, whether asset revaluation policies can enhance welfare. To answer these questions, first suppose the relevant welfare measure is Marshallian welfare (total expected surplus). Because no expected rent accrues to any players other than the entrepreneurs, Marshallian welfare in the basic setting is simply the expected surplus of the entrepreneur.[17] As Proposition 1 implies, the distressed entrepreneur may be forced to sell his asset for a price below its true value, just as the non-distressed entrepreneur may enjoy the advantage of selling his asset for a premium. Proposition 2 reports that rational pricing and risk neutrality ensure the expected losses for distressed entrepreneurs and the expected gains for non-distressed entrepreneurs are offsetting. Consequently, total expected surplus is maximized and first-best investment is induced when no revaluation mandate is imposed in the basic setting.

Proposition 2. *Surplus-maximizing revaluation policy in the basic setting induces no revaluation by setting $x_c(I)$ arbitrarily low for all I. This policy of strict adherence to historical cost valuation induces all risk-neutral entrepreneurs to undertake the first-best investment level (I^{FB}) and secures the first-best level of expected surplus.*

Proposition 2 implies that no substantive reporting requirement should be imposed in the basic setting if the relevant social objective is to maximize total (unweighted) expected surplus. Reporting regulations, however, often have the flavor (if not the stated goal) of protecting the less fortunate, the less sophisticated, or some other target population. For example, the *Securities Act of 1933* states (section 7): "The Commission shall prescribe special rules ... as ... necessary or appropriate in the public interest or for the protection of investors." This charge suggests the Securities and Exchange Commission should afford a firm's investors particular protection relative to, say, the firm's employees or its customers. Reflecting a corresponding orientation, the *Wall Street Journal* recently reported: "The nation's top securities regulator said he plans a "sustained and increasing focus" on protecting the assets of the aging baby-boomer population" (August 2, 2006). In a similar vein, the FASB emphasizes those with an information disadvantage: "The objectives in this Statement are those of general purpose external financial reporting by business enterprises.

[17] Our market pricing assumption ensures the buyers in the resale market face a fair game, and thus in expectation acquire zero rent or surplus. Similarly, competition eliminates rent for auditors.

The objectives stem primarily from the informational needs of external users who lack the authority to prescribe the financial information they want from an enterprise and therefore must use the information that management communicates to them" (CON 1, paragraph 28). Similarly, the recent FASB/IASB working draft of a unified conceptual framework reiterates this emphasis on "users who lack the ability to prescribe all the financial information they need."[18]

These observations suggest that, in practice, financial reporting policies may not be designed simply to maximize aggregate surplus. Instead, the welfare of some parties may be afforded greater consideration than the welfare of other parties. To capture this possibility in our model, we introduce differential weighting of the welfare of distressed and non-distressed entrepreneurs. This approach is not meant to suggest that GAAP is literally skewed toward protecting "distressed" individuals or entities. Rather, the suggestion is that differential concern for different interest groups often underlies the design of regulatory policy in practice.[19] To reflect this theme in our model, we identify the distressed investors, the ones who may be harmed by the market friction, as the ones that receive particular attention from the regulator.

Formally, suppose social welfare increases dollar for dollar with the net payoff of distressed entrepreneurs but increases by only $w \in [0, 1)$ dollars as the net payoff of non-distressed entrepreneurs increases by one dollar.[20] When the payoffs of non-distressed entrepreneurs are discounted in this manner, the gains they secure by selling their assets at a price above actual value confer a reduced social benefit. In this sense, the lemons problem imposes greater social losses. To limit these losses, the optimal asset revaluation regulation in this setting (which we call welfare-maximizing asset revaluation policy) imposes a non-trivial revaluation requirement on asset sellers when the incremental audit cost k is sufficiently small.

[18] See paragraph OB11 of the FASB's July 6, 2006 Preliminary Views titled "Conceptual Framework for Financial Reporting: Objective of Financial Reporting and Qualitative Characteristics of Decision-Useful Financial Reporting Information." *The Economist* (December 20, 2003) notes that private equity funds are disuaded from opening their doors to retail investors because doing so would entail increased transparency and force them "to clean up their balance sheets."

[19] The SEC's mission statement contains the mandate to "protect investors," the IRS claims to "provide America's taxpayers top quality service," and the FAA claims to provide the "safest ... aerospace system."

[20] Recall that no other rents arise in the model, because auditors earn only normal returns. Therefore, the reduced weighting of the surplus that accrues to non-distressed entrepreneurs enables identification of optimal policies that favor the distressed entrepreneur. More broadly, the key point is to document how the regulatory objectives that might underlie GAAP can affect the nature and consequences of GAAP.

Proposition 3. *When k is sufficiently small, the welfare-maximizing asset revaluation policy (for $0 \leq w < 1$) in the basic setting imposes non-trivial revaluation thresholds, $x_c(\cdot)$, that induce revaluation with strictly positive probability.*

Proposition 3 implies that departures from historical cost valuation, in the form of non-trivial asset revaluation requirements, can be optimal when the welfare of distressed entrepreneurs is valued more highly than the welfare of non-distressed entrepreneurs. Additional structure is required to identify the magnitude of audit costs that ensure non-trivial revaluation requirements and the specific investment distortions that arise from the requirements. The requisite additional structure is introduced in the next section.

5 Findings in the Structured Basic Setting

To obtain a more complete characterization of asset revaluation policies, we now introduce three forms of additional structure to the basic setting. First, as in Dye (2002), we assume investment I yields expected (gross) payoff $\hat{x}(I) = \frac{\beta}{\alpha} I^{\alpha}$, where α and β are strictly positive parameters. α is less than unity, reflecting diminishing expected payoffs to investment. Second, we assume the privately observed random variable, $\widetilde{\mu}$, is uniformly distributed on the interval $[-f, f]$, where f is a strictly positive constant. Thus, $h(\mu) = 1/[2f]$ for $\mu \in [-f, f]$. Consequently, the privately observed revision in value of the remaining cash flows, $v = E[\widetilde{x}|I, \mu] = \hat{x}(I) + \mu$, follows a uniform distribution between $\hat{x}(I) - f$ and $\hat{x}(I) + f$. For later reference, denote the end points of this distribution by $\underline{x}(I) = \hat{x}(I) - f$ and $\bar{x}(I) = \hat{x}(I) + f$. Also let $\phi(v|I)$ denote the induced density on revised asset value v.[21] Thus, $\phi(v|I) = 1/2f$ for all $v \in [\underline{x}(I), \bar{x}(I)]$.

Third, we examine two variants of the relatively simple truncation policies that are commonly observed in practice. *Proportional* policies, patterned after impairment regulations (e.g., lower of cost or market (LCM) or FAS 144), set the truncation threshold as a percentage of historical cost, i.e., $x_c(I) = zI$ for some specified percentage $z \geq 0$. *Targeted* policies, patterned after the seeming vagueness of FAS 5 where recognition is highly dependent on the nature of the transaction (a liability in this case) and requires considerable judgment on the part of the reporting firm and auditor, simply set $x_c(I)$ equal to a constant, x_c, for the type of asset or transaction in question. In particular, the truncation threshold is not linked rigidly to historical cost, and so an auditor might have some discretion in choosing the threshold to

[21] In the interest of limiting notation, we proceed from this point to denote the privately revised value of the asset as v, i.e., $v = E[\widetilde{x}|I, \mu] = \hat{x}(I) + \mu$. So the reported book value is either v or I in what follows.

maximize the regulatory objective. The term *targeted* reflects the interpretation that GAAP might be fine tuned to the asset or transaction type without an explicit link to historical cost. In this sense, relative to a proportional policy, a targeted policy might be interpreted to reflect more of a principle than an explicit rule (e.g., Schipper [2003]), although both policies clearly translate into particular mandates once the institutional setting is determined.[22]

The uniform density for $\widetilde{\mu}$ in this "structured basic setting" admits a relatively simple expression for the equilibrium price in the resale market. Recall that the non-distressed entrepreneur offers his asset for sale only when its expected value $E[\widetilde{x}|I,\mu] = \hat{x}(I) + \mu$ is intermediate between the critical cutoff value, x_c, and the equilibrium price, $P(I,I)$. Consequently, when $x_c \in [\underline{x}(I),\ \bar{x}(I)]$, the probability an asset with revised (private) value $E[\widetilde{x}|I,\mu] \geq x_c$ is offered for sale, given it is owned by a non-distressed entrepreneur, is:

$$\int_{x_c}^{P(I,I)} \phi(v|I)dv = \frac{P(I,I) - x_c}{2f} \ . \tag{6}$$

Because a distressed entrepreneur always sells his asset, the probability that a distressed entrepreneur offers an asset with book value $E[\widetilde{x}|I,\mu] \geq x_c$ for sale at price $P(I,I)$ is:

$$\int_{x_c}^{\overline{x}(I)} \phi(v|I)dv = \frac{\overline{x}(I) - x_c}{2f}. \tag{7}$$

Therefore, the probability that an asset with revised (private) value $E[\widetilde{x}|I,\mu] \geq x_c$ is traded at price $P(I,I)$ is:[23]

$$q(I) = \pi\left[\frac{\overline{x}(I) - x_c}{2f}\right] + [1-\pi]\left[\frac{P(I) - x_c}{2f}\right]. \tag{8}$$

Thus, the expected value of a non-revalued I-asset traded at price $P(I,I)$ is:

$$E[\widetilde{x}|S,I] = \frac{\pi\left[\overline{x}(I) - x_c\right]}{2fq(I)}\left[\frac{x_c + \overline{x}(I)}{2}\right] + \frac{[1-\pi]\left[P(I,I) - x_c\right]}{2fq(I)}\left[\frac{x_c + P(I,I)}{2}\right]. \tag{9}$$

Equating $E[\widetilde{x}|S,I]$ and $P(I,I)$ provides:

Lemma 1. *When $x_c(I) \in [\underline{x}(I),\ \bar{x}(I)]$ in the structured basic setting, the equilibrium price*

[22] Because targeted and proportional policies prescribe a region in which revaluation must take place, they resemble disclosure policies (e.g., Verrecchia [1983]), with the caveat that the disclosures here are driven by regulatory mandate. The policies also resemble common performance investigation policies (e.g., Townsend [1979], Baiman and Demski [1980], Lambert [1985], and Dye [1986]), again with the caveat of a regulatory mandate.

[23] This is the probability of event S in equations (2) and (3).

of an asset with book value I is:

$$P(I,I) = x_c(I) + \frac{\sqrt{\pi}\,[\,\overline{x}(I) - x_c(I)]}{1+\sqrt{\pi}} = x_c(I)\left[\frac{1}{1+\sqrt{\pi}}\right] + \overline{x}(I)\left[\frac{\sqrt{\pi}}{1+\sqrt{\pi}}\right]. \tag{10}$$

When $x_c(I) \leq \underline{x}(I)$, the corresponding equilibrium price is:

$$P(I,I) = \underline{x}(I) + \frac{\sqrt{\pi}\,[\,\overline{x}(I) - \underline{x}(I)\,]}{1+\sqrt{\pi}} = \widehat{x}(I) - f\,\left[\frac{1-\sqrt{\pi}}{1+\sqrt{\pi}}\right]. \tag{11}$$

Equation (11) in Lemma 1 reflects Proposition 1, revealing that if no asset revaluation is induced, the equilibrium price for assets with book value I will be less than the initial present value of expected future cash flows from the assets by the "lemons discount" of $f[1-\sqrt{\pi}]/[1+\sqrt{\pi}]$. This discount is larger the more pronounced is the information asymmetry about expected asset value (f) and the less likely is an entrepreneur to be distressed. When entrepreneurs are less likely to be distressed, there is an increased likelihood of opportunistic selling by non-distressed entrepreneurs which reduces the expected value, and thus the price, of assets with book value I.

Equation (10) in Lemma 1 provides corresponding insights for the case where non-trivial revaluation is induced in equilibrium. In this case, the equilibrium price of an asset with book value I is a weighted average of the specified revaluation threshold (x_c) and the largest possible expected value of the asset ($\overline{x}(I)$). Moreover, increasing $x_c(I) \in [\underline{x}(I),\ \bar{x}(I)]$ increases the equilibrium price of assets that escape revaluation. Also notice x_c is weighted more heavily than $\overline{x}(\cdot)$, and the weight on x_c increases as entrepreneurs become less likely to be distressed (so π declines). Again, an increased presence of non-distressed entrepreneurs implies an increased likelihood of opportunistic selling of assets with book value I, and thus a lower equilibrium price for these assets.

When he chooses his investment level, the entrepreneur anticipates the forthcoming equilibrium asset prices and potential revaluation costs. The entrepreneur's investment level typically is affected differently by targeted and proportional revaluation policies. The different impacts arise because the entrepreneur's investment level affects the truncation threshold under a proportional revaluation policy (i.e., $\frac{dx_c(I)}{dI} = z$) but not under a targeted policy (i.e., $\frac{dx_c(I)}{dI} = 0$). This difference provides the following observations.[24]

[24] Although $\frac{dx_c(I)}{dI} = 0$ under the targeted policy, $x_c(I)$ is not independent of I. As is evident in the proof of Proposition 4 (for example), $x_c(I)$ is set in rational anticipation of the investment scale it induces.

Proposition 4. *Suppose revaluation and trade at equilibrium price $P(I,I)$ both occur with strictly positive probability in the structured basic setting. Then under a targeted truncation policy, investment (weakly) increases above the first-best level as x_c increases, attaining a maximum at $\left[\frac{\beta[2f+\pi k]}{2f}\right]^{\frac{1}{1-\alpha}} > I^{FB}$.*

Proposition 5. *Suppose revaluation and trade at equilibrium price $P(I,I)$ both occur with strictly positive probability in the structured basic setting. Then under a proportional truncation policy, investment: (i) exceeds first-best investment (I^{FB}) if $z < 1$; (ii) is equal to I^{FB} if $z = 1$; and (iii) is less than I^{FB} if $z > 1$.*

Under a targeted policy, increased investment reduces the odds of having to bear a costly revaluation. This benefit of increased investment outweighs the corresponding cost of diminished net expected payoffs as I initially increases above I^{FB}. Eventually, however, the reduction in net payoffs from further over-investment outweighs the associated reduction in expected revaluation costs. Consequently, investment does not increase above a maximum level (identified in Proposition 4) as x_c increases further.

Under a proportional truncation policy, investment has little impact on the revaluation threshold when z is small. Consequently, equilibrium investment exceeds I^{FB}, just as under a targeted policy. In contrast, when z is large, a reduction in investment below I^{FB} serves to reduce substantially the critical threshold (zI) below which revaluation is mandated. The associated reduction in revaluation costs initially outweighs the corresponding reduction in net payoffs, leading to investment below I^{FB}. The dividing line between the two effects is the LCM point of $z = 1$, where the truncation point tracks the investment scale exactly (i.e., $x_c(I) = I$), and first-best investment is induced.[25]

Propositions 4 and 5 raise the possibility of investment distortions under both the targeted and proportional truncation policies. Propositions 6 and 7 report that welfare-maximizing regulations typically will induce these distortions under both policy types whenever revaluation costs are sufficiently small.

[25] The value of z in the proportional revaluation policy might be viewed as a tax on investment. The higher is z, the more pronounced is the increase in the upper bound of the region of mandated revaluation ($[\underline{x}(I), zI]$) as investment increases. Proposition 5 reveals that a high "investment tax" ($z > 1$) reduces investment below the first-best level. In contrast, a value of z below 1 spurs investment above the first-best level because the mandated revaluation region expands less rapidly than investment in this case.

Proposition 6. *When k is sufficiently small,*[26] *the welfare-maximizing revaluation policy* $(0 \leq w < 1)$ *in the structured basic setting with a targeted truncation policy specifies a threshold* x_c *that induces the maximum investment level* $I = \left[\frac{\beta[2f+\pi k]}{2f}\right]^{\frac{1}{1-\alpha}}$, *by setting* $x_c(I) \neq I$*) almost surely.*

When k is sufficiently small, nontrivial revaluation requirements benefit distressed investors by mitigating the lemons problem in the resale market without imposing large revaluation costs on the distressed investors. Furthermore, when k is small, the maximal investment distortion identified in Propositions 4 and 6 also is small, and so is optimally induced. This distortion is generically induced by a policy other than LCM in which $x_c = I$, reflecting the fact that this policy almost never balances exactly the gains from a less severe lemons problem and the losses from increased revaluation costs.

It is readily verified that the maximal distortion identified in Proposition 6 is $\widehat{I} = I^{FB}[1+\frac{\pi k}{2f}]^{\frac{1}{1-\alpha}}$ and the corresponding welfare-maximizing truncation point is $\widehat{x}_c = \overline{x}(\widehat{I}) - \frac{[1+\sqrt{\pi}]k}{[1-\sqrt{\pi}][1-w]}$. Consequently, $\frac{\partial \widehat{x}_c}{\partial w} < 0$ and $\frac{\partial \widehat{x}_c}{\partial f} > 0$ for large f, for example, which illustrate the more general principle that the optimal revaluation policy varies with the regulator's objective and with the environment in which the revaluation policy is being implemented.

The same is true when the proportional rule is employed. In this case, though, $x_c(I) = I$ is not only a generic impossibility, but also a literal impossibility (although $x_c(I)$ in the neighborhood of I is routinely possible). This conclusion, stated formally in Proposition 7, arises because $z = 1$ induces no investment distortion and thereby underutilizes the instruments in pursuing the regulatory objective.

Proposition 7. *When k is sufficiently small, the welfare-maximizing revaluation policy* $(0 \leq w < 1)$ *in the structured basic setting with a proportional truncation policy induces an investment distortion (by setting* $z \neq 1$*).*

A remaining question is whether the targeted policy might systematically deliver a higher level of expected welfare than the proportional policy or *vice versa.*[27] Proposition 8 reveals

[26] As is evident from the proof of this proposition, the critical value of k does not lend itself to algebraic isolation. Sufficient bounds on k can be specified, but are also complex. For example, it can be shown that when $\alpha = .5$, $k^* = \frac{2f\sqrt{\frac{[1-\sqrt{\pi}][1-w]}{[1+\sqrt{\pi}]}}}{\sqrt{\frac{[1+\sqrt{\pi}]}{[1-\sqrt{\pi}][1-w]}}+\sqrt{\frac{[\pi+w(1-\pi)]\pi I^{FB}}{f}}}$ is a sufficient upper bound on k.

[27] Of course, from Proposition 2, the optimal policy entails no revaluation when the objective is to maximize total (unweighted) expected surplus.

this is not the case. The targeted policy will ensure greater welfare than a proportional policy when, for example, high revaluation costs and a significant potential for negative cash flows render it optimal to induce revaluation only when the expected present value of future cash flows is (strictly) negative. A targeted policy can implement a negative threshold but a proportional policy (which expresses the critical threshold as a non-negative fraction of (non-negative) initial investment) cannot, though, as we shall see, this straightforward observation hardly exhausts the possibilities.

In contrast, a proportional revaluation policy can secure greater expected welfare than a targeted policy because the direct link between the revaluation threshold (x_c) and investment (I) in the proportional policy mitigates the incentives for over-investment that arises under the targeted policy. To illustrate, let $\alpha = .5$, $\beta = 10$, $\pi = .7$, $k = 20$, $w = 0$, and $f = 150$. The first-best level of investment is 100 in this setting. Under the optimal targeted policy, $x_c(I) = 134.445$, $I = 109.55$ and the expected payoff of the distressed investor is 87.28. Under the optimal proportional policy, $z = 1.2257$, $I = 98.02$, $x_c(98.02) = 120.14$ and the expected payoff to the distressed investor is 87.48.[28] Thus, the proportional revaluation policy secures greater expected welfare in this setting (and other similar settings) in part by inducing a less pronounced investment distortion.[29]

Proposition 8. *When k is sufficiently small that both the targeted and proportional policies are welfare improving, neither one systematically produces greater expected welfare than the other in all relevant settings.*[30]

Figures 1 - 4 provide additional insight into how the optimal policy varies with key parameters in the model when revaluation arises in equilibrium with strictly positive probability.

[28] It is also readily verified that the distressed investor receives a lower expected net payoff if no revaluation occurs than if revaluation occurs with probability 1.

[29] The opposite conclusion arises if $k = 80$ and $f = 500$. In this case, the optimal targeted policy sets $x_c(I) = -188.352$ and secures an expected payoff of 55.67 for the distressed investor, while the optimal proportional policy sets $z = 0$ so $x_c(I) = zI = 0$ and thereby secures an expected payoff of 54.09 for the distressed investor. The targeted policy also can be preferred when the $z \geq 0$ restriction is not binding. This is the case, for example, if $k = 5$ and $f = 150$.

[30] Many other illustrations of this more general conclusion are readily derived. For example, suppose $\alpha = .5$, $w = 0$, and k is sufficiently small that both the targeted and proportional policies are welfare improving. Define $I^{SR} \equiv I^{FB}[1 + \pi k/2f]^2$, $A \equiv \overline{x}(I^{SR}) - \frac{[1+\sqrt{\pi}]k}{1-\sqrt{\pi}}$, $\widehat{P} \equiv \frac{A+\overline{x}(I^{SR})\sqrt{\pi}}{1+\sqrt{\pi}}$, $B \equiv \frac{-\pi k}{2f}\widehat{x}(I^{SR})$, and $G \equiv -B\underline{x}(I^{SR}) + [B - I^{SR}]\widehat{P} + \overline{x}(I^{SR})[\frac{I^{SR}+\sqrt{\pi}B}{1+\sqrt{\pi}}] - \frac{k}{2f}[I^{SR} - B] + 2\pi k I^{SR}$. It can be shown that the optimal proportional policy secures greater expected welfare than the optimal targeted policy when $A = 0$ and $G > 0$. In contrast, the optimal targeted policy secures greater expected welfare than the optimal proportional policy when $A < 0$ and $G < 0$.

Unless otherwise noted, the Figures consider the basic structured setting in which $\alpha = .5$, $\beta = 10$, $\pi = .2$, $f = 1,000$, $k = 80$ and $w = .2$. (In this setting, $I^{FB} = 100$, $\widehat{x}(I^{FB}) = 200$, $\overline{x}(I^{FB}) = 1,200$ and $\underline{x}(I^{FB}) = -800$.) The optimal truncation point is presented as a fraction of the induced investment $(x_c(I)/I)$ in each of the figures. Figure 1 depicts the optimal (normalized) truncation point as the welfare weight on the non-distressed investor, w, varies. The Figure reveals that the targeted policy outperforms the proportional policy for the smallest and largest values of w, while the reverse is true for intermediate values of w.[31] Figures 2 - 4 presents corresponding findings for variations in the range of uncertainty (f), the likelihood of financial distress (π), and auditing costs (k), respectively. Together, Figures 1 - 4 illustrate the general conclusion that the preferred form of revaluation policy and the optimal truncation value can vary substantially with regulatory objectives and with the environment in which the policies are implemented.

6 Extensions of the Model

We now consider extensions of the model in order to ascertain the roles played by the key simplifying assumptions that have been maintained to this point: no voluntary disclosure, a single type of entrepreneur, observable investment, and risk neutrality. To make most apparent the importance of the maintained assumptions, we focus the ensuing analysis on policies that maximize expected (unweighted) surplus. (Recall from Proposition 2 that no meaningful role for revaluation arises in this setting under the assumptions maintained above.) For simplicity, we also emphasize the targeted revaluation policy, where appropriate.

6.1 Voluntary Disclosure of (Audited) Asset Value

To begin, consider the possibility of voluntary audited disclosure. To do so most simply, we maintain all the features of the structured basic setting except auditors are now assumed able to verify accurately at cost k the value of any asset, regardless of its underlying value.

In this setting, if an entrepreneur chooses to do so, he can voluntarily disclose the expected value of his asset via audited revaluation of the asset at personal cost k. A distressed entrepreneur will undertake voluntary revaluation if the revised estimated value of his asset exceeds the equilibrium price for non-certified assets by more than the cost of revaluation.

[31] When w is sufficiently close to 1, it is optimal to induce little or no revaluation. (Recall Proposition 2.) Revaluation can be eliminated in the present setting by a targeted policy with $x_c < 0$, but cannot be eliminated with a proportional policy (since $z \geq 0$). When w is sufficiently close to 0, the targeted policy can induce considerable revaluation while avoiding the under-investment that arises under the proportional policy when $z > 1$. (Recall Proposition 5.)

Such voluntary revaluation is profitable because it allows the entrepreneur to increase the revenue from the sale of his relatively valuable asset by more than the cost of revaluation.

In contrast, a non-distressed entrepreneur will never undertake (disclosure via) revaluation in this setting because revaluation generates a net expected payoff that is k below the expected net payoff from retaining the asset. The non-distressed entrepreneur will continue to opportunistically sell his asset with book value I whenever its privately observed value is between x_c and the prevailing price for assets with book value I.[32]

Proposition 9 considers the impact of voluntary (revaluation) disclosure when no mandatory revaluation is imposed. The proposition reveals that the opportunity to voluntarily revalue assets can be detrimental for investors. When the cost of voluntary revaluation is sufficiently low, investors will find it profitable to revalue assets with sufficiently high value. By removing the high-value assets from the pool of assets with book value I, such voluntary certification reduces the expected value, and thus the equilibrium price, of an asset with book value I. The combination of revaluation costs for high-value assets and a lower price for assets with book value I causes aggregate expected surplus to decline. Thus, investors would be better off *ex ante* if they could credibly promise to never avail themselves of the opportunity to voluntarily revalue assets.[33]

Proposition 9. *Suppose entrepreneurs can voluntarily revalue their assets at cost* $k < \frac{2f}{1+\sqrt{\pi}}$ *in the structured basic setting, but no mandatory certification is imposed. Then entrepreneurs will voluntarily revalue assets with expected values above* $\underline{x}(I^{FB}) + k\left[1+\sqrt{\pi}\right]$*, undertake the first-best level of investment* (I^{FB})*,*[34] *and achieve expected surplus* $\hat{x}(I^{FB}) - I^{FB} - \frac{\pi k}{2f}\left[2f - k\left(1+\sqrt{\pi}\right)\right] < V^{FB}$*.*

Conceivably, mandatory revaluation of low-value assets could increase expected surplus in this setting with voluntary revaluation. Mandatory revaluation would both raise the equilibrium price of assets with book value I and reduce the incidence of (costly) voluntary revaluation. However, mandatory revaluation only reduces voluntary revaluation costs by

[32] We continue to assume an entrepreneur who chooses to sell an asset with actual value below x_c must certify the value of the asset (at personal cost k).

[33] Consider FAS 2.

[34] Investment below the first-best level might be anticipated in this setting because increased investment increases the likelihood of high asset values and the associated cost of voluntary revaluation. However, there is an offsetting benefit of increased investment in this setting. Increased investment reduces the likelihood of low asset values and the associated low price for non-revalued assets. This price is particularly low when voluntary revaluation removes the highest-value assets from the market.

introducing corresponding mandatory revaluation costs. Furthermore, although any diminution of the lemons problem can benefit distressed entrepreneurs, it can harm non-distressed entrepreneurs. On balance, as Proposition 10 reports, mandatory revaluation affects neither the entrepreneur's investment level nor his expected net payoff in this setting.

Proposition 10. *If entrepreneurs can voluntarily revalue their assets at cost $k < \frac{2f}{1+\sqrt{\pi}}$ in the structured basic setting, non-trivial mandatory revaluation affects neither the entrepreneur's investment level (I^{FB}) nor his expected surplus.*

The "irrelevance" of mandatory revaluation suggested by Proposition 10 should be interpreted with caution because the conclusion reflects all of the maintained simplifying assumptions, including constant revaluation costs. If high asset values are significantly more costly to audit than low asset values, voluntary revaluation of high-value assets generally will not exactly offset mandated revaluation of low-value assets. Consequently, a meaningful role for non-trivial revaluation mandates can emerge even when voluntary revaluation is feasible in this setting. The more robust conclusion is that the design of asset revaluation regulations can become more complex and more subtle when voluntary revaluation is a viable option for investors, and must deal with subgame temptations to engage in ex ante inefficient voluntary disclosure via the revaluation option.

6.2 Unobserved Investor Type

To this point, we have assumed all entrepreneurs are identical and the productivity of each entrepreneur's investment is common knowledge. We now consider the possibility that entrepreneurs differ and each entrepreneur is privately informed about the expected payoff from his investment.

For simplicity, suppose each entrepreneur is either a high (H) or a low productivity (L) type. The present value of the future cash flows from investment I by a type $i \in \{L, H\}$ investor is $x_i(I) = \frac{\beta_i}{\alpha} I^\alpha + \mu + \varepsilon$, where $\beta_H > \beta_L$, and μ and ε are the realizations of zero-mean independent random variables $\widetilde{\mu}$ and $\widetilde{\varepsilon}$. The random variable $\widetilde{\mu}$ is again uniformly distributed on $[-f,\ f]$ and $\widetilde{\varepsilon}$ has unbounded support (to preclude perfect inference of the entrepreneur's type from the realized expected payoff from investment).

Recall the surplus-maximizing revaluation policy in the basic setting effectively imposes no revaluation requirement (as in Proposition 2). Absent any such requirement, all (identical) entrepreneurs undertake the first-best investment level and the equilibrium price for assets with book value $v = I^{FB}$ is indicated in (11) of Lemma 1). The parallel outcome in the present setting would entail first-best investment by both investor types

($I_i^{FB} \equiv \arg\max_I\{\frac{\beta_i}{\alpha}I^\alpha - I\}$ for $i = L, H$) and the equilibrium prices for assets with book value I_i^{FB} is:

$$P(I_i^{FB}, I_i^{FB}) = \widehat{x}(I_i^{FB}) - f\left[\frac{1-\sqrt{\pi}}{1+\sqrt{\pi}}\right], \text{ where } \widehat{x}(I_i^{FB}) = \frac{\beta_i}{\alpha}(I_i^{FB})^\alpha, \text{ for } i = L, H. \tag{12}$$

This outcome may not arise in the absence of revaluation mandates in the present setting, though. L types may undertake investment I_H^{FB} in order to masquerade as H types and thereby effectively exaggerate the expected value of their assets.

The equilibrium ability and incentive of L types to masquerade as H types depends in part on the inferences drawn from off-equilibrium behavior. Absent any asset revaluation, asset buyers in the resale market are assumed to believe any investment at or above I_H^{FB} was undertaken by H types, while any investment below I_H^{FB} was undertaken by L types. Further assume

$$\widehat{x}_H(I) - \widehat{x}_L(I) < \frac{2f}{1+\sqrt{\pi}} \text{ for all } I \geq I_H^{FB}, \tag{13}$$

so that the difference in expected payoff from investment according to the investor's type is not too pronounced. In addition, for convenience, suppose $\alpha = 0.5$, $f = 2\beta_H^2$, and β_L is sufficiently close to zero, so the L type is productive but much less productive than the H type. These conditions ensure that L types will either invest I_L^{FB} or I_H^{FB} in this setting with unknown investor type.

Lemma 2. *If no revaluation mandate is imposed in the setting with unknown investor types: (i) a separating equilibrium (with $I_L = I_L^{FB} < I_H^{FB} = I_H$) arises when π is sufficiently close to zero; whereas (ii) a pooling equilibrium (with $I_L = I_H = I_H^{FB}$) arises when π is sufficiently close to unity.*

When π is close to zero, investors are unlikely to become distressed. The prevalence of non-distressed investors creates a severe lemons problem that leads to a low price for assets with book value I_H^{FB}. In light of the meager expected payoff L types anticipate from undertaking investment I_H^{FB}, they prefer to reduce their investment cost by undertaking the lower investment, I_L^{FB}. In contrast, when π is close to unity, few opportunistic non-distressed entrepreneurs are present, and so the market price for assets with book value I_H^{FB} will be relatively high. This relatively high price induces L types to invest I_H^{FB}, leading to the pooling equilibrium identified in conclusion (ii) of Lemma 2.

When pooling would otherwise arise, revaluation requirements can increase expected surplus, as Proposition 11 reports. The proposition considers a revaluation policy that specifies revaluation thresholds ($x_c(\cdot)$) that vary (nonlinearly) with the observed investment level, in effect employing the targeted policy with two distinct asset scale categories. This flexibility in the revaluation policy is valuable in the presence of asymmetric knowledge of both the *ex ante* and the *ex post* expected payoff from investment.

Proposition 11. *Suppose pooling would arise in the setting with unknown investor types in the absence of any revaluation requirement. Then when audit costs k are sufficiently large, a surplus-maximizing revaluation policy sets $x_c(I_L^{FB}) << 0$, $x_c(I_L^{FB}) = \widehat{x}_H(I_H^{FB}) - f$, and $x_c(I)$ arbitrarily high for $I \notin \{I_L^{FB},\, I_H^{FB}\}$. In equilibrium, separation occurs, first-best investment and surplus obtain, and no revaluation ever occurs.*

The revaluation policy identified in Proposition 11 relieves an investor of all revaluation obligations if he invests I_L^{FB}. In contrast, entrepreneurs who invest I_H^{FB} are required to revalue assets with updated value below $\widehat{x}_H(I_H^{FB}) - f$. This requirement imposes revaluation costs (when $E[x|I_H^{FB}, \mu] \in [\widehat{x}_L(I_H^{FB}) - f,\, \widehat{x}_H(I_H^{FB}) - f))$ on L types who invest I_H^{FB} without imposing any corresponding revaluation costs on H types who invest I_H^{FB}. H types are certain to avoid revaluation costs in this setting because the (interim) expected value of an asset derived from investment I_H^{FB} by an H type is never below $\widehat{x}_H(I_H^{FB}) - f$ (since $\mu \geq -f$). When revaluation costs k are sufficiently large, the policy described in Proposition 11 reduces the expected payoff the L type anticipates from undertaking investment I_H^{FB} below the corresponding expected payoff from investing I_L^{FB}. Consequently, the policy will induce a separating equilibrium in which first-best investments are undertaken, the first-best level of expected surplus is achieved, and, in equilibrium, revaluation never occurs.

The uniform distribution for $\widetilde{\mu}$ and the corresponding moving support for $E[x|I, \mu]$ in this setting with unknown investor type admit a revaluation policy that secures the first-best level of expected surplus. More generally, revaluation policies like the one described in Proposition 11 may impose revaluation costs on H types in equilibrium and thereby fail to secure the first-best level of surplus. However, the key features of the revaluation in Proposition 11 will persist more generally. By imposing more stringent revaluation requirements on assets with large underlying investment than on assets with small underlying investment, low-productivity entrepreneurs can be deterred from over-investing in an attempt to exaggerate the value of their assets. Notice that the more stringent revaluation policy imposed on large investments does not reflect any special concern with the welfare of "small" investors here. Instead, the policy reflects the gain in total surplus that arises when low-productivity types are deterred from over-investing in an attempt to exaggerate the value of their assets.

6.3 Unobserved Investment

We have also assumed investment is observable (as in, say, Newman, Patterson and Smith [2005] but not in, say, Dye and Sridhar [2004] or Liang and Wen [2006]). In principle, the critical investment in question could entail effort that is inherently difficult to measure. To briefly consider this possibility formally, return to the basic setting but now suppose the scale of investment is privately known by the entrepreneur and cannot be communicated. Proposition 12 reports that entrepreneurs will under-invest when the scale of their investment is unobservable.

Proposition 12. *If investment is not observed and no revaluation is possible in the basic setting, equilibrium investment will be below first-best investment.*

Proposition 12 implies that for k sufficiently small, surplus maximizing regulation will induce non-trivial revaluation in order to increase equilibrium investment and the resulting expected surplus. Therefore, revaluation policies can enhance total surplus when investment is unobservable even though they do not do so in the basic setting where investment is observable.

6.4 Risk Aversion

To this point, we have also assumed all entrepreneurs are risk neutral and asset resale pricing reflects risk neutrality. Return, now, to the basic setting (where investment is observed publicly) but suppose entrepreneurs are strictly risk averse, while pricing continues to reflect risk neutrality. An entrepreneur's expected cash flow from investment I in this setting is $\hat{x}(I) - I$ in the absence of any revaluation requirements. If revaluation were both costless and always mandated in this setting, an entrepreneur would receive the same expected cash flow from the same investment, I. However, the cash flow stream under revaluation would be more risky because it would be a mean preserving spread of the corresponding stream under no revaluation (Rothschild and Stiglitz [1970]). The same holds for any truncated revaluation requirement. Therefore, no revaluation would be imposed if the objective were to maximize the expected surplus of strictly risk averse entrepreneurs.

Mandated revaluation is not optimal here because it exposes the risk-averse asset seller to revaluation risk, a risk he does not face in the absence of mandated revaluation. Of course, asset portfolios, options and other risk-mitigating devices warrant consideration in any complete analysis of revaluation policies in such a setting.[35]

[35] A tension arises even when $k = 0$ because mandated revaluation to assist distressed entrepreneurs can

7 Conclusions

GAAP contains a bewildering array of prescriptions, ranging from seemingly principle-based to explicit rule-based mandates coupled with a seemingly inconsistent approach to valuation. While many explanations for this consistently inconsistent set of prescriptions have been offered, we have shown that virtually any element in the array of prescriptions in GAAP can surface as optimal regulatory prescription in the simplest of economic models. For example, either targeted (e.g., principle-based) or proportional (e.g., rule-based) revaluation policies can be optimal. Furthermore, the critical threshold in familiar truncation policies can assume a wide variety of optimal levels, depending upon prevailing objectives and economic forces.

This is not to say GAAP is unaffected by political gamesmanship or human foibles. Nor do we claim the regulatory objectives and economic forces exhibited in our simple model capture the bulk of objectives and forces at play in the world of financial reporting regulation. Rather, we offer a transparent setting in which the interplay between objectives and forces can be identified and in which this interplay exhibits the wide variety of GAAP policies and prescriptions.

Many avenues for research remain, including the consideration of multiple assets, alternative audit cost structures,[36] the possibility of evading revaluation regulations, and capital market tensions (e.g., Whittred and Chan [1992]). Combinations of regulatory issues, such as asset revaluation and revenue recognition, are also of considerable interest. Future research also might extend the basic models analyzed here by incorporating managerial incentives (along the lines of Kirschenheiter and Melumad (2002) and Arya and Glover (2003), for example).

increase the risk the entrepreneurs face.

[36] Notice, for example, that optimal revaluation policies may no longer mandate revaluation of only the least valuable assets if revaluation costs are lower for highly valuable assets than for less valuable assets.

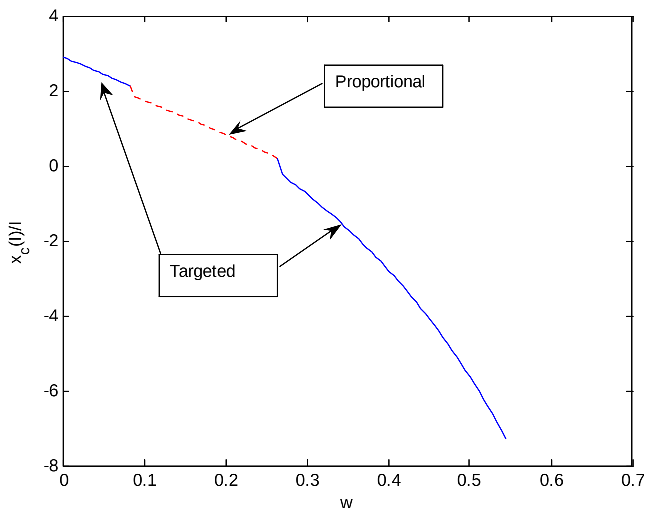

Figure 1: The Effect of w on the Optimal Revaluation Threshold $x_c(I)/I$.

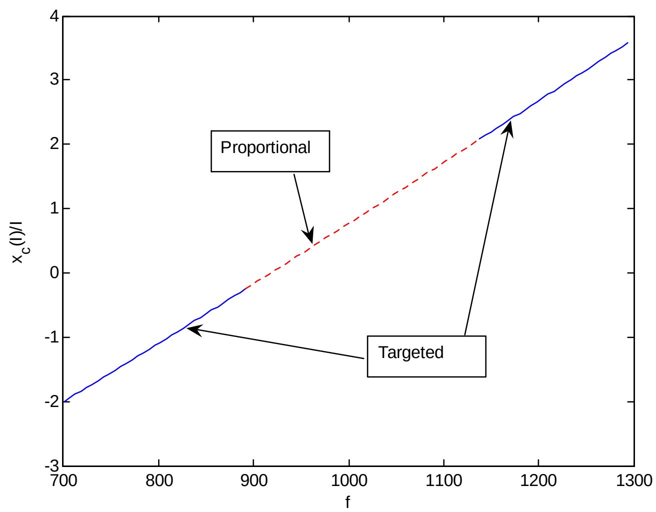

Figure 2: The Effect of f on the Optimal Revaluation Threshold $x_c(I)/I$.

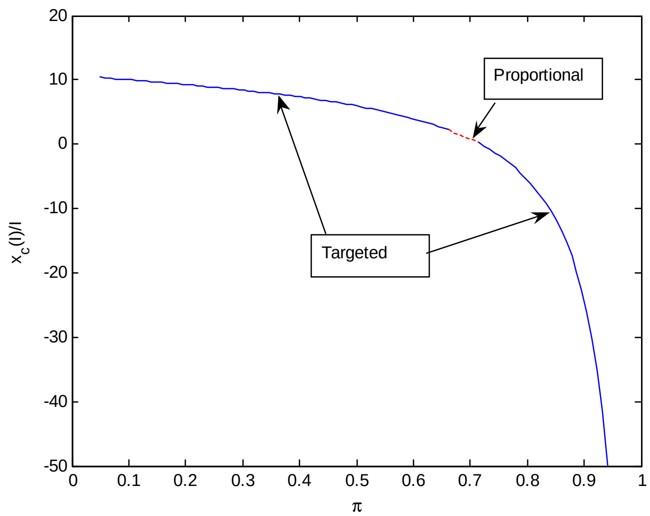

Figure 3: The Effect of π on the Optimal Revaluation Threshold $x_c(I)/I$.

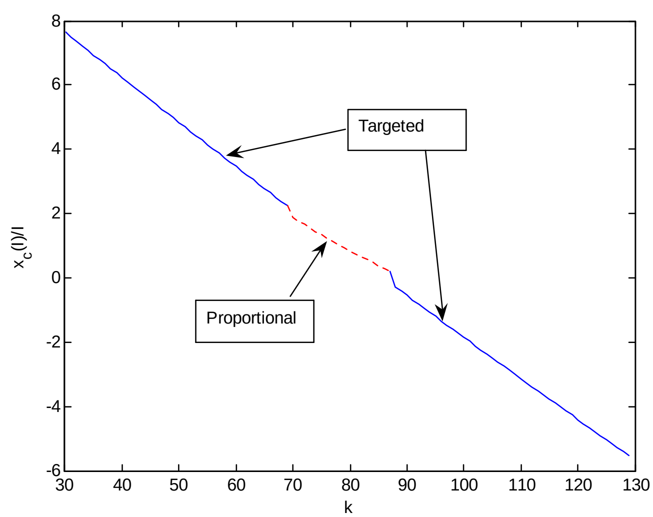

Figure 4: The Effect of k on the Optimal Revaluation Threshold $x_c(I)/I$.

Appendix

Proof of Proposition 1.

From (4) and (2) in the text:

$$P(I,I) = \hat{x}(I) + \int_{\mu} \mu h(\mu|S) d\mu < \hat{x}(I). \quad \text{(A1)}$$

The inequality in (A1) follows from (3) in the text because:

$$\int_{\mu} \mu h(\mu|S) d\mu = \int_{x_c - \hat{x}(I) \le \mu \le P(I,I) - \hat{x}(I)} \mu h(\mu) d\mu + \int_{\mu \ge P(I,I) - \hat{x}(I)} \pi \mu h(\mu) d\mu$$

$$< \int_{\mu} \mu h(\mu) d\mu = 0. \quad \blacksquare \quad \text{(A2)}$$

Proof of Proposition 2.

With no revaluation requirement, (5) in the text reduces to:

$$V(I) = \pi P(I,I) + [1-\pi] \left[\int_{\mu \le P(I,I) - \hat{x}(I)} P(I,I) h(\mu) d\mu + \int_{\mu \ge P(I,I) - \hat{x}(I)} [\hat{x}(I) + \mu] h(\mu) d\mu \right] - I$$

$$= P(I,I) \left[\pi + [1-\pi] \int_{\mu \le P(I,I) - \hat{x}(I)} h(\mu) d\mu \right] + [1-\pi] \int_{\mu \ge P(I,I) - \hat{x}(I)} [\hat{x}(I) + \mu] h(\mu) d\mu - I. \quad \text{(A3)}$$

Substituting from (A1) into (A3) provides:

$$V(I) = \left[\hat{x}(I) + \int_{\mu} \mu h(\mu|S) d\mu \right] \left[\pi + [1-\pi] \int_{\mu \le P(I,I) - \hat{x}(I)} h(\mu) d\mu \right]$$

$$+ [1-\pi] \int_{\mu \ge P(I,I) - \hat{x}(I)} [\hat{x}(I) + \mu] h(\mu) d\mu - I. \quad \text{(A4)}$$

Because $h(\mu|S) = h(\mu, S) / \int_{\mu} h(\mu, S) d\mu$ and $\int_{\mu} h(\mu, S) d\mu = \pi + [1-\pi] \int_{\mu \le P(I,I) - \hat{x}(I)} h(\mu) d\mu$,

$$V(I) = \hat{x}(I) \left[\pi + [1-\pi] \int_{\mu \le P(I,I) - \hat{x}(I)} h(\mu) d\mu \right] + \int_{\mu} \mu h(\mu, S) d\mu$$

$$+ \quad [1-\pi] \int_{\mu \geq P(I,I)-\hat{x}(I)} [\hat{x}(I)+\mu] h(\mu) d\mu - I$$

$$= \hat{x}(I) + \int_{\mu \leq P(I,I)-\hat{x}(I)} \mu h(\mu) d\mu + \int_{\mu \geq P(I,I)-\hat{x}(I)} \pi \mu h(\mu) d\mu$$

$$+ \quad [1-\pi] \int_{\mu \geq P(I,I)-\hat{x}(I)} \mu h(\mu) d\mu - I \quad = \quad \widehat{x}(I) - I. \tag{A5}$$

I^{FB} maximizes (A5). Also, the set $\{x_c | x_c(I^{FB}) < \widehat{x}(I^{FB}) + \mu, \forall \mu\}$ is nonempty. ■

Proof of Proposition 3.

From (1) and (4) in the text, if $x_c(I) \geq \widehat{x}(I) + \mu$ for all μ, all assets offered for sale are revalued and sold at price $P(v, I) = \widehat{x}(I) + \mu$. For a distressed entrepreneur, the expected payoff from investment I is $\widehat{x}(I) - I - k$. The expected payoff from investment I for a non-distressed entrepreneur is $\widehat{x}(I) - I$, because a non-distressed entrepreneur will never sell his asset, since a sale entails audit cost k. Thus, expected welfare is:

$$W^{FR}(x_c) \;=\; \pi[\widehat{x}(I) - I - k] + w[1-\pi][\widehat{x}(I) - I]. \tag{A6}$$

Notice that I^{FB} maximizes (A6). When $k = 0$, (A6) becomes:

$$W^{FR}(x_c)|_{k=0} \;=\; [\pi + w(1-\pi)][\widehat{x}(I) - I]. \tag{A7}$$

It is readily verified that (A7) strictly exceeds the corresponding measure when no revaluation is induced, $W^{NR}(x_c) = \pi[P(I,I) - I] + w[1-\pi][\gamma - I]$, where, from Proposition 2, $\gamma = \frac{\hat{x}(I) - \pi P(I,I)}{1-\pi} > \hat{x}(I)$, and from Proposition 1, $P(I,I) < \widehat{x}(I)$. ■

Proof of Lemma 1.

Equating $P(I,I)$ with (9) in the text provides:

$$P(I,I) \;=\; \frac{\pi\left[\overline{x}(I) - x_c(I)\right]}{2fq}\left[\frac{x_c(I) + \overline{x}(I)}{2}\right] + \frac{[1-\pi]\left[P(I,I) - x_c(I)\right]}{2fq(I)}\left[\frac{x_c(I) + P(I,I)}{2}\right]. \tag{A8}$$

Rearranging terms in (A8) provides:

$$4fqP(I,I) \;=\; \pi[\overline{x}(I) - x_c(I)][x_c(I) + \overline{x}(I)] + [1-\pi][P^2(I,I) - x_c^2(I)]. \tag{A9}$$

From (8) in the text:

$$4fq \;=\; 2\pi[\overline{x}(I) - x_c(I)] + 2[1-\pi][P(I,I) - x_c(I)]. \tag{A10}$$

Substituting (A10) into (A9) yields:

$$2\pi[\overline{x}(I) - x_c(I)]P(I,I) + 2[1-\pi][P(I,I) - x_c(I)]P(I,I)$$

$$= \; \pi[\overline{x}^2(I) - x_c^2(I)] + [1-\pi][P^2(I,I) - x_c^2(I)]. \tag{A11}$$

Rearranging terms in (A11) provides:

$$2\pi[\overline{x}(I) - x_c(I)]P(I,I) + [1-\pi][P^2(I,I) - 2P(I,I)x_c(I) + x_c^2(I)] = \; \pi[\overline{x}^2(I) - x_c^2(I)]$$

$$\Leftrightarrow \quad [1-\pi][P(I,I) - x_c(I)]^2 + 2\pi[\overline{x}(I) - x_c(I)][P(I,I) - x_c(I)] + 2\pi[\overline{x}(I) - x_c(I)]x_c(I)$$

$$= \; \pi[\overline{x}^2(I) - x_c^2(I)]$$

$$\Leftrightarrow \quad [1-\pi][P(I,I) - x_c(I)]^2 + 2\pi[\overline{x}(I) - x_c(I)][P(I,I) - x_c(I)] - \pi[\overline{x}(I) - x_c(I)]^2 = 0. \tag{A12}$$

Solving (A12) provides:

$$P(I,I) - x_c(I) = \frac{-2\pi[\overline{x}(I) - x_c(I)] \pm 2\sqrt{\pi}[\overline{x}(I) - x_c(I)]}{2[1-\pi]} = \frac{[-\pi \pm \sqrt{\pi}][\overline{x}(I) - x_c(I)]}{1-\pi}, \text{ or}$$

$$P(I,I) = x_c(I) + \frac{\sqrt{\pi}[\overline{x} - x_c(I)]}{1+\sqrt{\pi}} \quad \text{or} \quad P(I,I) = x_c(I) - \frac{\sqrt{\pi}[\overline{x}(I) - x_c(I)]}{1-\sqrt{\pi}}. \tag{A13}$$

The second possibility in (A13) cannot hold because the expected value of the non-revalued asset cannot be below $x_c(I)$. Substituting $x_c(I) = \underline{x}(I)$ into the first expression in (A13) provides (11) in the text. ■

Proofs of Proposition 4 and 5.

The entrepreneur's expected payoff from investment I if revaluation is induced with strictly positive probability is:

$$V^{SR}(I; x_c(I)) \;=\; \pi\left[\int_{\underline{x}(I)}^{x_c(I)} [v-k]\phi(v)dv + \int_{x_c(I)}^{\overline{x}(I)} P(I,I)\phi(v)dv - I\right]$$

$$+ \ [1-\pi]\left[\int_{\underline{x}(I)}^{x_c(I)} v\phi(v)dv + \int_{x_c(I)}^{P(I)} P(I,I)\phi(v)dv + \int_{P(I)}^{\overline{x}(I)} v\phi(v)dv - I\right]. \tag{A14}$$

Substituting from (10) in the text into (A14) provides:

$$V^{SR}(I; x_c(I)) \ = \ \widehat{x}(I) - I - \frac{\pi k[x_c(I) - \underline{x}(I)]}{2f}. \tag{A15}$$

To prove Proposition 4, where revaluation is a nontrivial possibility, notice that the entrepreneur takes $x_c(I) = x_c$ as given and chooses I to $Maximize_I \ V^{SR}(\cdot)$ *subject to* $\widehat{x}(I) - f \leq x_c \leq \widehat{x}(I) + f$, where $V^{SR}(\cdot)$ is as specified in (A15). The solution to this problem varies according to whether I is such that x_c is interior or at a boundary of the possible realizations of v. If $I = I^{SR}$ is indeed such that x_c is interior, then from (A15):

$$\frac{dV^{SR}(\cdot)}{dI} \ = \ \widehat{x}'(I^{SR})\left[1 + \frac{\pi k}{2f}\right] - 1 \ = \ 0, \ \text{which implies} \tag{A16}$$

$$\widehat{x}'(I^{SR}) \ = \ \frac{1}{1 + \frac{\pi k}{2f}} \ \ \text{and} \ \ I^{SR} \ = \ \left[\frac{\beta[2f + \pi k]}{2f}\right]^{\frac{1}{1-\alpha}}. \tag{A17}$$

Therefore, expected surplus is:

$$V^{SR}(\cdot) = \left\{ \begin{array}{ll} \widehat{x}(I^{SR}) - I^{SR} - \frac{\pi k}{2f}[x_c(I^{SR}) - \underline{x}(I^{SR})] & if \ \underline{x}(I^{SR}) \leq x_c \leq \overline{x}(I^{SR}) \\ \widehat{x}(I^{SRL}) - I^{SRL}, \ where \ I^{SRL} = [\frac{\alpha}{\beta}(x_c + f)]^{\frac{1}{\alpha}} & if \ x_c \leq \widehat{x}(I^{SRL}) - f \\ \widehat{x}(I^{SRU}) - I^{SRU} - \pi k, \ where \ I^{SRU} = [\frac{\alpha}{\beta}(x_c - f)]^{\frac{1}{\alpha}} & if \ x_c \geq \widehat{x}(I^{SRU}) + f \end{array} \right\}. \tag{A18}$$

The optimal investment level is derived by comparing the expected surplus in these three regions. The entrepreneur will prefer I^{SR} to I^{SRL} if:

$$\begin{aligned} x_c + f - I^{SRL} \ &\leq \ \widehat{x}(I^{SR}) - I^{SR} - \left[\frac{\pi k[x_c - \underline{x}(I^{SR})]}{2f}\right] \\ \Leftrightarrow \quad f - \widehat{x}(I^{SR}) + I^{SR} - \frac{\pi k \underline{x}(I^{SR})}{2f} \ &\leq \ I^{SRL} - x_c - \frac{\pi k x_c}{2f} \\ \Leftrightarrow \quad I^{SR} - \left[1 + \frac{\pi k}{2f}\right]\underline{x}(I^{SR}) \ &\leq \ I^{SRL} - \left[1 + \frac{\pi k}{2f}\right]x_c. \end{aligned} \tag{A19}$$

(A19) holds if the following is true:

$$x_c \ \geq \ \frac{\beta}{\alpha}\left[\frac{\beta[2f + \pi k]}{2f}\right]^{\frac{\alpha}{1-\alpha}} - f. \tag{A20}$$

(A19) holds as an equality when (A20) holds as an equality, and (A20) is consistent with $x_c \geq \underline{x}(I^{SR})$. Notice that the left hand side of (A19) is independent of x_c. The right hand side of (A19) is nondecreasing in x_c because (A20) implies:

$$\frac{d}{dx_c}\left(I^{SRL} - [1 + \frac{\pi k}{2f}]x_c\right) \;\geq\; 0. \tag{A21}$$

The entrepreneur will prefer I^{SR} to I^{SRU} if:

$$x_c - f - I^{SRU} - \pi k \;\leq\; \widehat{x}(I^{SR}) - I^{SR} - \left[\frac{\pi k[x_c - \underline{x}(I^{SR})]}{2f}\right]$$

$$\Leftrightarrow \qquad I^{SR} - \widehat{x}(I^{SR}) - \frac{\pi k\underline{x}(I^{SR})}{2f} - f - \pi k \;\leq\; I^{SRU} - \left[1 + \frac{\pi k}{2f}\right] x_c$$

$$\Leftrightarrow \qquad I^{SR} - \left[1 + \frac{\pi k}{2f}\right] \underline{x}(I^{SR}) \;\leq\; [2f + \pi k] + I^{SRU} - \left[1 + \frac{\pi k}{2f}\right] x_c. \tag{A22}$$

(A22) holds if the following is true:

$$x_c \;\leq\; \frac{\beta}{\alpha}\left[\frac{\beta[2f + \pi k]}{2f}\right]^{\frac{\alpha}{1-\alpha}} + f. \tag{A23}$$

(A22) holds as an equality if (A23) holds as an equality, and (A23) is consistent with $x_c \leq \overline{x}(I^{SR})$. Also notice that (A23) ensures the following:

$$\frac{d}{dx_c}\left([2f + \pi k] + \left[\frac{\alpha}{\beta}(x_c - f)\right]^{\frac{1}{\alpha}} - \left[1 + \frac{\pi k}{2f}\right] x_c\right) \;\leq\; 0. \tag{A24}$$

The right hand side of (A22) is nonincreasing in x_c. Also $I^{SR} > \; I^{FB} = \; \beta^{\frac{1}{1-\alpha}}$.

To prove Proposition 5, given $x_c(I) = zI$, the entrepreneur chooses I to $Maximize_I \; V(I;z)$ *subject to* $\widehat{x}(I) - f \;\leq\; zI \;\leq\; \widehat{x}(I) + f$, where expected surplus, $V(I;z)$, is specified by (A15). Differentiation of (A15) provides:

$$\frac{dV(I;z)}{dI} \;=\; \widehat{x}'(I) - 1 - \frac{\pi k[z - \widehat{x}'(I)]}{2f} \;=\; \widehat{x}'\left[1 + \frac{\pi k}{2f}\right] - 1 - \frac{\pi kz}{2f} \;=\; 0. \tag{A25}$$

$$\Leftrightarrow \qquad \widehat{x}'(I) = \frac{1 + \frac{\pi kz}{2f}}{1 + \frac{\pi k}{2f}} \;=\; \frac{2f + \pi kz}{2f + \pi k} \;=\; \beta I^{\alpha - 1}. \tag{A26}$$

$$\Leftrightarrow \qquad I \;=\; [\frac{2f + \pi kz}{\beta(2f + \pi k)}]^{\frac{1}{\alpha - 1}} \;=\; [\frac{\beta(2f + \pi k)}{2f + \pi kz}]^{\frac{1}{1-\alpha}}. \tag{A27}$$

Also, (A27) satisfies the constraint $\widehat{x}(I) - f \leq zI \leq \widehat{x}(I) + f$ in some neighborhood of $z = 1$. ■

Proof of Proposition 6.

When $x_c(I) = x_c$, the regulator's problem is:

$$\underset{x_c}{Maximize}\ W(x_c) \quad subject\ to \quad I(x_c) = \arg\max_I \ V(I; x_c). \tag{A28}$$

The proof follows from the following five conclusions (which are proved following their statement).

Conclusion 1: If the regulator sets x_c sufficiently low, no revaluation is induced and expected welfare is:

$$W^{NR}(x_c) \ = \ [\pi + w(1-\pi)][\widehat{x}(I^{FB}) - I^{FB}] - \frac{[1-w][1-\sqrt{\pi}]\pi f}{1+\sqrt{\pi}}. \tag{A29}$$

Because $P(I, I) = \widehat{x}(I) - \frac{[1-\sqrt{\pi}]f}{1+\sqrt{\pi}}$, the entrepreneur's expected surplus given investment I in this case is:

$$\begin{aligned} V^{NR}(I; x_c) \ &= \ \pi P(I, I) + [1-\pi]\left[\int_{\underline{x}(I)}^{P(I,I)} P(I, I)\phi(v)dv + \int_{P(I,I)}^{\overline{x}(I)} v\phi(v)dv\right] - I \\ &= \ \widehat{x}(I) - I \qquad if \ \ x_c \leq \underline{x}(I). \end{aligned} \tag{A30}$$

(A30) can be written as:

$$V^{NR}(I; x_c) = \left\{ \begin{array}{ll} \widehat{x}(I^{FB}) - I^{FB}, & if\ x_c \leq \underline{x}(I^{FB}) \\ x_c + f - I,\ where\ I = [\frac{\alpha}{\beta}(x_c + f)]^{\frac{1}{\alpha}}, & otherwise \end{array} \right\}. \tag{A31}$$

When x_c is set sufficiently low, the first-best investment is induced and expected welfare is:

$$W^{NR}(x_c) \ = \ \pi[P(I, I) - I] + w[1-\pi]\left[\int_{\underline{x}}^{P} P(I, I)\phi(v)dv + \int_{P}^{\overline{x}} v\phi(v)dv - I\right]. \tag{A32}$$

Straightforward algebraic manipulation provides (A29).

Conclusion 2: If the regulator sets x_c sufficiently high, revaluation is always induced and expected welfare is:

$$W^{FR}(x_c) = [\pi + w(1-\pi)][\widehat{x}(I^{FB}) - I^{FB}] - \pi k. \tag{A33}$$

Using logic analogous to that employed in the proof of Conclusion 1, expected surplus in the present case is readily shown to be:

$$V^{FR}(x_c) = \left\{ \begin{array}{ll} \widehat{x}^{FB} - I^{FB} - \pi k, & if\ x_c \geq \overline{x}(I^{FB}) \\ x_c + f - I - \pi k,\ where\ I = [\frac{\alpha}{\beta}(x_c - f)]^{1/\alpha}, & otherwise \end{array} \right\}. \tag{A34}$$

The first-best level of investment is induced in this case, given x_c sufficiently high..

Conclusion 3: If the regulator sets $x_c > \underline{x}(I^{FB})$, then for $k \in (0, \frac{2f[1-\sqrt{\pi}][1-w]}{1+\sqrt{\pi}}]$, expected welfare will be:

$$W^{SR}(x_c) \ = \ [\pi + w(1-\pi)][\widehat{x}(I^{SR}) - I^{SR}] + \frac{\pi}{2f}\left[-2fk + \frac{[1+\sqrt{\pi}]k^2}{2[1-\sqrt{\pi}][1-w]}\right]. \tag{A35}$$

The optimal investment I^{SR} is characterized by (A17) in the proof of Proposition 4.

Expected welfare of a given x_c is:

$$\begin{aligned} W^{SR}(x_c) = \ & \pi\left[\int_{\underline{x}}^{x_c}[v-k]\phi(v)dv + \int_{x_c}^{\overline{x}} P\phi(v)dv - I\right] \\ + \ & w[1-\pi]\left[\int_{\underline{x}}^{x_c} v\phi(v)dv + \int_{x_c}^{P} P\phi(v)dv + \int_{P}^{\overline{x}} v\phi(v)dv - I\right], \end{aligned} \tag{A36}$$

where P is as specified in (10) in the text. Performing the integration in (A36) and rearranging terms provides:

$$\begin{aligned} W^{SR}(x_c) \ = \ & \frac{\pi}{2f}\left[\frac{1}{2}(x_c^2 - \underline{x}^2) - k(x_c - \underline{x}) + P(\overline{x} - x_c)\right] \\ + \ & \frac{w(1-\pi)}{2f}\left[\frac{1}{2}(x_c^2 - \underline{x}^2) + P(P - x_c) + \frac{1}{2}(\overline{x}^2 - P^2)\right] - [\pi + w(1-\pi)]I \\ = \ & \frac{\pi}{2f}\left[\frac{1}{2}(x_c^2 - \underline{x}^2) - k(x_c - \underline{x}) + P(\overline{x} - x_c)\right] \\ + \ & \frac{w(1-\pi)}{2f}\left[\frac{1}{2}(x_c^2 - \underline{x}^2) - Px_c + \frac{1}{2}(\overline{x}^2 + P^2)\right] - [\pi + w(1-\pi)]I. \end{aligned} \tag{A37}$$

As shown in the proof of Proposition 4, three outcomes are conceivable in this case. If (A20) is violated, $x_c \leq \frac{\beta}{\alpha}\left[\frac{\beta[2f+\pi k]}{2f}\right]^{\frac{\alpha}{1-\alpha}} - f$, so precisely the minimum investment required to avoid the audit cost is induced. Investment increases with x_c. Similarly, if (A23) is violated, $x_c \geq \frac{\beta}{\alpha}\left[\frac{\beta[2f+\pi k]}{2f}\right]^{\frac{\alpha}{1-\alpha}} + f$ and revaluation is always induced. Investment increases with x_c in this case. Otherwise, revaluation is induced with strictly positive probability less than 1 and the investment does not vary with x_c. These three regions are now considered in turn.

In the region where investment increases with x_c and $x_c \leq \frac{\beta}{\alpha}\left[\frac{\beta[2f+\pi k]}{2f}\right]^{\frac{\alpha}{1-\alpha}} - f$, substituting $\underline{x} = x_c$, $\overline{x} = x_c + 2f$ and $P(I,I) = x_c + \frac{\sqrt{\pi}[\overline{x}-x_c]}{1+\sqrt{\pi}}$ into (A37) provides:

$$\begin{aligned} W^{SR}(x_c) = {} & \pi\left[x_c + \frac{2\sqrt{\pi}f}{1+\sqrt{\pi}}\right] + \frac{w[1-\pi]}{2f} - x_c\left[x_c + \frac{2\sqrt{\pi}f}{1+\sqrt{\pi}}\right] \\ & + \frac{1}{2}[x_c+2f]^2 + \frac{1}{2}\left[x_c + \frac{2\sqrt{\pi}f}{1+\sqrt{\pi}}\right]^2 - [\pi + w(1-\pi)]I \\ = {} & \pi\left[x_c + \frac{2\sqrt{\pi}f}{1+\sqrt{\pi}}\right] + w[1-\pi]\left[f + x_c + \frac{\pi f}{(1+\sqrt{\pi})^2}\right] - [\pi + w(1-\pi)]I \\ = {} & [\pi + w(1-\pi)][x_c - I] + \frac{2\pi\sqrt{\pi}f}{1+\sqrt{\pi}} + w[1-\pi]\left[f + \frac{\pi f}{(1+\sqrt{\pi})^2}\right], \end{aligned} \tag{A38}$$

where $I = I^{SRL} = [\frac{\alpha}{\beta}(x_c+f)]^{\frac{1}{\alpha}}$. Differentiating (A38) with respect to x_c provides:

$$\frac{dW^{SR}(x_c)}{dx_c} = [\pi + w(1-\pi)][1 - \frac{dI^{SRL}}{dx_c}] = [\pi + w(1-\pi)]\left[1 - \frac{1}{\beta}[\frac{\alpha}{\beta}(x_c+f)]^{\frac{1-\alpha}{\alpha}}\right]. \tag{A39}$$

Notice that $\frac{dW^{SR}(x_c)}{dx_c} = 0$ if $x_c = \underline{x}(I^{FB})$. When $x_c = \underline{x}(I^{FB}) + \epsilon$ for $\epsilon > 0$, $\frac{dW^{SR}(x_c)}{dx_c} < 0$. The expression in (A38) decreases with x_c for $\underline{x}(I^{FB}) < x_c \leq \frac{\beta}{\alpha}\left[\frac{\beta[2f+\pi k]}{2f}\right]^{\frac{\alpha}{1-\alpha}} - f$. Thus, $x_c > \underline{x}(I^{SRL})$. The expected welfare, given $x_c = \underline{x}(I^{FB}) < \frac{\beta}{\alpha}\left[\frac{\beta[2f+\pi k]}{2f}\right]^{\frac{\alpha}{1-\alpha}} - f$ and $I^{SRL} = I^{FB}$, coincides with (A29).

In the region where investment increases with x_c and $x_c \geq \frac{\beta}{\alpha}\left[\frac{\beta[2f+\pi k]}{2f}\right]^{\frac{\alpha}{1-\alpha}} + f$, substituting $\overline{x} = x_c$, $\underline{x} = x_c - 2f$ and $P(I,I) = x_c$ into (A37) provides:

$$\begin{aligned} W^{SR}(x_c) &= \pi[x_c - f - k] + w(1-\pi)(x_c - f) - [\pi + w(1-\pi)]I \\ &= [\pi + w(1-\pi)][x_c - f - I] - \pi k, \end{aligned} \tag{A40}$$

where $I = I^{SRU} = [\frac{\alpha}{\beta}(x_c-f)]^{\frac{1}{\alpha}}$. Differentiating (A40) with respect to x_c provides:

$$\begin{aligned} \frac{dW^{SR}(x_c)}{dx_c} &= [\pi + w(1-\pi)]\left[1 - \frac{dI^{SRU}}{dx_c}\right] \\ &= [\pi + w(1-\pi)]\left[1 - \frac{1}{\beta}[\frac{\alpha}{\beta}(x_c-f)]^{\frac{1-\alpha}{\alpha}}\right] < 0. \end{aligned} \tag{A41}$$

The inequality in (A41) holds because $x_c \geq \frac{\beta}{\alpha}[\frac{\beta(2f+\pi k)}{2f}]^{\frac{\alpha}{1-\alpha}} + f$. Thus, $x_c < \overline{x}(I^{SRU})$.

In the region where investment does not vary with x_c, the equilibrium price is $P(I,I) = \frac{x_c + \sqrt{\pi}\overline{x}}{1+\sqrt{\pi}}$.

Consequently, $\frac{dP(I,I)}{dx_c} = \frac{1}{1+\sqrt{\pi}}$. Investment is as specified in (A17). Differentiating (A37) provides:

$$\begin{aligned}
\frac{dW^{SR}(x_c)}{dx_c} &= \frac{\pi}{2f}\left[x_c - k + \frac{\overline{x}-x_c}{1+\sqrt{\pi}} - \frac{x_c+\sqrt{\pi}\overline{x}}{1+\sqrt{\pi}}\right] \\
&\quad + \frac{w[1-\pi]}{2f}\left[x_c - \frac{x_c+\sqrt{\pi}\overline{x}}{1+\sqrt{\pi}} - \frac{x_c}{1+\sqrt{\pi}} + \frac{x_c+\sqrt{\pi}\overline{x}}{(1+\sqrt{\pi})^2}\right] \\
&= \frac{\pi}{2f}\left[-k + \frac{[1-w]\,[1-\sqrt{\pi}]\,[\overline{x}-x_c]}{1+\sqrt{\pi}}\right]. \qquad \text{(A42)}
\end{aligned}$$

Setting the expression in (A42) equal to 0 identifies the welfare-maximizing threshold:

$$x_c = \overline{x}(I^{SR}) - \frac{[1+\sqrt{\pi}]k}{[1-\sqrt{\pi}][1-w]}. \qquad \text{(A43)}$$

(A43) implies $x_c \geq \underline{x}(I^{SR})$ if and only if $k \leq \frac{2f[1-\sqrt{\pi}][1-w]}{1+\sqrt{\pi}}$. To derive expected welfare in this case, let $S \equiv \frac{k}{[1-\sqrt{\pi}][1-w]}$ and employ (A43) to obtain:

$$P(I,I) = \frac{\overline{x} - \frac{[1+\sqrt{\pi}]k}{[1-\sqrt{\pi}][1-w]} + \sqrt{\pi}\overline{x}}{1+\sqrt{\pi}} = \overline{x} - \frac{k}{[1-\sqrt{\pi}][1-w]} = \overline{x} - S, \text{ and} \qquad \text{(A44)}$$

$$\begin{aligned}
W^{SR}(x_c) &= \frac{\pi}{2f}\left\{\frac{1}{2}[\overline{x}-(1+\sqrt{\pi})S]^2 - \frac{1}{2}\underline{x}^2 - k[\overline{x}-(1+\sqrt{\pi})S-\underline{x}]\right. \\
&\quad \left. +[\overline{x}-S][1+\sqrt{\pi}]S\right\} + \frac{w[1-\pi]}{2f}\left\{\frac{1}{2}[\overline{x}-(1+\sqrt{\pi})S]^2 - \frac{1}{2}\underline{x}^2\right. \\
&\quad \left. -[\overline{x}-S][\overline{x}-(1+\sqrt{\pi})S] + \frac{1}{2}\overline{x}^2 + \frac{1}{2}[\overline{x}-S]^2\right\} - [\pi+w(1-\pi)]I^{SR}.
\end{aligned}$$

$$\begin{aligned}
&= \frac{\pi}{2f}\left[2f\widehat{x} - 2fk + \frac{1}{2}[\pi-1]S^2 + [1+\sqrt{\pi}]kS\right] \\
&\quad + \frac{w[1-\pi]}{2f}\left[2f\widehat{x} + \frac{1}{2}\pi S^2\right] - [\pi+w(1-\pi)]I^{SR} \\
&= \frac{\pi}{2f}\left[2f\widehat{x} - 2fk + \frac{1}{2}[\pi-1]\frac{k^2}{[1-\sqrt{\pi}]^2[1-w]^2} + [1+\sqrt{\pi}]k\frac{k}{[1-\sqrt{\pi}][1-w]}\right] \\
&\quad + \frac{w[1-\pi]}{2f}\left[2f\widehat{x} + \frac{1}{2}\pi\frac{k^2}{[1-\sqrt{\pi}]^2[1-w]^2}\right] - [\pi+w(1-\pi)]I^{SR}. \\
\Leftrightarrow \quad W^{SR}(x_c) &= [\pi+w(1-\pi)][\widehat{x}(I^{SR}) - I^{SR}] + \frac{\pi}{2f}\left[-2fk + \frac{[1+\sqrt{\pi}]k^2}{2[1-\sqrt{\pi}][1-w]}\right]. \qquad \text{(A45)}
\end{aligned}$$

(A45) is the maximum level of expected welfare that can be achieved by inducing revaluation with strictly positive probability less than 1 and investment I^{SR} when $k \in (0, \frac{2f[1-\sqrt{\pi}][1-w]}{1+\sqrt{\pi}}]$.

Conclusion 4: There exists a $k^* \in (0, \frac{2f[1-\sqrt{\pi}][1-w]}{1+\sqrt{\pi}}]$ such that for $k \in (0, k^*]$, $W^{SR}(\cdot)$ as identified in Conclusion 3 exceeds both $W^{NR}(\cdot)$ and $W^{FR}(\cdot)$ as identified in Conclusions 1 and 2, respectively.

Comparing (A35) and (A29) provides:

$$[\pi + w(1-\pi)][\widehat{x}(I^{SR}) - I^{SR}] + \frac{\pi}{2f}\left[-2fk + \frac{[1+\sqrt{\pi}]k^2}{2[1-\sqrt{\pi}][1-w]}\right]$$

$$- [\pi + w(1-\pi)][\widehat{x}(I^{FB}) - I^{FB}] + \frac{[1-w][1-\sqrt{\pi}]\pi f}{1+\sqrt{\pi}}$$

$$= [\pi + w(1-\pi)][\widehat{x}(I^{SR}) - I^{SR} - \widehat{x}(I^{FB}) + I^{FB}] - \pi k + \frac{\pi[1+\sqrt{\pi}]k^2}{4f[1-\sqrt{\pi}][1-w]} + \frac{[1-w][1-\sqrt{\pi}]\pi f}{1+\sqrt{\pi}}$$

$$= [\pi + w(1-\pi)][\widehat{x}(I^{SR}) - I^{SR} - \widehat{x}(I^{FB}) + I^{FB}] + \frac{\pi[1+\sqrt{\pi}]}{4f[1-\sqrt{\pi}][1-w]}\left[k - \frac{2f[1-w][1-\sqrt{\pi}]}{1+\sqrt{\pi}}\right]^2. \tag{A46}$$

Recall:

$$\widehat{x}(I^{FB}) - I^{FB} = \left[\frac{1}{\alpha} - 1\right] I^{FB}, \text{ and} \tag{A47}$$

$$\widehat{x}(I^{SR}) - I^{SR} = \left[\frac{2f}{\alpha\,[2f+\pi k]} - 1\right] I^{SR} = \left[\frac{2f}{\alpha[2f+\pi k]} - 1\right]\left[\frac{2f+\pi k}{2f}\right]^{\frac{1}{1-\alpha}} I^{FB}. \tag{A48}$$

Substituting (A47) and (A48) into (A46) provides:

$$\frac{\pi[1+\sqrt{\pi}]}{4f[1-\sqrt{\pi}][1-w]}\left[k - \frac{2f[1-w][1-\sqrt{\pi}]}{1+\sqrt{\pi}}\right]^2$$

$$- [\pi + w(1-\pi)]\left[\left[\frac{1}{\alpha} - 1\right] - \left[\frac{2f}{\alpha[2f+\pi k]} - 1\right]\left[\frac{2f+\pi k}{2f}\right]^{\frac{1}{1-\alpha}}\right] I^{FB}$$

$$= \frac{\pi[1+\sqrt{\pi}]}{4f[1-\sqrt{\pi}][1-w]}\left[k - \frac{2f[1-w][1-\sqrt{\pi}]}{1+\sqrt{\pi}}\right]^2$$

$$- [\pi + w(1-\pi)]\left[\left[\frac{1}{\alpha} - 1\right] + \left[\frac{2f+\pi k}{2f} - \frac{1}{\alpha}\right]\left[\frac{2f+\pi k}{2f}\right]^{\frac{\alpha}{1-\alpha}}\right] I^{FB}. \tag{A49}$$

When $k = 0$, (A49) becomes:

$$\frac{\pi[1+\sqrt{\pi}]}{4f[1-\sqrt{\pi}][1-w]}\left[\frac{2f[1-w][1-\sqrt{\pi}]}{1+\sqrt{\pi}}\right]^2 = \frac{\pi f[1-w][1-\sqrt{\pi}]}{[1+\sqrt{\pi}]} > 0. \tag{A50}$$

When $k = \frac{2f[1-w][1-\sqrt{\pi}]}{1+\sqrt{\pi}}$, (A49) becomes:

$$-[\pi + w(1-\pi)] \left[\left[\frac{1}{\alpha} - 1 \right] + \left[1 + \frac{\pi[1-w][1-\sqrt{\pi}]}{1+\sqrt{\pi}} - \frac{1}{\alpha} \right] \left[1 + \frac{\pi[1-w][1-\sqrt{\pi}]}{1+\sqrt{\pi}} \right]^{\frac{\alpha}{1-\alpha}} \right] I^{FB} < 0. \tag{A51}$$

(A50) and (A51) imply there exists a $k^* \in (0, \frac{2f[1-w][1-\sqrt{\pi}]}{1+\sqrt{\pi}}]$ such that for $k \in (0, k^*]$, $W^{SR}(\cdot)$ as specified in (A35) (weakly) exceeds $W^{NR}(\cdot)$ as specified in (A29).

Comparing (A35) and (A33) provides:

$$\begin{aligned}
&[\pi + w(1-\pi)][\widehat{x}(I^{SR}) - I^{SR}] + \frac{\pi}{2f} \left[-2fk + \frac{[1+\sqrt{\pi}]k^2}{2[1-\sqrt{\pi}][1-w]} \right] \\
&\qquad - [\pi + w(1-\pi)][\widehat{x}(I^{FB}) - I^{FB}] + \pi k \\
= \; & [\pi + w(1-\pi)][\widehat{x}(I^{SR}) - I^{SR} - \widehat{x}(I^{FB}) + I^{FB}] + \frac{\pi[1+\sqrt{\pi}]k^2}{4f[1-\sqrt{\pi}][1-w]} \\
&\qquad - [\pi + w(1-\pi)] \left[1 + \left[\frac{2f+\pi k}{2f} - 2 \right] \left[\frac{2f+\pi k}{2f} \right] \right] I^{FB} \\
= \; & \frac{\pi[1+\sqrt{\pi}]k^2}{4f[1-\sqrt{\pi}][1-w]} - [\pi + w(1-\pi)] \left[\left[\frac{1}{\alpha} - 1 \right] + \left[\frac{2f+\pi k}{2f} - \frac{1}{\alpha} \right] \left[\frac{2f+\pi k}{2f} \right]^{\frac{\alpha}{1-\alpha}} \right] I^{FB}.
\end{aligned} \tag{A52}$$

The expression in (A52) exceeds the corresponding expression in (A49).

<u>Conclusion 5</u>: $x_c(I^{SR}) = I^{SR}$ is measure zero occurrence.

Suppose k is such that truncation is interior, $I^{SR} = \left[\frac{\beta[2f+\pi k]}{2f} \right]^{\frac{1}{1-\alpha}}$, and $x_c(I^{SR}) = I^{SR}$. Then (A43) provides:

$$I^{SR} \; = \; \overline{x}(I^{SR}) - \frac{[1+\sqrt{\pi}]k}{[1-\sqrt{\pi}][1-w]} \; = \frac{\beta}{\alpha}(I^{SR})^{\alpha} + f - \frac{[1+\sqrt{\pi}]k}{[1-\sqrt{\pi}][1-w]}. \tag{A53}$$

This has zero mass in the space of parameters. A parallel argument applies to the case where truncation is at a boundary. ■

<u>**Proof of Proposition 7.**</u>

When $x_c(I) = zI$, the regulator's problem is:

$$\underset{z}{Maximize} \; W(z) \;\; subject\ to \;\; I(z) = \arg\max_{I} V(I; z), \;\; \text{where } z \geq 0. \tag{A54}$$

If $z \geq 0$ is set sufficiently low or sufficiently high, no revaluation or always revaluation occurs in equilibrium. The first-best level of investment is induced. Expected welfare is independent of truncation policy types. That is, expected welfare is as specified in (A29) for $z \leq \frac{\widehat{x}(I^{FB})-f}{I^{FB}}$; whereas as specified in (A33) for $z \geq \frac{\widehat{x}(I^{FB})+f}{I^{FB}}$.

If $z \in [\frac{\widehat{x}(I^{FB})-f}{I^{FB}}, \frac{\widehat{x}(I^{FB})+f}{I^{FB}}]$, the regulator maximizes:

$$\begin{aligned} W^{SR}(z) = \ & \pi \left[\int_{\underline{x}}^{zI} [v-k]\phi(v)dv + \int_{zI}^{\overline{x}} P\phi(v)dv - I \right] \\ & + \ w[1-\pi] \left[\int_{\underline{x}}^{zI} v\phi(v)dv + \int_{zI}^{P} P\phi(v)dv + \int_{P}^{\overline{x}} v\phi(v)dv - I \right]. \end{aligned} \tag{A55}$$

Substituting $P(I,I) = zI + \frac{\sqrt{\pi}\ [\overline{x}-zI]}{1+\sqrt{\pi}}$ into (A55) and simplifying provides:

$$\begin{aligned} W^{SR}(z) = \ & \frac{\pi}{2f} \left[-(\frac{1}{2} - \frac{\sqrt{\pi}}{1+\sqrt{\pi}})z^2I^2 - \frac{1}{2}\underline{x}^2 - k[zI-\underline{x}] + (1-\frac{2\sqrt{\pi}}{1+\sqrt{\pi}})zI\overline{x} + \frac{\sqrt{\pi}}{1+\sqrt{\pi}}\overline{x}^2 \right] \\ & + \frac{w(1-\pi)}{2f} \left[2f\widehat{x} + \frac{\pi(\overline{x}^2 + z^2I^2 - 2zI\overline{x})}{2(1+\sqrt{\pi})^2} \right] - [\pi + w(1-\pi)]I \end{aligned} \tag{A56}$$

$$\begin{aligned} = \ & \frac{\pi}{2f} \left[-\frac{1-\sqrt{\pi}}{2[1+\sqrt{\pi}]}z^2I^2 + 2f\widehat{x} - k[zI-\underline{x}] + \left(1 - \frac{2\sqrt{\pi}}{1+\sqrt{\pi}}\right) zI\overline{x} + \left(\frac{\sqrt{\pi}}{1+\sqrt{\pi}} - \frac{1}{2} \right) \overline{x}^2 \right] \\ & + \ \frac{w[1-\pi]}{2f} \left[2f\widehat{x} + \frac{\pi(\overline{x}^2 + z^2I^2 - 2zI\overline{x})}{2(1+\sqrt{\pi})^2} \right] - [\pi + w(1-\pi)]I \end{aligned} \tag{A57}$$

$$\begin{aligned} = \ & \frac{\pi}{2f} \left[-\frac{1-\sqrt{\pi}}{2(1+\sqrt{\pi})}z^2I^2 - k[zI-\underline{x}] + \frac{1-\sqrt{\pi}}{1+\sqrt{\pi}}zI\overline{x} - \frac{1-\sqrt{\pi}}{2(1+\sqrt{\pi})}\overline{x}^2 \right] \\ & + \ \frac{w[1-\pi]}{2f} \frac{\pi[\overline{x}^2 + z^2I^2 - 2zI\overline{x}]}{2[1+\sqrt{\pi}]^2} + [\pi + w(1-\pi)][\widehat{x} - I], \end{aligned} \tag{A58}$$

$$= \ \frac{\pi k}{2f}[\widehat{x} - zI - f] - \frac{\pi[1-\sqrt{\pi}][1-w][\widehat{x} - zI + f]^2}{4f[1+\sqrt{\pi}]} + [\pi + w(1-\pi)][\widehat{x} - I], \tag{A59}$$

where I is as specified in (A27) in the proof of Proposition 5.

Differentiating (A59) provides:

$$\begin{aligned} \frac{dW^{SR}(z)}{dz} = \ & \frac{\pi k}{2f} \left[[\widehat{x}' - z]\frac{dI}{dz} - I \right] - \frac{\pi(1-\sqrt{\pi})(1-w)2(\widehat{x} - zI + f)}{4f(1+\sqrt{\pi})} \left[[\widehat{x}' - z]\frac{dI}{dz} - I \right] \\ & + [\pi + w(1-\pi)][\widehat{x}' - 1]\frac{dI}{dz}. \end{aligned} \tag{A60}$$

Substituting $\frac{dI}{dz} = \frac{\pi k I}{[2f+\pi kz][\alpha-1]}$ and (A26) into (A60) provides:

$$\begin{aligned}
\frac{dW^{SR}(z)}{dz} &= \frac{\pi k}{2f}\left[\left(\frac{2f+\pi kz}{2f+\pi k} - z\right)\frac{\pi k I}{[2f+\pi kz][\alpha-1]} - I\right] \\
&\quad - \frac{\pi[1-\sqrt{\pi}][1-w][\widehat{x} - zI + f]}{2f[1+\sqrt{\pi}]}\left[\left(\frac{2f+\pi kz}{2f+\pi k} - z\right)\frac{\pi k I}{[2f+\pi kz][\alpha-1]} - I\right] \\
&\quad + [\pi + w(1-\pi)]\left[\frac{2f+\pi kz}{2f+\pi k} - 1\right]\frac{\pi k I}{[2f+\pi kz][\alpha-1]}, \\
&= \frac{\pi k}{2f}\left[\frac{2f\pi k I[1-z]}{[2f+\pi k][2f+\pi kz][\alpha-1]} - I\right] \\
&\quad - \frac{\pi[1-\sqrt{\pi}][1-w][\widehat{x} - zI + f]}{2f(1+\sqrt{\pi})}\left[\frac{2f\pi k I[1-z]}{[2f+\pi k][2f+\pi kz][\alpha-1]} - I\right] \\
&\quad - [\pi + w(1-\pi)]\frac{\pi^2 k^2 I[1-z]}{[2f+\pi k][2f+\pi kz][\alpha-1]}, \\
&= \frac{[1-\pi][1-w]\pi^2 k^2 I[1-z]}{[2f+\pi k][2f+\pi kz][\alpha-1]} - \frac{\pi k I}{2f} \\
&\quad - \frac{\pi[1-\sqrt{\pi}][1-w][\widehat{x} - zI + f]}{2f(1+\sqrt{\pi})}\left[\frac{2f\pi k I[1-z]}{[2f+\pi k][2f+\pi kz][\alpha-1]} - I\right]. \qquad \text{(A61)}
\end{aligned}$$

The welfare maximizing policy, presuming $z \geq 0$, is identified by $\frac{dW^{SR}(z)}{dz} = 0$. If $z = 1$ is optimal, then since $I = I^{FB}$ in this case, (A61) implies:

$$\frac{dW^{SR}(z)}{dz}|_{z=1} = -\frac{\pi k I^{FB}}{2f} + \frac{\pi[1-\sqrt{\pi}][1-w][\widehat{x}+f]I^{FB}}{2f(1+\sqrt{\pi})} = 0. \qquad \text{(A62)}$$

Further simplifying (A62) provides:

$$-k + \frac{[1-\sqrt{\pi}][1-w][\widehat{x}+f]}{(1+\sqrt{\pi})} = 0. \qquad \text{(A63)}$$

Substituting $\widehat{x} = \frac{\beta^{\frac{1}{1-\alpha}}}{\alpha}$ into (A63) provides:

$$k = \frac{[1-\sqrt{\pi}][1-w]}{1+\sqrt{\pi}}\left[\frac{\beta^{\frac{1}{1-\alpha}}}{\alpha} + f\right]. \qquad \text{(A64)}$$

Substituting $z = 1$ and I^{FB} into (A59) reveals:

$$W^{SR}|_{z=1} = \frac{\pi k}{2f}[\widehat{x} - I^{FB} - f] - \frac{\pi[1-\sqrt{\pi}][1-w][\widehat{x} - I^{FB} + f]^2}{4f[1+\sqrt{\pi}]} + [\pi + w(1-\pi)][\widehat{x} - I^{FB}]. \quad \text{(A65)}$$

It remains to show that expected welfare as depicted in (A65) is not optimal. Suppose it is optimal. Then the following contradiction arises from comparing (A65) and (A29).

$$W^{SR}|_{z=1} \geq W^{NR}(z) \quad \text{(A66)}$$

$$\Leftrightarrow \frac{\pi k}{2f}[\widehat{x} - I^{FB} - f] - \frac{\pi[1-\sqrt{\pi}][1-w][\widehat{x} - I^{FB} + f]^2}{4f[1+\sqrt{\pi}]} + [\pi + w(1-\pi)][\widehat{x} - I^{FB}] \geq [\pi + w(1-\pi)][\widehat{x} - I^{FB}] - \frac{[1-w][1-\sqrt{\pi}]\pi f}{1+\sqrt{\pi}} \quad \text{(A67)}$$

$$\Leftrightarrow \frac{k}{2f}[\widehat{x} - I^{FB} - f] - \frac{[1-\sqrt{\pi}][1-w][\widehat{x} - I^{FB} + f]^2}{4f[1+\sqrt{\pi}]} \geq -\frac{[1-w][1-\sqrt{\pi}]f}{1+\sqrt{\pi}} \quad \text{(A68)}$$

$$\Leftrightarrow k \geq \frac{2f[1-\sqrt{\pi}][1-w]}{[1+\sqrt{\pi}][\widehat{x} - I^{FB} - f]}\left[\frac{[\widehat{x} - I^{FB} + f]^2}{4f} - f\right]. \quad \text{(A69)}$$

From (A64):

$$\frac{[1-\sqrt{\pi}][1-w]}{1+\sqrt{\pi}}[\widehat{x} + f] \geq \frac{2f[1-\sqrt{\pi}][1-w]}{[1+\sqrt{\pi}][\widehat{x} - I^{FB} - f]}\left[\frac{[\widehat{x} - I^{FB} + f]^2}{4f} - f\right] \quad \text{(A70)}$$

$$\Leftrightarrow \widehat{x} + f \geq \frac{2f}{[\widehat{x} - I^{FB} - f]}\left[\frac{[\widehat{x} - I^{FB} + f]^2}{4f} - f\right], \quad \text{(A71)}$$

$$\Leftrightarrow [\widehat{x} + f][\widehat{x} - I^{FB} - f] \geq \frac{[\widehat{x} - I^{FB} + f]^2}{2} - 2f^2, \quad \text{(A72)}$$

$$\Leftrightarrow [\widehat{x} - f + I^{FB}][\widehat{x} - f - I^{FB}] \geq 0. \quad \text{(A73)}$$

But $zI^{FB} \geq \widehat{x}(I^{FB}) - f$. ■

Proof of Proposition 8.

The proof follows immediately from the discussion in the text. ■

Proofs of Propositions 9 and 10.

If assets offered for sale are voluntarily revalued with strictly positive probability less than 1 in equilibrium when no mandatory revaluation is imposed, the probability that a non-revalued asset is traded at price $P = P(I, I)$ is:

$$q^{vn} = \pi \int_{\underline{x}}^{P+k} \frac{dv}{2f} + [1-\pi] \int_{\underline{x}}^{P} \frac{dv}{2f} = \pi \left[\frac{P+k-\underline{x}}{2f} \right] + [1-\pi] \left[\frac{P-\underline{x}}{2f} \right]$$

$$= \frac{P - \underline{x}(I^{vn}) + \pi k}{2f}, \tag{A74}$$

where I^{vn} is the level of investment induced in this setting.

We first prove that when $k < \frac{2f}{1+\sqrt{\pi}}$, voluntary revaluation will be undertaken with strictly positive probability and the equilibrium price of non-revalued assets will be:

$$P(I^{vn}, I^{vn}) = \underline{x}(I^{vn}) + k\sqrt{\pi}. \tag{A75}$$

This is the case because the equilibrium price of a non-revalued asset will be its expected value:

$$P(I^{vn}, I^{vn}) = EV^N \equiv \frac{1}{q^{vn}} \left[\pi \left[\frac{P+k-\underline{x}}{2f} \right] \left[\frac{P+k+\underline{x}}{2} \right] + [1-\pi] \left[\frac{P-\underline{x}}{2f} \right] \left[\frac{\underline{x}+P}{2} \right] \right]. \tag{A76}$$

Rearranging terms in (A76) provides:

$$P = \frac{1}{4fq^{vn}} \left[\pi [P - \underline{x} + k][P + \underline{x} + k] + [1-\pi] [P - \underline{x}] [P + \underline{x}] \right]. \tag{A77}$$

Solving for P in (A77) provides:

$$4fq^{vn}P = \pi [P - \underline{x}] [P + \underline{x}] + \pi k [P - \underline{x}] + \pi k [P + \underline{x} + k] + [1-\pi] [P - \underline{x}] [P + \underline{x}]. \tag{A78}$$

Substituting from (A74) and simplifying (A78) provides:

$$2P[P - \underline{x} + \pi k] = [P - \underline{x}] [P + \underline{x}] + \pi k [2P + k]$$

$$\Leftrightarrow \quad 2P^2 - 2P\underline{x} + 2P\pi k = P^2 - \underline{x}^2 + 2P\pi k + \pi k^2$$

$$\Leftrightarrow \quad P^2 - 2P\underline{x} + \underline{x}^2 - \pi k^2 = 0. \tag{A79}$$

Solving (A79) for P provides:

$$P = \frac{1}{2} \left[2\underline{x} \pm \sqrt{4\underline{x}^2 - 4[\underline{x}^2 - \pi k^2]} \right] = \underline{x} \pm k\sqrt{\pi}. \tag{A80}$$

The equilibrium price is $P = \underline{x} + k\sqrt{\pi}$ since P must exceed $\underline{x}$. Thus, voluntary revaluation occurs in equilibrium when:

$$x_u \equiv P + k = \underline{x} + k[1 + \sqrt{\pi}] < \overline{x}, \tag{A81}$$

which is implied by $k < \frac{2f}{1+\sqrt{\pi}}$.

Given $k \in (0, \frac{2f}{1+\sqrt{\pi}})$, the expected surplus from investment I is:

$$V^{vn}(I) \;=\; \pi E[v^{Dn}] + [1-\pi]E[v^{Nn}], \tag{A82}$$

where $E[v^{Dn}]$ and $E[v^{Nn}]$ are the expected surplus for a distressed and a non-distressed entrepreneur, respectively, given investment I. Notice:

$$\begin{aligned}
E[v^{Dn}] &= \frac{1}{2f}\left[\int_{\underline{x}}^{x_u} P dv \;+\; \int_{x_u}^{\overline{x}} (v-k)dv\right] - I \\
&= \frac{1}{2f}\left[P[x_u - \underline{x}] \;+\; \frac{1}{2}[\overline{x}^2 - x_u^2] - k[\overline{x} - x_u]\right] - I \\
&= \frac{1}{2f}\left[2f\widehat{x} - 2fk + \frac{1}{2}k^2(1+\sqrt{\pi})^2\right] - I \\
&= \widehat{x} - I - k + \frac{1}{4f}[1+\sqrt{\pi}]^2 k^2 \;,\; \text{ and}
\end{aligned} \tag{A83}$$

$$\begin{aligned}
E[v^{Nn}] &= \frac{1}{2f}\left[\int_{\underline{x}}^{P} P dx + \int_{P}^{\overline{x}} x dx\right] - I \;=\; \frac{1}{2f}\left[P[P - \underline{x}] + \frac{1}{2}[\overline{x}^2 - P^2]\right] - I \\
&= \frac{1}{2f}\left[\frac{1}{2}[\overline{x}^2 - \underline{x}^2] + \frac{1}{2}\pi k^2\right] - I \;=\; \widehat{x} - I + \frac{\pi k^2}{4f}.
\end{aligned} \tag{A84}$$

Substituting (A83) and (A84) into (A82) provides:

$$\begin{aligned}
V^{vn}(I) &= \pi\left[\widehat{x} - I - k + \frac{1}{4f}(1+\sqrt{\pi})^2 k^2\right] + [1-\pi]\left[\widehat{x} - I + \frac{\pi k^2}{4f}\right] \\
&= \widehat{x} - I - \frac{\pi k}{2f}\left[2f - k[1+\sqrt{\pi}]\right].
\end{aligned} \tag{A85}$$

Differentiating (A85) with respect to I provides $\frac{dV^{vn}(I)}{dI} = \hat{x}'(I) - 1$, which implies the first-best level of investment is induced. Furthermore, $2f - k\,[1+\sqrt{\pi}] > 0$ since $k < \frac{2f}{1+\sqrt{\pi}}$. Therefore, (A85) implies $V^{vn}(I^{FB}) < \hat{x}(I^{FB}) - I^{FB}$, and so the entrepreneur's expected surplus is reduced by the possibility of voluntary revaluation.

To prove Proposition 10, suppose revaluation occurs with strictly positive probability for a given $x_c(I)$. As voluntary revaluation is present, the probability a non-revalued asset is traded at price $P = P(I, I)$ is:

$$q^v \equiv \pi\left[\frac{P + k - x_c(I)}{2f}\right] + [1-\pi]\left[\frac{P - x_c(I)}{2f}\right] = \frac{P - x_c(I) + \pi k}{2f}. \tag{A86}$$

The equilibrium price of a non-revalued asset will be its expected value:

$$P = EV \equiv \frac{1}{q^v}\left[\pi[\frac{P+k-x_c}{2f}][\frac{P+k+x_c}{2}] + [1-\pi][\frac{P-x_c}{2f}][\frac{x_c+P}{2}]\right]. \tag{A87}$$

Using logic analogous to that employed in the proof of Proposition 9, the equilibrium price is:

$$P = x_c + k\sqrt{\pi}. \tag{A88}$$

As in (A81), voluntary revaluation occurs in equilibrium when:

$$x_u = P + k = x_c + k[1+\sqrt{\pi}] > x_c. \tag{A89}$$

Also, $x_u < \overline{x}$ gives the upper bound for k as $\frac{\overline{x}-x_c}{1+\sqrt{\pi}}$. The expected payoff from investment I in this setting is:

$$V^v(I; x_c) = \pi E[v^d] + [1-\pi]E[v^n], \tag{A90}$$

where:

$$E[v^d] = \frac{1}{2f}\left[\int_{\underline{x}}^{x_c}[v-k]dx + \int_{x_c}^{x_u} P dx + \int_{x_u}^{\overline{x}}(v-k)dx\right] - I \text{ ; and} \tag{A91}$$

$$E[v^n] = \frac{1}{2f}\left[\int_{\underline{x}}^{x_c} v dx + \int_{x_c}^{P} P dx + \int_{P}^{\overline{x}} v dx\right] - I. \tag{A92}$$

Straightforward algebraic manipulation provides $V^v(I; x_c(I)) = V^{vn}(I^{FB})$, where $V^{vn}(I^{FB})$ is as specified in (A85). Notice that $V^v(I; x_c(I))$ is independent of $x_c(I)$. Finally, $x_c(I^{FB}) \leq \overline{x}(I^{FB}) - k[1+\sqrt{\pi}]$ because $k \leq \frac{\overline{x}-x_c}{1+\sqrt{\pi}}$. Since $x_c(I^{FB}) \geq \underline{x}(I^{FB})$ is necessary for nontrivial revaluation, $\overline{x}(I^{FB}) - k[1+\sqrt{\pi}] \geq \underline{x}(I^{FB})$ gives $k \leq \frac{2f}{1+\sqrt{\pi}}$. ■

Proofs of Lemma 2 and Proposition 11.

Under the assumed off-equilibrium beliefs, the price anticipated by the entrepreneur at the time of investment is:

$$P(v, I) = \begin{cases} v & \text{if } v \neq I; \\ \hat{x}_L(I) - f\left[\frac{1-\sqrt{\pi}}{1+\sqrt{\pi}}\right] & \text{if } v = I \text{ and } I < I_H^{FB}; \\ \hat{x}_H(I) - f\left[\frac{1-\sqrt{\pi}}{1+\sqrt{\pi}}\right] & \text{if } v = I \text{ and } I \geq I_H^{FB}. \end{cases} . \tag{A93}$$

For an H type entrepreneur, the expected surplus from investment I, absent any revaluation, is:

$$V^{H}(I) \equiv \pi P(\cdot) + \frac{1-\pi}{2f} P(\cdot)[P(\cdot) - \hat{x}_{H}(I) + f] + \frac{1-\pi}{4f}\left[[\hat{x}_{H}(I) + f]^{2} - P^{2}(\cdot)\ \right] - I. \qquad \text{(A94)}$$

When $I \geq I_{H}^{FB}$, (A94) becomes:

$$V^{H}(I) \ = \ \widehat{x}_{H}(I) - I, \qquad \text{(A95)}$$

which implies $I = I_{H}^{FB}$. Conversely, for $I < I_{H}^{FB}$, differentiating (A94) with respect to P provides:

$$\begin{aligned} \frac{dV^{H}}{dP} &= \pi + \frac{1-\pi}{2f}[2P(\cdot) - \widehat{x}_{H}(I) + f] - \frac{1-\pi}{2f}P(\cdot) \\ &= \pi + \frac{1-\pi}{2f}[P(\cdot) - \widehat{x}_{H}(I) + f] > 0, \qquad \text{(A96)} \end{aligned}$$

where the inequality in (A96) holds because $P(\cdot) > \underline{x}(I)$. The price is lower in this case because:

$$\hat{x}_{L}(I) - \frac{f[1-\sqrt{\pi}]}{1+\sqrt{\pi}} \ < \ \hat{x}_{H}(I) - \frac{f[1-\sqrt{\pi}]}{1+\sqrt{\pi}}. \qquad \text{(A97)}$$

Continuing, we readily derive $I = I_{H}^{FB}$.

For an L type entrepreneur, the expected surplus from investment I is:

$$V^{L}(I) \equiv \pi P(\cdot) + \frac{1-\pi}{2f} P(\cdot)[P(\cdot) - \hat{x}_{L}(I) + f] + \frac{1-\pi}{4f}[(\hat{x}_{L}(I) + f)^{2} - P^{2}(\cdot)] - I. \qquad \text{(A98)}$$

For $I < I_{H}^{FB}$, (A98) becomes:

$$V^{L}(I) = \widehat{x}_{L}(I) - I, \qquad \text{(A99)}$$

which implies $I = I_{L}^{FB} < I_{H}^{FB}$. For $I \geq I_{H}^{FB}$, $\frac{dP}{dI} = \frac{d\widehat{x}_{H}(I)}{dI}$. Differentiating (A98) with respect to I provides:

$$\begin{aligned} \frac{dV^{L}(I)}{dI} &= \frac{1-\pi}{2f}[\hat{x}_{L}(I) + f - P]\frac{d\hat{x}_{L}(I)}{dI} + \left[\pi + \frac{1-\pi}{2f}[P - \hat{x}_{L}(I) + f]\right]\frac{dP}{dI} - 1 \\ &= \frac{1-\pi}{2f}[\hat{x}_{L}(I) + f - P]\frac{d\hat{x}_{L}(I)}{dI} + \left[\pi + \frac{1-\pi}{2f}[P - \hat{x}_{L}(I) + f]\right]\frac{d\widehat{x}_{H}(I)}{dI} - 1 \\ &= \frac{1-\pi}{2f}[\hat{x}_{L}(I) + f - P][\frac{d\hat{x}_{L}(I)}{dI} - \frac{d\widehat{x}_{H}(I)}{dI}] + \frac{d\widehat{x}_{H}(I)}{dI} - 1 \ < \ 0. \qquad \text{(A100)} \end{aligned}$$

The inequality in (A100) follows from (13) in the text because $\frac{d\widehat{x}}{dI}$ increases in β and $\frac{d\widehat{x}_{H}}{dI} \leq 1$ for $I \geq I_{H}^{FB}$. The L type will either "separate" via $I = I_{L}^{FB} < I_{H}^{FB}$ or "mimic" via $I = I_{H}^{FB}$. If he mimics, the L type will sell his asset at price $P^{H} = \widehat{x}_{H}(I_{H}^{FB}) - f\left[\frac{1-\sqrt{\pi}}{1+\sqrt{\pi}}\right]$, generating expected surplus:

$$V^{Lm} = \pi P^{H} + \frac{1-\pi}{2f} P^{H}[P^{H} - \hat{x}_{L}\left(I_{H}^{FB}\right) + f] + \frac{1-\pi}{4f}\left[[\hat{x}_{L}\left(I_{H}^{FB}\right) + f]^{2} - [P^{H}]^{2}\ \right] - I_{H}^{FB}. \qquad \text{(A101)}$$

If $V^L(I_L^{FB}) = \widehat{x}_L(I_L^{FB}) - I_L^{FB} \geq V^{Lm}$, separation occurs in equilibrium. Otherwise, the L type prefers to mimic.

The L type prefers to mimic rather than separate if, for example, π is arbitrarily close to unity, implying $V^{Lm} \approx P^H - I_H^{FB} \approx \hat{x}_H\left(I_H^{FB}\right) - I_H^{FB} > \hat{x}_L\left(I_L^{FB}\right) - I_L^{FB}$. The L type prefers to separate rather than mimic if, for example, π is arbitrarily close to zero, $\alpha = .5$, $f \approx 2\beta_H^2$, and $\beta_L = \varepsilon$. This implies $P^H \approx \hat{x}_H\left(I_H^{FB}\right) - f$ and $V^{Lm} \approx \frac{1}{2f}P^H[P^H - f] + \frac{1}{4f}[f^2 - P^{H2}] - I_H^{FB} = -\beta_H^2/2 < V^L(I_L^{FB}) \approx 0$.

Finally, suppose $V^{Lm} > V^L(I_L^{FB})$. Further suppose the revaluation point is set at $x_c(I_L^{FB}) << 0$, $x_c(I_H^{FB}) = \hat{x}_H\left(I_H^{FB}\right) - f$, and arbitrarily high otherwise (to avoid off-equilibrium issues). (Notice this requires use of the targeted policy.) If the L type chooses not to mimic, revaluation never occurs. If he elects to mimic, he incurs revaluation with probability $[\hat{x}_H\left(I_H^{FB}\right) - \hat{x}_L\left(I_H^{FB}\right)]/2f$. k sufficiently large ensures $V^{Lm} < \hat{x}_L\left(I_L^{FB}\right) - I_L^{FB}$. ■

Proof of Proposition 12.

Let I^e denote the market's conjecture about the level of investment undertaken by the (identical) entrepreneurs. Also let $P(I^e)$ denote the equilibrium price of an asset offered for sale. The expected expected surplus from investment I in this setting with no revaluation and unobserved investment is:

$$V(I) = \pi P(I^e) + [1-\pi]\{\int_{\mu \leq P(I^e) - \hat{x}(I)} P(I^e)h(\mu)d\mu + \int_{\mu \geq P(I^e) - \hat{x}(I)} [\hat{x}(I) + \mu]h(\mu)d\mu\} - I. \quad \text{(A102)}$$

Differentiating (A102) provides:

$$\begin{aligned} \frac{dV(I)}{dI} &= [1-\pi]\{-\hat{x}'(I)\,P(I^e)h(P(I^e) - \hat{x}(I)) + \hat{x}'(I)\,P(I^e)h(P(I^e) - \hat{x}(I)) \\ &\quad + [1 - H(P(I^e) - \hat{x}(I))]\hat{x}'(I)\} - 1 \\ &= [1-\pi][1 - H(P(I^e) - \hat{x}(I)]\hat{x}'(I) - 1. \qquad \text{(A103)} \end{aligned}$$

Hence, $\hat{x}'(I) > 1$ at $I = I^e$ because the conjectured and the actual investment coincide in equilibrium. ■

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