Causal effects in controlled experiments

There are many advantages to controlled (laboratory) experiments. Chief among them are knowledge/control of salient conditions and randomization eases the challenge of identifying causal effects. Of course, important elements remain outside the purview of the analyst leading to stochastic data and statistical inference for interpreting the results.

Controlled experimental design often draws on analysis of variance (ANOVA) (or analysis of covariance, ANCOVA) to interpret the results. In this note, we explore experiments with one or two randomly-assigned treatments (factors) as well as exogenous or endogenous covariates. Potential endogeneity arises when one group of subjects' choices create conditions (covariates) that possibly impact other subjects' choices (outcomes). If outcome depends on the covariate and the unobservable components of the subjects' utilities (underlying their choices) are correlated then the data suffer an omitted, correlated variable bias. This bias arises from (nonrandom) selective sampling.

We discuss two strategies for addressing selective sampling bias. First, we consider exhaustive covariate sampling. Then, we visit instrumental variables employing only data from strategies played by the subjects.

1 Exhaustive covariate sampling

A strategy involving elicitation of responses from the subject at all levels of the covariate mitigates selective sampling. We refer to this as an exhaustive covariate strategy. Nothing comes for free and this strategy is no exception. The strategy may alter the behavior of the subjects introducing unintended effects. For example, suppose we intend to explore a sequential game. This strategy effectively transforms the game to one of simultaneous play.

Next, we report examples illustrating these issues and the exhaustive covariate strategy. The examples consider an experiment in which one subject (say, employer) offers payment (extrinsic reward) to another subject and the second subject (employee) makes a choice that affects the extrinsic reward for each subject. Employer's choice can be the covariate and employee's choice the outcome or vice versa. The covariate involves three discrete levels encoded as indicator variables X_j, $j = 1, 2, 3$.

The experimental design is ANOVA. For the single treatment setting, the ANOVA is

$$Y = \alpha_1 X_1 + \alpha_2 X_2 + \alpha_3 X_3 + \varepsilon_1$$

where $\alpha_j = E[Y \mid X_j]$. For the two treatment setting, the ANOVA is

$$Y = \alpha_1 X_1 T_1 + \alpha_2 X_2 T_1 + \alpha_3 X_3 T_1 + \gamma_1 X_1 T_2 + \gamma_2 X_2 T_2 + \gamma_3 X_3 T_2 + \varepsilon_2$$

where $\alpha_j = E[Y \mid X_j T_1]$, $\gamma_j = E[Y \mid X_j T_2]$, and T_k refers to treatment $k = 1, 2$.

Example 1 (One treatment, exogenous covariate) *Suppose the data generating process (DGP) has the following joint distribution with outcome Y and discrete covariate levels X_1, X_2, X_3.*

Y	X_1	X_2	X_3	$\Pr(Y=j)$
1	$\frac{5}{12}$	$\frac{2}{9}$	$\frac{1}{12}$	$\frac{13}{18}$
2	$\frac{1}{12}$	$\frac{1}{9}$	$\frac{1}{18}$	$\frac{1}{4}$
3	0	0	$\frac{1}{36}$	$\frac{1}{36}$
$E[Y \mid X_j]$	$\frac{7}{6}$	$\frac{4}{3}$	$\frac{5}{3}$	$E[Y] = \frac{47}{36}$

DGP

Further, suppose 36 *trials produce the following outcomes played by the two subjects.*

Y	X_1	X_2	X_3	$\Pr(Y=j)$
1	15	8	3	$\frac{13}{18}$
2	3	4	2	$\frac{1}{4}$
3	0	0	1	$\frac{1}{36}$
$E[Y \mid X_j]$	$\frac{7}{6}$	$\frac{4}{3}$	$\frac{5}{3}$	$E[Y] = \frac{47}{36}$

$D = 1$, *strategies played*

If the exhaustive covariate strategy is employed we also have the following (counterfactual) data.

Y	X_1	X_2	X_3	$\Pr(Y=j)$
1	30	16	6	$\frac{13}{18}$
2	6	8	4	$\frac{1}{4}$
3	0	0	2	$\frac{1}{36}$
$E[Y \mid X_j]$	$\frac{7}{6}$	$\frac{4}{3}$	$\frac{5}{3}$	$E[Y] = \frac{47}{36}$

$D = 0$, *counterfactual strategies*

There is no sample selection bias as the strategies played mirror the population at large and standard ANOVA designs identify the reported conditional means, $E[Y \mid X_j]$.[1]

	X_1	X_2	X_3
$\alpha_j = E[Y \mid X_j]$	$\frac{7}{6}$	$\frac{4}{3}$	$\frac{5}{3}$

[1] In standard mean difference form the ANOVA is

$$E[Y \mid X] = \frac{7}{6} + \frac{1}{6} X_2 + \frac{1}{2} X_3$$

The simplest sampling failure is incomplete common support. This is illustrated in the next example.

Example 2 (One treatment, full support lacking) *Suppose the DGP is the same as example 1 with conditional population means*

	X_1	X_2	X_3
$E[Y \mid X_j]$	$\frac{7}{6}$	$\frac{4}{3}$	$\frac{5}{3}$

and 30 *trials produce the following outcomes played by the two subjects.*

Y	X_1	X_2	X_3	$\Pr(Y=j)$
1	15	8	0	$\frac{23}{30}$
2	3	4	0	$\frac{7}{30}$
3	0	0	0	$\frac{0}{30}$
$E[Y \mid X_j]$	$\frac{7}{6}$	$\frac{4}{3}$	NA	$E[Y] = \frac{217}{180}$

$D = 1$*, strategies played*

If the exhaustive covariate strategy is employed we also have the following (counterfactual) data.

Y	X_1	X_2	X_3	$\Pr(Y=j)$
1	30	16	9	$\frac{55}{78}$
2	6	8	6	$\frac{20}{78}$
3	0	0	3	$\frac{3}{78}$
$E[Y \mid X_j]$	$\frac{7}{6}$	$\frac{4}{3}$	$\frac{5}{3}$	$E[Y] = \frac{575}{468}$

$D = 0$*, counterfactual strategies*

There is sample selection bias as the strategies played don't involve support at all levels of the covariate and accordingly no longer mirror the population at large. ANOVA designs based on strategies played properly reflect population means at factor levels X_1 *and* X_2 *but offer no evidence on* X_3*.*

	X_1	X_2	X_3
$\alpha_j = E[Y \mid X_j]$	$\frac{7}{6}$	$\frac{4}{3}$	NA

Hence, data garnered from strategies played are not misleading but rather incomplete.

While lack of complete common support can limit inference, endogeneity poses a different, unobservable challenge. The next example describes a simple, single treatment version of this challenge.

Example 3 (One treatment, endogenous covariate) *Suppose the DGP is the same as example 1 with conditional population means*

	X_1	X_2	X_3
$E[Y \mid X_j]$	$\frac{7}{6}$	$\frac{4}{3}$	$\frac{5}{3}$

But, 36 *trials produce the following outcomes played by the two subjects.*

Y	X_1	X_2	X_3	$\Pr(Y = j)$
1	16	9	4	$\frac{29}{36}$
2	2	3	1	$\frac{1}{6}$
3	0	0	1	$\frac{1}{36}$
$E[Y \mid X_j]$	$\frac{10}{9}$	$\frac{5}{4}$	$\frac{3}{2}$	

$D = 1$, *strategies played*

If the exhaustive covariate strategy is employed we also have the following (counterfactual) data.

Y	X_1	X_2	X_3	$\Pr(Y = j)$
1	14	15	14	$\frac{43}{72}$
2	4	9	11	$\frac{1}{3}$
3	0	0	5	$\frac{5}{72}$
$E[Y \mid X_j]$	$\frac{11}{9}$	$\frac{11}{8}$	$\frac{17}{10}$	

$D = 0$, *counterfactual strategies*

There is sample selection bias as the strategies played no longer mirror the population at large. However, a standard ANOVA design that ignores the distinction between strategies played and counterfactual strategies identifies the conditional population means via the reported conditional means

	X_1	X_2	X_3
$\alpha_j = E[Y \mid X_j]$	$\frac{7}{6}$	$\frac{4}{3}$	$\frac{5}{3}$

while ANOVA for the strategies played yields

	X_1	X_2	X_3
$\widehat{\alpha}_j$	$\frac{10}{9}$	$\frac{5}{4}$	$\frac{3}{2}$

The latter is contaminated by sample selection bias.

Example 4 (one treatment, more extreme selection bias) *Suppose the DGP is the same as example 1 with conditional population means*

	X_1	X_2	X_3
$E[Y \mid X_j]$	$\frac{7}{6}$	$\frac{4}{3}$	$\frac{5}{3}$

But, 36 trials produce the following outcomes played by the two subjects.

Y	X_1	X_2	X_3	$\Pr(Y=j)$
1	1	5	0	$\frac{1}{6}$
2	0	12	12	$\frac{2}{3}$
3	0	0	6	$\frac{1}{6}$
$E[Y \mid X_j]$	1	$\frac{29}{17}$	$\frac{7}{3}$	

$D=1$, *strategies played*

If the exhaustive covariate strategy is employed we also have the following (counterfactual) data.

Y	X_1	X_2	X_3	$\Pr(Y=j)$
1	29	19	18	$\frac{11}{12}$
2	6	0	0	$\frac{1}{12}$
3	0	0	0	0
$E[Y \mid X_j]$	$\frac{41}{35}$	1	1	

$D=0$, *counterfactual strategies*

The sample selection bias is mitigated if a standard ANOVA design that ignores the distinction between strategies played and counterfactual strategies is employed to report the conditional means

	X_1	X_2	X_3
$\alpha_j = E[Y \mid X_j]$	$\frac{7}{6}$	$\frac{4}{3}$	$\frac{5}{3}$

while ANOVA for the strategies played yields

	X_1	X_2	X_3
$\widehat{\alpha}_j$	1	$\frac{29}{17}$	$\frac{7}{3}$

The latter is contaminated by more extreme sample selection bias than in the previous example.

Next, we discuss cases with two treatments.

Example 5 (two treatments, exogenous covariate) *Suppose the DGP for treatment one is the same as example 1 with conditional population means*

	X_1, T_1	X_2, T_1	X_3, T_1
$E[Y \mid X_j, T_1]$	$\frac{7}{6}$	$\frac{4}{3}$	$\frac{5}{3}$

and the DGP for treatment two is

$Y \mid T_2$	X_1, T_2	X_2, T_2	X_3, T_2	$\Pr(Y = j \mid T_2)$
1	$\frac{1}{3}$	$\frac{1}{9}$	$\frac{1}{36}$	$\frac{17}{36}$
2	$\frac{1}{12}$	$\frac{1}{9}$	$\frac{1}{18}$	$\frac{1}{4}$
3	$\frac{1}{12}$	$\frac{1}{9}$	$\frac{1}{12}$	$\frac{5}{18}$
$E[Y \mid X_j, T_2]$	$\frac{3}{2}$	2	$\frac{7}{3}$	$E[Y \mid T_2] = \frac{65}{36}$

DGP for T_2

Further, suppose 72 *trials played by the two subjects produce the following outcomes.*

Y	X_1, T_1	X_2, T_1	X_3, T_1	X_1, T_2	X_2, T_2	X_3, T_2
1	15	8	3	12	4	1
2	3	4	2	3	4	2
3	0	0	1	3	4	3
$E[Y \mid X_j, T_k]$	$\frac{7}{6}$	$\frac{4}{3}$	$\frac{5}{3}$	$\frac{3}{2}$	2	$\frac{7}{3}$

$D = 1$*, strategies played*

If the exhaustive covariate strategy is employed we also have the following (counterfactual) data.

Y	X_1, T_1	X_2, T_1	X_3, T_1	X_1, T_2	X_2, T_2	X_3, T_2
1	30	16	6	12	8	5
2	6	8	4	3	8	10
3	0	0	2	3	8	15
$E[Y \mid X_j, T_k]$	$\frac{7}{6}$	$\frac{4}{3}$	$\frac{5}{3}$	$\frac{3}{2}$	2	$\frac{7}{3}$

$D = 0$*, counterfactual strategies*

There is no sample selection bias as the strategies played mirror the population at large and standard ANOVA designs identify the reported conditional means, $E[Y \mid X_j, T_k]$*.*[2]

	X_1, T_k	X_2, T_k	X_3, T_k
$\alpha_j = E[Y \mid X_j, T_1]$	$\frac{7}{6}$	$\frac{4}{3}$	$\frac{5}{3}$
$\gamma_j = E[Y \mid X_j, T_2]$	$\frac{3}{2}$	2	$\frac{7}{3}$

[2] In standard mean difference, interaction form the ANOVA is

$$\begin{aligned} E[Y \mid X, T] &= \frac{7}{6} + \frac{1}{6}X_2 + \frac{1}{2}X_3 \\ &\quad + \frac{1}{3}T_2 + \frac{2}{3}X_2 \times T_2 + \frac{2}{3}X_3 \times T_2 \end{aligned}$$

Mean differences in treatment conditional on the level of the covariate are straightforward.

$\gamma_j - \alpha_j =$	X_1	X_2	X_3
$E[Y \mid X_j, T_2] - E[Y \mid X_j, T_1]$	$\frac{3}{2} - \frac{7}{6} = \frac{1}{3}$	$2 - \frac{4}{3} = \frac{2}{3}$	$\frac{7}{3} - \frac{5}{3} = \frac{2}{3}$

Hence, the treatment effect depends on the level of the covariate in this setting, for instance, $E[Y \mid X_1, T_2] - E[Y \mid X_1, T_1] \neq E[Y \mid X_2, T_2] - E[Y \mid X_2, T_1]$ *or* $E[Y \mid X_1, T_2] - E[Y \mid X_1, T_1] \neq E[Y \mid X_3, T_2] - E[Y \mid X_3, T_1]$

Example 6 (Two treatments, full support lacking) *Suppose the DGP is the same as example 5 with conditional population means*

	X_1, T_1	X_2, T_1	X_3, T_1
$E[Y \mid X_j, T_1]$	$\frac{7}{6}$	$\frac{4}{3}$	$\frac{5}{3}$

and the DGP for treatment two is

$Y \mid T_2$	X_1, T_2	X_2, T_2	X_3, T_2	$\Pr(Y = j \mid T_2)$
1	$\frac{1}{3}$	$\frac{1}{9}$	$\frac{1}{36}$	$\frac{17}{36}$
2	$\frac{1}{12}$	$\frac{1}{9}$	$\frac{1}{18}$	$\frac{1}{4}$
3	$\frac{1}{12}$	$\frac{1}{9}$	$\frac{1}{12}$	$\frac{5}{18}$
$E[Y \mid X_j, T_2]$	$\frac{3}{2}$	2	$\frac{7}{3}$	$E[Y \mid T_2] = \frac{65}{36}$

DGP for T_2

Further, suppose 66 *trials played by the two subjects produce the following outcomes.*

Y	X_1, T_1	X_2, T_1	X_3, T_1	X_1, T_2	X_2, T_2	X_3, T_2
1	15	8	0	12	4	1
2	3	4	0	3	4	2
3	0	0	0	3	4	3
$E[Y \mid X_j, T_k]$	$\frac{7}{6}$	$\frac{4}{3}$	NA	$\frac{3}{2}$	2	$\frac{7}{3}$

$D = 1$, *strategies played*

If the exhaustive covariate strategy is employed we also have the following (counterfactual) data.

Y	X_1, T_1	X_2, T_1	X_3, T_1	X_1, T_2	X_2, T_2	X_3, T_2
1	30	16	9	12	8	5
2	6	8	6	3	8	10
3	0	0	3	3	8	15
$E[Y \mid X_j, T_k]$	$\frac{7}{6}$	$\frac{4}{3}$	$\frac{5}{3}$	$\frac{3}{2}$	2	$\frac{7}{3}$

$D = 0$, *counterfactual strategies*

There is sample selection bias as the strategies played fail to reflect full support and accordingly don't mirror the population at X_3. Standard ANOVA designs identify the reported conditional means, $E[Y \mid X_j, T_k]$ for $j = 1, 2$ but not for $j = 3$.[3]

	X_1, T_k	X_2, T_k	X_3, T_k
$\alpha_j = E[Y \mid X_j, T_1]$	$\frac{7}{6}$	$\frac{4}{3}$	NA
$\gamma_j = E[Y \mid X_j, T_2]$	$\frac{3}{2}$	2	$\frac{7}{3}$

Mean differences in treatment conditional on the level of the covariate are straightforward.

$\gamma_j - \alpha_j =$	X_1	X_2	X_3
$E[Y \mid X_j, T_2] - E[Y \mid X_j, T_1]$	$\frac{3}{2} - \frac{7}{6} = \frac{1}{3}$	$2 - \frac{4}{3} = \frac{2}{3}$	$\frac{7}{3} - NA = NA$

Again, the treatment effect depends on the level of the covariate in this setting, $E[Y \mid X_1, T_2] - E[Y \mid X_1, T_1] \neq E[Y \mid X_2, T_2] - E[Y \mid X_2, T_1]$.

Again, lack of complete common support limits inference, but the unobservability of endogeneity poses a different challenge. The next example describes a dual treatment version of this challenge.

Example 7 (Two treatments, endogenous covariate) *Suppose the DGP remains the same as in example 5.*

Now, suppose 72 trials played by the two subjects produce the following outcomes.

Y	X_1, T_1	X_2, T_1	X_3, T_1	X_1, T_2	X_2, T_2	X_3, T_2
1	16	9	4	15	4	3
2	2	3	1	2	4	2
3	0	0	1	1	4	1
$E[Y \mid X_j, T_k]$	$\frac{10}{9}$	$\frac{5}{4}$	$\frac{3}{2}$	$\frac{11}{9}$	2	$\frac{5}{3}$

$D = 1$, *strategies played*

If the exhaustive covariate strategy is employed we also have the following

[3] In standard mean difference, interaction form the ANOVA is

$$E[Y \mid X, T] = \frac{7}{6} + \frac{1}{6}X_2 + \frac{7}{6}X_3 + \frac{1}{3}T_2 + \frac{2}{3}X_2 \times T_2$$

where the treatment effect at X_3 is unknown as the strategies played fail to supply evidence.

(counterfactual) data.

Y	X_1, T_1	X_2, T_1	X_3, T_1	X_1, T_2	X_2, T_2	X_3, T_2
1	14	15	14	9	8	3
2	4	9	11	4	8	10
3	0	0	5	5	8	17
$E[Y \mid X_j, T_k]$	$\frac{11}{9}$	$\frac{11}{8}$	$\frac{17}{10}$	$\frac{16}{9}$	2	$\frac{37}{15}$

$D = 0$, *counterfactual strategies*

A standard ANOVA design that ignores the distinction between strategies played and counterfactual strategies identifies the conditional population means via the reported conditional means

	X_1, T_k	X_2, T_k	X_3, T_k
$\alpha_j = E[Y \mid X_j, T_1]$	$\frac{7}{6}$	$\frac{4}{3}$	$\frac{5}{3}$
$\gamma_j = E[Y \mid X_j, T_2]$	$\frac{3}{2}$	2	$\frac{7}{3}$

while ANOVA for the strategies played yields

	X_1, T_k	X_2, T_k	X_3, T_k
$\widehat{\alpha}_j \mid X_j, T_1$	$\frac{10}{9}$	$\frac{5}{4}$	$\frac{3}{2}$
$\widehat{\gamma}_j \mid X_j, T_2$	$\frac{11}{9}$	2	$\frac{5}{3}$

Again, the latter is contaminated by sample selection bias.

The treatment effect conditional on the covariate levels are the same as in example **??**.

$\gamma_j - \alpha_j =$	X_1	X_2	X_3
$E[Y \mid X_j, T_2] - E[Y \mid X_j, T_1]$	$\frac{3}{2} - \frac{7}{6} = \frac{1}{3}$	$2 - \frac{4}{3} = \frac{2}{3}$	$\frac{7}{3} - \frac{5}{3} = \frac{2}{3}$

However, the estimated treatment effect conditional on the covariate levels based on the strategies played are biased.

	X_1	X_2	X_3
$\widehat{\gamma}_j - \widehat{\alpha}_j$	$\frac{11}{9} - \frac{10}{9} = \frac{1}{9}$	$2 - \frac{5}{4} = \frac{3}{4}$	$\frac{5}{3} - \frac{3}{2} = \frac{1}{6}$

2 Instrumental variables strategies

The foregoing exhaustive covariate sampling strategy suffers the drawback that eliciting responses at all levels of the other subject's potential strategy may change the game for the subject (effectively transforming sequential play into simultaneous play). Next, we consider experimental strategies that only utilize the strategies played by the subjects (in the above examples, the bottom third of the data). This eliminates the concern expressed immediately above regarding the exhaustive covariate sampling strategy, however it raises serious challenges.

It is typically difficult to identify instrumental variables and this setting presents similar challenges. The ANOVA design described above seeks to identify means and mean differences across factor levels. However, mean estimates are potentially confounded by nonrandom sampling. Effective instruments can mitigate this problem. To illustrate, we return to example 3 and discuss appropriate instruments as well as implications of poor instruments. Poor instruments can result in greater selection bias than exogenous treatment (ignoring the potential covariate endogeneity).

2.1 ANOVA design with instruments

Example 8 (One treatment, IV strategy) *Return to the DGP for treatment two in example 5 except for clarity we write out the representative data including the unobservable or error component, ε, as well as the number of occurrences of each case, m, and appropriate instruments, z_1 and z_3 (in this case $z_2 = X_2$).*

Y	X_1	X_2	X_3	ε	m	z_1	z_3
1	1	0	0	$-\frac{1}{2}$	15	$\frac{32}{41}$	$-\frac{4}{41}$
2	1	0	0	$\frac{1}{2}$	2	$\frac{50}{41}$	$\frac{4}{41}$
3	1	0	0	$\frac{3}{2}$	1	$\frac{68}{41}$	$\frac{12}{41}$
1	0	1	0	-1	4	$\frac{23}{41}$	$-\frac{8}{41}$
2	0	1	0	0	4	1	0
3	0	1	0	1	4	$\frac{59}{41}$	$\frac{8}{41}$
1	0	0	1	$-\frac{4}{3}$	3	$\frac{17}{41}$	$\frac{91}{123}$
2	0	0	1	$-\frac{1}{3}$	2	$\frac{35}{41}$	$\frac{115}{123}$
3	0	0	1	$\frac{2}{3}$	1	$\frac{53}{41}$	$\frac{139}{123}$

Selective sampling (some covariate levels are over-represented while others are under-represented) produces an omitted,correlated variables bias (X_j is correlated with ε, for $j = 1, 3$) in the mean or mean differences estimated by standard ANOVA procedures (say, OLS regression). Standard 2SLS-IV effectively identifies the means or mean differences.

	X_1	X_2	X_3
$\alpha_j = E[Y \mid X_j]$	$\frac{3}{2}$	2	$\frac{7}{3}$

However, exogenous treatment yields biased estimates.

	X_1	X_2	X_3
$\widehat{\alpha}_j$	$\frac{11}{9}$	2	$\frac{5}{3}$
$\widehat{\alpha}_j - \alpha_j$	$\frac{11}{9} - \frac{3}{2} = -\frac{5}{18}$	$2 - 2 = 0$	$\frac{5}{3} - \frac{7}{3} = -\frac{2}{3}$

Not only is it challenging to identify as many instruments as covariate levels (for instance, for two treatment with three levels each, this involves six potential instruments) but also poor instruments can produce greater bias than ignoring endogeneity. This fragility is illustrated next.

Example 9 (One treatment, poor instruments) *Suppose everything is the same as in example 8 except the instruments (w_1, w_2, and w_3 in place of z_1, z_2, and z_3).*

Y	X_1	X_2	X_3	ε	m	w_1	w_2	w_3
1	1	0	0	$-\frac{1}{2}$	15	$\frac{81}{82}$	$-\frac{2}{41}$	$\frac{1}{41}$
2	1	0	0	$\frac{1}{2}$	2	$\frac{83}{82}$	$\frac{2}{41}$	$-\frac{1}{41}$
3	1	0	0	$\frac{3}{2}$	1	$\frac{85}{82}$	$\frac{6}{41}$	$-\frac{3}{41}$
1	0	1	0	-1	4	$\frac{40}{41}$	$\frac{37}{41}$	$\frac{2}{41}$
2	0	1	0	0	4	1	1	0
3	0	1	0	1	4	$\frac{42}{41}$	$\frac{45}{41}$	$-\frac{2}{41}$
1	0	0	1	$-\frac{4}{3}$	3	$\frac{119}{123}$	$-\frac{16}{123}$	$\frac{131}{123}$
2	0	0	1	$-\frac{1}{3}$	2	$\frac{122}{123}$	$-\frac{4}{123}$	$\frac{125}{123}$
3	0	0	1	$\frac{2}{3}$	1	$\frac{125}{123}$	$\frac{8}{123}$	$\frac{119}{123}$

While the deviations in w_j relative to z_j may seem rather small, the impact is disastrous. 2SLS-IV based on w yields

	X_1	X_2	X_3
$\widetilde{\alpha}_j$	$\frac{912}{767}$	$\frac{877}{412}$	$\frac{546}{355}$
$\widetilde{\alpha}_j - \alpha_j$	$\frac{912}{767} - \frac{3}{2} = -\frac{38}{127}$	$\frac{877}{412} - 2 = \frac{53}{412}$	$\frac{546}{355} - \frac{7}{3} = -\frac{610}{767}$

which involves greater bias than exogenous treatment.

	X_1	X_2	X_3
$\widehat{\alpha}_j$	$\frac{11}{9}$	2	$\frac{5}{3}$
$\widehat{\alpha}_j - \alpha_j$	$\frac{11}{9} - \frac{3}{2} = -\frac{5}{18}$	$2 - 2 = 0$	$\frac{5}{3} - \frac{7}{3} = -\frac{2}{3}$
$\lvert\widetilde{\alpha}_j - \alpha_j\rvert - \lvert\widehat{\alpha}_j - \alpha_j\rvert$	$\frac{14}{653}$	$\frac{53}{412}$	$\frac{53}{412}$

2.2 Strategic choice with instruments

An alternative instrumental variable strategy utilizes a strategic choice modeling approach. That is, model the joint strategies of the players as a random utility model. This implies nine outcomes for the setting in example 8 Y_{ij} where i refers to one player's choice (X) and j refers to the second player's choice (Y). Conveniently, we don't require a link function in this setting as a linear probability model is not confounded by the usual problems as everything is binary (bounded between zero and one) and without additional covariates we're simply exploring frequencies. While this addresses the idea that each player's strategy depends on the other player's strategy (endogenous play), by itself it fails to address the selective sampling problem described in the foregoing discussion. To address selective sampling we add an instrument. An example helps clarify.

Example 10 (One treatment, strategic choice) *Suppose the DGP is the same as example 8 but the experimental design is one of strategic choice with instruments. A representative* 36 *observation sample is below.*

Y_{ij}	m	ε	z
$Y_{11}=1$	15	$1-\frac{1}{3}=\frac{2}{3}$	$\frac{48}{79}$
$Y_{12}=1$	2	$1-\frac{1}{12}=\frac{11}{12}$	$\frac{157}{135}$
$Y_{13}=1$	1	$1-\frac{1}{12}=\frac{11}{12}$	$\frac{1777}{764}$
$Y_{21}=1$	4	$1-\frac{1}{9}=\frac{8}{9}$	$\frac{383}{494}$
$Y_{22}=1$	4	$1-\frac{1}{9}=\frac{8}{9}$	$\frac{383}{494}$
$Y_{23}=1$	4	$1-\frac{1}{9}=\frac{8}{9}$	$\frac{383}{494}$
$Y_{31}=1$	3	$1-\frac{1}{36}=\frac{35}{36}$	$\frac{23}{89}$
$Y_{32}=1$	2	$1-\frac{1}{18}=\frac{17}{18}$	$\frac{383}{494}$
$Y_{33}=1$	1	$1-\frac{1}{12}=\frac{11}{12}$	$\frac{1777}{764}$

Without the instrument the estimated joint probabilities are simply the sample frequencies, f, while employment of the instrument recovers the joint distribution for the DGP, p.

Y_{ij}	f	p	$bias=f-p$
$Y_{11}=1$	$\frac{15}{36}$	$\frac{1}{3}$	$\frac{15}{36}-\frac{1}{3}=\frac{1}{12}$
$Y_{12}=1$	$\frac{2}{36}$	$\frac{1}{12}$	$\frac{2}{36}-\frac{1}{12}=-\frac{1}{36}$
$Y_{13}=1$	$\frac{1}{36}$	$\frac{1}{12}$	$\frac{1}{36}-\frac{1}{12}=-\frac{1}{18}$
$Y_{21}=1$	$\frac{4}{36}$	$\frac{1}{9}$	$\frac{4}{36}-\frac{1}{9}=0$
$Y_{22}=1$	$\frac{4}{36}$	$\frac{1}{9}$	$\frac{4}{36}-\frac{1}{9}=0$
$Y_{23}=1$	$\frac{4}{36}$	$\frac{1}{9}$	$\frac{4}{36}-\frac{1}{9}=0$
$Y_{31}=1$	$\frac{3}{36}$	$\frac{1}{36}$	$\frac{3}{36}-\frac{1}{36}=\frac{1}{18}$
$Y_{32}=1$	$\frac{2}{36}$	$\frac{1}{18}$	$\frac{2}{36}-\frac{1}{18}=0$
$Y_{33}=1$	$\frac{1}{36}$	$\frac{1}{12}$	$\frac{1}{36}-\frac{1}{12}=-\frac{1}{18}$

This bias in probability, of course, readily translates into bias in mean estimates as described above.

As with any instrumental variable strategy, poor instruments in the strategic choice design can produce greater bias than exogenous treatment.

3 Conclusions

We have discussed exogenous and endogenous treatment effects in controlled experiments as well as common support challenges. Of course, both lack of common support and endogeneity can coexist. Unobservability of the latter is primary impetus for our proposed strategy to elicit responses at all levels of the covariates.

Causal effects rarely (if ever) come for free which applies even in a controlled experimental setting. If we ignore potential endogeneity, sample selection bias may seriously undermine our efforts to recover the DGP. If we employ exhaustive covariate sampling we may change the way the game is played. If we employ instruments and they prove to be poor instruments we may induce more selection bias than exogenous treatment. Every setting is unique and calls for the analyst to balance these concerns in selecting an experimental design — a design judgment that is properly settled at the outset of the experiment.