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The Accounting System as an Information Channel

We now deliver on our oft repeated claim that accounting uses the language and algebra of valuation to convey information. The key step is to revisit our earlier depiction of accounting, but now formally add uncertainty to the story. The resulting accounting stock, or value, and flow, or income, functions become mappings from states into signals, precisely how we have modeled information. Two important themes follow. First, any substantive information delivered in this fashion is conveyed by the accruals. Second, using the structure of "the language and algebra of valuation" is without loss of generality, it imposes no significant restriction on the accounting apparatus. It is a wonderfully flexible information channel.

To this point we have put a model of a reporting firm in place, reviewed the classical approach to economic and accounting based measures of stocks and flows, introduced the conceptual notion of information as a partition of some underlying set of states, and reviewed the essentials of consistent decision making in the presence of uncertainty and information. It is now time to expand our thinking, to formally, not metaphorically, treat accounting as a source of information. The immediate issue is how to go about formally treating accounting as a source of information. Subsequent chapters deal with the question of what information is to be conveyed in this fashion.

We begin with the example of the three period firm used in our earlier exploration of economic and accounting measurement. Expanding it to include uncertain cash flows, and paying careful attention to the accounting system in place, provides an illustration of what it means for an accounting system to serve as a source of information. We then provide an enlarged interpretation of this example, and discuss two important questions: what information is conveyed by a particular accounting system and what, if any, are the limits, in principle, to an accounting system's ability to convey information? We then examine the difference between the information content and the valuation themes that are used to describe accounting measurement.

Keep in mind the fact theory focuses on essential details, on important features or first order effects, in its role of putting structure on a class of phenomena. We now formally treat accounting as a source of information. This is accomplished by retaining the accounting structure of Chapter 4, the stock and flow functions, and adding uncertainty to the story. We continue in streamlined fashion, with an asset total being reported along with a "bottom line" income number; and, as before, this focus on essential details means we continue to assume, for convenience, we are dealing with an all equity firm.

An Old Friend

In Chapters 3 and 4 we relied on an illustration in which distinct products were produced and sold in each of three periods. The technology and prices were such that a profit maximizing choice by the firm had $q_1 = 75$ being sold at $t = 1$ (at unit price 132), $q_2 = 100$ sold at $t = 2$ (at unit price 193.6), and $q_3 = 125$ sold at $t = 3$ (at unit price 266.2). Remaining details included a sequence of labor expenditures and physical capital with an initial cost of 25,000.

Exhibit 4 in Chapter 3 summarizes the various details, and we repeat that Exhibit below. Remember that the inflows are payments from customers, the initial outflow is the payment for the physical capital, and the subsequent outflows are the payments for the labor in the respective periods.

Now, to provide a setting in which information is present we must introduce some uncertainty. To keep things uncluttered, suppose the selling price of the first product will be 132 ± 10 per unit and the selling price of the third product will be 266.2 ± 10 per unit. In each case +10 and -10 are equally likely events, and the two price events are also independent. This implies we have four equally likely combinations to worry about: first up and third down, first up and third up, first down and third down, and first down and third up.

	$t = 0$	$t = 1$	$t = 2$	$t = 3$
inflows		132(75) = 9,900	193.6(100) = 19,360	266.2(125) = 33,275
outflows	200(125) = 25,000	110(45) = 4,950	121(80) = 9,680	133.1(125) = 16,637.50
net, CF_t	- 25,000	4,950	9,680	16,637.50

Exhibit 1: Cash Inflows and Outflows for Continuing Illustration

Further suppose the setting is otherwise identical to that assumed before (e.g., the interest rate is 10%, capital is limited to $K^{max} = 125$, the spot price of first period labor is $p_{L1} = 110$, etc.) and that the firm behaves in risk neutral fashion. In addition, to keep the story as simple as possible, the firm must commit to its production schedule at time $t = 0$. This means the firm will design its activities to maximize the expected value of the present value of its cash flows.

If you lay all of this out, you will notice the expected value of each selling price is equal to the assumed known price in the original story. So the firm will settle on precisely the same input and output schedules as before. The only difference is the inflows will depend on which prices prevail, and two of these prices are uncertain.[1]

Naturally, we model the four price combinations as associated with four (equally likely) states, denoted $S = \{s_1, s_2, s_3, s_4\}$. With our many assumptions, then, the firm's cash flow will depend on which prices prevail, i.e., on which state prevails. The calculations, for each state, are displayed in Exhibit 2.

	***t* = 0**	***t* = 1**	***t* = 2**	***t* = 3**
inflows		142(75) = 10,650	193.6(100) = 19,360	256.2(125) = 32,025
outflows	200(125) = 25,000	110(45) = 4,950	121(80) = 9,680	133.1(125) = 16,637.50
net, CF_t	**-** 25,000	5,700	9,680	15,387.50

Exhibit 2A: Cash Flow Calculations for State s_1 **-** $(P_1, P_3) = (142, 256.2)$

	***t* = 0**	***t* = 1**	***t* = 2**	***t* = 3**
inflows		142(75) = 10,650	193.6(100) = 19,360	276.2(125) = 34,525
outflows	200(125) = 25,000	110(45) = 4,950	121(80) = 9,680	133.1(125) = 16,637.50
net, CF_t	- 25,000	5,700	9,680	17,887.50

Exhibit 2B: Cash Flow Calculations for State s_2 **-** $(P_1, P_3) = (142, 276.2)$

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[1]At the risk of being over bearing, let $P_t(1 + r)^t$ denote the selling price of product t (in period t, of course). The first and third of these selling prices are uncertain, all other prices and the technology are both known and identical to what was initially assumed. With technology and factor prices all known, we arrive at the cost curve as originally derived. For example, staying in the region where the $K \le K^{max}$ constraint is not binding, the cost curve is what we derived earlier:

$$C(q_1,q_2,q_3) = 200\sqrt{2(q_1^2 + q_2^2 + q_3^2)}\,.$$

In turn, the expected value of the present value of the receipts is

$$E[P_1(1.1)^{-1}]q_1 + P_2(1.1)^{-1}q_2 + E[P_3(1.1)^{-1}]q_3 = 193.6q_2(1.1)^{-2} + 266.2q_3(1.1)^{-3} = 120q_1 + 160q_2 + 200q_3.$$

And we wind up with the maximizing exercise of:

$$\underset{q_1,\, q_2,\, q_3 \ge 0}{\text{maximize}}\ 120q_1 + 160q_2 + 200q_3 - C(q_1,q_2,q_3)$$

which is what we had in the initial example.

	$t = 0$	$t = 1$	$t = 2$	$t = 3$
inflows		122(75) = 9,150	193.6(100) = 19,360	256.2(125) = 32,025
outflows	200(125) = 25,000	110(45) = 4,950	121(80) = 9,680	133.1(125) = 16,637.50
net, CF_t	- 25,000	4,200	9,680	15,387.50

Exhibit 2C: Cash Flow Calculations for State s_3 - $(P_1, P_3) = (122, 256.2)$

	$t = 0$	$t = 1$	$t = 2$	$t = 3$
inflows		122(75) = 9,150	193.6(100) = 19,360	276.2(125) = 34,525
outflows	200(125) = 25,000	110(45) = 4,950	121(80) = 9,680	133.1(125) = 16,637.50
net, CF_t	- 25,000	4,200	9,680	17,887.50

Exhibit 2D: Cash Flow Calculations for State s_4 - $(P_1, P_3) = (122, 276.2)$

In turn, we summarize these calculations with the state dependent cash flow specification, $CF(s)$, in Exhibit 3. Notice the expected value of each period's cash flow, denoted $E[CF_t]$, is, given our various assumptions, the cash flow amount we earlier assumed to be present with certainty.

$CF_t(s)$	s_1	s_2	s_3	s_4	$E[CF_t]$
$CF_0(s)$	- 25,000	- 25,000	- 25,000	- 25,000	- 25,000
$CF_1(s)$	5,700	5,700	4,200	4,200	4,950
$CF_2(s)$	9,680	9,680	9,680	9,680	9,680
$CF_3(s)$	15,387.50	17,887.50	15,387.50	17,887.50	16,637.50
$\sum_t CF_t(s)$	5,767.50	8,267.50	4,267.50	6,767.50	

Exhibit 3: Cash Flow Vector as a Function of State

Don't miss the larger picture. The firm must decide, in the face of uncertainty, which combination of products, quantities and factors is best.[2] Having so decided, it faces an uncertain cash flow sequence. And, as we see in Exhibit 3, one of four such sequences will materialize.

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[2]Just beneath the surface, then, is some outcome structure, described via $x = p(s,a)$ in the prior chapter. Here, that outcome is a sequence of cash flows. And any given choice or decision implies some cash flow sequence that depends on the state.

Earlier, in the certainty world, we specified the economic facts with a cash flow vector, *CF*, and an interest rate, *r*. Here, the cash flow vector depends on the state. Naturally, the interest rate might also depend on the state, but we will forego that refinement.[3] So the economic facts now take the form of a state dependent cash flow vector and an interest rate.

Importantly, our original story has now expanded to include an uncertain cash flow sequence. We model this uncertainty using a state descriptor, $s \in S$. So the firm's cash flow now depends on the state, and we write it $CF(s)$. Moreover, this $CF(s)$ specification reflects underlying choices by the firm.

cash flow is a source of information

Initially, we remind ourselves cash flow itself may and often does tell us something.[4] Would it, for example, be good news if a start up company's cash flow from operations was unexpectedly large (and positive)? Would it be bad news if an established firm's cash flow from operations was unexpectedly low (and negative)?

This simple idea is also present in our example. We know the first and third period's selling prices might be high or low. Observing the $t = 1$ cash flow, given our knowledge of the firm's environment and activities, allows us to figure out whether the first period's selling price was, indeed, high or low. A parallel comment applies to the third price and the $t = 3$ cash flow.

It will, however, be useful to address this in a more formal matter. Closely examine Exhibit 3. Each $CF_t(s)$ is a mapping from states, from S, into possible cash flows. For example, $CF_0(s)$ is constant across the states, while $CF_1(s)$ is 5,700 for the first two states, but 4,200 for the last two. Thus, observing $t = 1$ cash flow of 5,700 tells us $s \in \{s_1, s_2\}$. Likewise, observing $t = 1$ cash flow of 4,200 tells us $s \in \{s_3, s_4\}$. Observing the $CF_1(s)$ realization at time $t = 1$ is equivalent to observing the partition $\{\{s_1, s_2\},\{s_3, s_4\}\}$ at time $t = 1$. Cash flow is a source of information!

This follows, in a formal sense, simply because $CF_t(s)$ is, literally, a function of the state. So all we do is interpret the realization of period t cash flow as a signal. At time $t = 1$ that signal is either 5,700 or 4,200. It's as simple as that.

Continuing, $CF_2(s)$ is constant across s, but $CF_3(s)$ is another story. Observing $t = 3$ cash flow of 15,387.50 tells us $s \in \{s_1, s_3\}$. Likewise, observing $t = 3$ cash flow of 17,887.50 tells us $s \in \{s_2, s_4\}$. Observing the $CF_3(s)$ realization at time $t = 3$ is equivalent to observing the partition $\{(s_1, s_3\},\{s_2, s_4\}\}$ at $t = 3$.

Of course, observing partition $\{(s_1, s_3\},\{s_2, s_4\}\}$ at $t = 3$ having already observed partition $\{\{s_1, s_2\},\{s_3, s_4\}\}$ tells us exactly which state was present. For example, knowing $s \in \{s_1, s_2\}$ and subsequently learning $s \in \{s_1, s_3\}$ implies $s = s_1$. In temporal terms, then, the story in Exhibit 3 is equivalent to the following sequence of partitions, or information sources:

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[3]Interest rates would, presumably, reflect macro forces in the economy, as well as a market-based synthesis of all available information in the economy.

[4]Remember our story began in a perfect market setting, with no uncertainty. There was, therefore, no reason to hold cash on hand, so cash flow to and from the owners was stressed. Now treating this cash flow as a source of information is akin to treating dividends as a source of information. Enlarging the story to distinguish cash flow from operations would allow us to treat that construction as a source of information, but the expanded detail would not bring additional insight.

$t = 0$: null;
$t = 1$: $\{\{s_1, s_2\},\{s_3, s_4\}\}$;
$t = 2$: $\{\{s_1, s_2\},\{s_3, s_4\}\}$; and
$t = 3$: $\{\{s_1\},\{s_2\},\{s_3\},\{s_4\}\}$.

Our story gradually grows in information from null to perfect. Given the underlying details, this gradual refinement to perfect information is no surprise. The only uncertainty is the selling price of the first and third period's products. With known quantities, observing cash flow as we march through time inevitably reveals all there is to know. But this reflects our particularly simple story. The important point is the cash flow series is a source of information.

In a more subtle sense, we are working with a great deal of structure. We know the structure of the firm's decision problem, and we know or anticipate its decisions will culminate in the cash flow possibilities, literally, the cash flow functions, in Exhibit 3. Subsequently observing the cash flow realizations, knowing the mapping from states to cash flows, reveals something about the states. We begin, in other words, with a sound understanding of the firm's fundamentals. We therefore know the partition before us, we know Exhibit 3. We simply do not know which element of that partition is about to be revealed.[5]

accrual accounting is a source of information

Now turn to accrual reporting of this story. Initially recall the way we calculated accounting income for these transactions, in the world of certainty. Revenue was recorded at the time of delivery to the customer (and payment from the customer). Labor cost was easily matched to revenue, thanks to the technology and payment timing assumptions. And the physical capital was depreciated over the three year horizon, using a generic depreciation schedule of d_1, d_2 and 25,000 - d_1 - d_2. This earlier rendering, originally displayed in Exhibit 2 of Chapter 4, is repeated below.

	$t = 1$	$t = 2$	$t = 3$
revenue	9,900	19,360	33,275
labor expense	4,950	9,680	16,637.50
depreciation	d_1	d_2	25,000 - d_1 - d_2
accounting income	4,950 - d_1	9,680 - d_2	$d_1 + d_2$ - 8,362.50

Exhibit 4: Accounting Income Calculations for Transactions in Exhibit 1

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Suppose we adopt the same tack here. Revenue will be recognized, at the time of delivery. Labor cost is matched, as noted, and depreciation is again identified in generic terms. This

[5]Return to the underlying story. We know the firm's technology, the factor prices it faces, and the uncertain product prices it faces. We know it behaves in risk neutral fashion. We infer from all of this that its decisions lead to the cash flow possibilities in Exhibit 3, possibilities that turn on what the uncertain prices turn out to be.

accounting system is installed at time t = 0, and from that point on merely records transactions (and depreciation) as they occur. Of course, selling prices, and therefore revenue, depend on which state materializes. So the income sequence will depend on the state. You should be able to replicate the accounting income calculations summarized in Exhibit 5 below. (To check our work and your intuition, notice the total income in each state equals the total cash flow in that state, and the expected value of the income in each period equals the income for that period in Exhibit 4 above.)

$\hat{I}_t$	s_1	s_2	s_3	s_4	$E[\hat{I}_t]$
$\hat{I}_1$	5,700 - d_1	5,700 - d_1	4,200 - d_1	4,200 - d_1	4,950 - d_1
$\hat{I}_2$	9,680 - d_2	9,680 - d_2	9,680 - d_2	9,680 - d_2	9,680 - d_2
$\hat{I}_3$	$d_1 + d_2$ - 9,612.50	$d_1 + d_2$ - 7,112.50	$d_1 + d_2$ - 9,612.50	$d_1 + d_2$ - 7,112.50	$d_1 + d_2$ - 8,362.50
$\sum\hat{I}_t$	5,767.50	8,267.50	4,267.50	6,767.50	6,267.50

Exhibit 5: Possible Accounting Income Calculations for Data in Exhibit 3.

For example, under state s_1 it is learned at t = 1, when the first product's revenue is recognized, that the product's selling price is 10 above expectation, resulting in first period income that exceeds what was expected (i.e., 5,700 - d_1) by 75 units at 10 extra per unit, or 750. Likewise, at time t = 3 when the third product's revenue is recognized, a price of 10 below expectation per unit is recorded, resulting in a third period income that falls short of expectation by 125 units at 10 per unit, or 1,250. Naturally, with no price uncertainty associated with the second product we have no surprise in the second period's income. And, just as the cash flow each period is a mapping from states to possible cash flows, we see in Exhibit 5 that income each period, given a specific accounting method, is a mapping from states to possible income realizations.

Now turn to the information side of our work. Given the specified accounting treatment (i.e., the depreciation pattern of d_1 and d_2) and given our understanding of the firm's fundamentals (i.e., Exhibit 3) we know the details of Exhibit 5. We know, for example, first period income will be 5,700 - d_1 or 4200 - d_1. Think of this as a mapping from states to signals. When we see a signal, an income, of 5,700 - d_1 we know, since we know Exhibit 5, $s \in \{s_1, s_2\}$. Likewise, when we see a signal, an income, of 4,200 - d_1, we know that $s \in \{s_3, s_4\}$. Thus, observing accounting income at time t = 1 is equivalent to observing the partition $\{\{s_1, s_2\},\{s_3, s_4\}\}$. Simply working through Exhibit 5 should convince you that regardless of the choice of d_1 and d_2, observing accounting income as calculated in the Exhibit is equivalent to the following partition sequence:[6]

[6]We are focusing on the income measure because it strikes us as most intuitive in this particular setting. The same approach could be used by focusing on the balance sheet, though in this case we would want to focus on the point in time just before the cash balance is distributed to the firm's owners. Shortly, however, we will combine cash basis and accrual basis reporting, and this concern over the time at which the balance sheet is reckoned will be moot.

$t = 0$: null;
$t = 1$: $\{\{s_1, s_2\},\{s_3, s_4\}\}$;
$t = 2$: $\{\{s_1, s_2\},\{s_3, s_4\}\}$; and
$t = 3$: $\{\{s_1\},\{s_2\},\{s_3\},\{s_4\}\}$.

Accounting measurement, then, provides a mapping from states into, in this case, accounting flows, or income. This mapping induces a partition on the set of states, and we thus have an information source, a partition. Cash and accrual basis measurement, that is, are simply two different ways of doing the accounting, and both, in principle, are sources of information.

Euphoria, though, needs to be checked, as we have a slight problem with our example. The sequence of partitions induced by the accrual method in Exhibit 5 is identical to that induced by the cash basis (or cash flow) reporting in Exhibit 3. This is hardly surprising, though, as the depreciation pattern is confined to the constants d_1 and d_2. There is no way that such constants can carry any information to the accrual process. If accounting accruals reveal nothing, information-wise, that is not discernible from the cash flow series itself, we have arrived at a particularly embarrassing, nebulous view of accounting.

A Richer Reporting Structure

There are two keys to moving forward. One is to endow the accrual process with the ability to reveal more than simply tracking the cash flow. The accruals must access some underlying information and carry it forward into the accounts. For example, when a troubled receivable is written off, this write-off is based on additional information suggesting the receivable will not be collected. Likewise, the usual accrual story based on the going concern assumption is actually a story based on the lack of information suggesting the going concern assumption is problematic. Our accrual rendering, then, cannot be as mechanical as that depicted in Exhibit 5 if we are to capture this essential feature of the accrual process.

The second key is to remember we typically report cash and accrual-based measures. Treating both as information sources, then, requires we carry along the substance and the fabric of multiple information sources.

Return to our example, but further suppose the accrual system can record a restructuring charge at time $t = 1$, in the amount d, if the first period's selling price is high, but the third period's price will be low.[7] Intuitively, the firm is experiencing good times (a high selling price) in the first period, but also has learned those good times will not continue (a low selling price will be forthcoming in the third period). So the asset is written down via the additional charge of d. Glancing back at Exhibits 2 and 3 you should notice this combination of high initial followed by low subsequent price is the state s_1 story. We further assume, simply for convenience, that the firm cannot act on this foreknowledge of the third price. This means we stay with the original production plan and the cash flow details in Exhibits 2 and 3. But the accrual story is affected by

[7]Remember, this illustration is purposely kept simple. The only uncertainty is the selling prices in the first and third period. With everything else kept constant, merely observing the cash flows will reveal what prices actually prevailed. So, if the accounting system is to tell us more than can be gleaned by observing cash flow, it must somehow be observing and acting upon something over and above the cash flows themselves.

this possibility. Details are in Exhibit 6.

$\hat{I}_t$	s_1	s_2	s_3	s_4	$E[\hat{I}_t]$
$\hat{I}_1$	5,700 - d_1 - d	5,700 - d_1	4,200 - d_1	4,200 - d_1	4,950 - d_1 - .25d
$\hat{I}_2$	9,680 - d_2	9,680 - d_2	9,680 - d_2	9,680 - d_2	9,680 - d_2
$\hat{I}_3$	d_1 + d_2 + d - 9,612.50	d_1 + d_2 - 7,112.50	d_1 + d_2 - 9,612.50	d_1 + d_2 - 7,112.50	d_1 + d_2 + .25d - 8,362.50
$\sum\hat{I}_t$	5,767.50	8,267.50	4,267.50	6,767.50	6,267.50

Exhibit 6: Revised Accounting Income Calculations

Note well. Everything is as before, with the single exception an additional expense, in the amount d, is recorded in state s_1. For *any* non-zero restructuring charge d and any depreciation pattern (d_1 and d_2), at time $t = 1$ we have three distinct income reports: 5,700 - d_1 - d, 5,700 - d_1 and 4,200 - d_1. And these three possible reports correspond, respectively, to $\{s_1\}$, $\{s_2\}$ and $\{s_3, s_4\}$. Just as before, the second period's income is constant across the states. And the third period's report will be one of three distinct reports (again, for any non-zero d and completely arbitrary d_1 and d_2). Overall, then, the proposed accounting system is equivalent to the following sequence of partitions:[8]

$t = 0$: null;
$t = 1$: $\{\{s_1\},\{s_2\},\{s_3, s_4\}\}$;
$t = 2$: $\{\{s_1\},\{s_2\},\{s_3, s_4\}\}$; and
$t = 3$: $\{\{s_1\},\{s_2\},\{s_3\},\{s_4\}\}$.

The $t = 0$, $t = 2$ and $t = 3$ partitions are identical to those provided by the cash flow itself, but the $t = 1$ partition is a subpartition of that provided by the cash flow measure. (That is, partition $\{\{s_1\},\{s_2\},\{s_3, s_4\}\}$ is a subpartition of $\{\{s_1, s_2\},\{s_3, s_4\}\}$.) Moreover, the $t = 2$ partition is identical to that of $t = 1$, as we accumulate information over time, we never forget. Initially we know the depreciation and restructuring charge patterns. Given that, we interpret the observed accounting income by translating it into an element of the partition that is induced by the underlying process. We thereby learn more about which state of the world we are facing. This illustrates the idea of an accrual based system carrying more information than a cash basis system.

Several points are in order. First, the movement from the accounting income report to the

[8]Indeed, cash flow itself is redundant here in the presence of accounting income. But that need not necessarily be the case. This is why we craft the exercise in terms of cash flow and accounting accruals being informationally equivalent to cash flow and whatever else the accounting system is able to access in its calculations. To appreciate this, suppose we set $t = 1$ depreciation at $d_1 = 2{,}400$ in the first two states, but at $d_1 = 900$ in the second two states. This equalizes accounting income across the four states, except for the restructuring charge in state s_1. In this case, we would combine the cash flow and accounting income reports to de facto observe the noted partition at time $t = 1$.

underlying partition is, as usual, a decoding exercise. It presumes we understand the firm's fundamentals and its choice of accounting method. The typical financial report, recall, details in the first note what accounting policies (e.g., depreciation and inventory valuation) are being used. In parallel fashion, we would never use an accounting system's measurement of product cost without knowledge of the costing procedures used to produce that datum. So the decoding begins with knowledge of the firm's fundamentals and knowledge of what accounting procedures the firm is employing. With this in place, the details in Exhibit 6 are rationalized, and we have a mapping from states to signals, an information source in our language.[9]

Second, and closely related, is the issue of scaling. The task assigned to the accrual process here is to report the above noted sequence of partitions. We are accustomed to, say, Celsius and Fahrenheit temperatures scales, essentially different scales used to measure the same phenomenon. And scaling options are present here as well. As we have stressed, in Exhibit 6 d_1 and d_2 are arbitrary constants and d can be any number other than zero.[10] Such enormous freedom in the scaling is driven by the simplicity of the example, but scaling is an important issue that will remain throughout our study. It is also an important issue in the world of affairs. From an information perspective, it makes no difference whether goodwill is written off immediately, or amortized over 10, 20 or 50 years. It makes no difference if all depreciation schedules are straight line or accelerated so long as an utterly mechanical approach is taken. It makes no difference what percentage of sales we accrue as uncollectible, as long as that percentage is a constant. In all of these cases, we would see through the particular imposed regularity in the recording.[11]

Third, the essence of the story in Exhibit 6 is the accrual process is reporting something that cannot be discerned by merely observing the cash flow. The critical addition, of course, occurs at time $t = 1$. There, cash flow reveals the partition $\{\{s_1, s_2\},\{s_3, s_4\}\}$, but the income measure reveals the subpartition $\{\{s_1\},\{s_2\},\{s_3, s_4\}\}$. The accrual process is here able to report some type of "forward looking" information. Implicitly, then, the firm is somehow observing the partition $\{\{s_1\},\{s_2, s_3, s_4\}\}$, as this is essential if the firm is going to know enough to make the advertized restructuring entry or adjustment. In other words, if the accrual process is to report something over and above what is reported by the cash flow itself at that point in time, the firm must have access, somehow, to that additional underlying information.

Finally, the corresponding asset balances directly follow, given the cash balance is

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[9]In turn, the firm's fundamentals are determined at its inception, and any change in fundamentals is a random event that is captured by a well specified state description. Similarly, the firm's accounting methods are chosen, or specified, at inception. Any change in those methods is, again, captured by a well specified state description. So far, of course, we have not explicitly dealt with selecting the accounting method. That will be addressed in due course. For the moment we concern ourselves with how to model the notion that accounting is, indeed, a source of information.

[10]This is not a cynical comment. In our example, the depreciation schedule itself is not called upon to carry any information. It is essentially arbitrary. And the restructuring charge is merely an indicator, simply because the price revelation it is reporting can be but one of two values.

[11]A subtle point is also present here. In, say, the goodwill case, if we write it off immediately, we have destroyed the option to write it off later. Conversely, if we write it off over, say, 20 years but prior to the end of that horizon it turns out this venture has gone sour we have the option of accelerating the remaining writeoff. But this is again letting the accounting system access an underlying information source and report accordingly.

immediately distributed to the owners and given we insist on articulation. See Exhibit 7. (For example, at time t = 1, no restructuring charge is recorded under states s_2, s_3 or s_4 and the depreciation expense totals d_1, so the net asset value is 25,000 - d_1.)

$A_t(s)$	s_1	s_2	s_3	s_4
$A_1(s)$	25,000 - d_1 - d	25,000 - d_1	25,000 - d_1	25,000 - d_1
$A_2(s)$	25,000 - d_1 - d_2 - d	25,000 - d_1 - d_2	25,000 - d_1 - d_2	25,000 - d_1 - d_2
$A_3(s)$	0	0	0	0

Exhibit 6: Companion Asset Values for Story in Exhibit 6

Accounting Stocks and Flows

We use the device of an unknown state variable to model uncertainty, and we use the idea of observing a partition (technically, an element of a partition) of the set of possible states as our model of information. In turn, this information partition can be thought of as being produced by some mapping from states into signals. Cash flow, we have stressed, is such a mapping. It is a source of information. Accrual accounting, formally understood, is as well. That is why understanding Exhibits 5 and 6 is so important.

Now, earlier, in Chapter 4, we introduced accounting stock and flow measures as cousins, so to speak, of their economic counterparts. There, accounting valuation was depicted as a mapping that took some specific cash flow vector, CF, and interest rate, r, and converted those fundamentals into a sequence of book value or stock reports. We wrote this as follows:

$$A(CF,r) = [A_0,A_1, ...,A_T]. \qquad [1]$$

The only restriction we placed on this mapping was the boundary conditions at birth and death of the firm: $A_0(CF,r) = - CF_0$ and $A_T(CF,r) = 0$. In parallel fashion, we depicted accounting income as a mapping that took this CF and r into a sequence of income or flow reports:

$$\hat{I}(CF,r) = [\hat{I}_1,\hat{I}_2, ...,\hat{I}_T]. \qquad [2]$$

The only restriction here was that everything add up in the end.[12] But this adding up, in turn, became superfluous when we imposed articulation:

$$\hat{I}_t = A_t - A_{t-1} + CF_t, \text{ for } t = 1, 2, ..., T. \qquad [3]$$

[12]That is, $\sum_{t=1}^{T} \hat{I}_t = \sum_{t=0}^{T} CF_t$.

This, of course, all depends on certainty, the very antithesis of an information perspective. And it is vital we understand how this characterization extends to our uncertain setting.

The trick, quite simply, is to carry along the state variable. Glance back at Exhibit 6, where we have the restructuring charge of d if state s_1 prevails. The net asset balance clearly depends on the state. So, depending on the firm's business plan or strategy (Exhibits 2 and 3) and accounting method (here choice of d, d_1 and d_2 along with the noted recognition and matching conventions), the time t accounting value or stock report further depends on the realization of $s \in S$. This is the $A_t(s)$ series given in Exhibit 6.

So we now express the accounting stock measure as some function that maps the firm's fundamentals, $CF(s)$, the interest rate r, and the state $s \in S$ into a sequence of reported values:

$$A(CF,r,s) = [A_0(s),A_1(s), ...,A_T(s)]. \qquad [4]$$

And we naturally continue to respect the boundary conditions, now written as $A_0(CF,r,s) = -CF_0(s)$ and $A_T(CF,r,s) = 0$, for each $s \in S$. Exhibit 6 is illustrative.

Likewise, the accounting income or flow measures are now depicted in state dependent fashion as

$$\hat{I}(CF,r,s) = [\hat{I}_1(s),\hat{I}_2(s), ...,\hat{I}_T(s)]. \qquad [5]$$

And articulation naturally requires

$$\hat{I}_t(s) = A_t(s) - A_{t-1}(s) + CF_t(s), \text{ for } t = 1, 2, ..., T \qquad [6]$$

which, again, ensures the previously noted "adding up" requirement, now on a state by state basis.

This "visualization" in [4], [5] and [6], this extension of the certainty setting of Chapter 4 to a setting that includes uncertainty, is central to our information content approach. At time t, cash basis reporting reveals the realization of $CF_t(s)$, and the accounting system reveals the realization of its stock, $A_t(s)$, and flow, $\hat{I}_t(s)$, calculations. We have three mappings from states to signals, three information sources, $CF_t(s)$, $A_t(s)$ and $\hat{I}_t(s)$. (Though given the duality imposed by articulation, one of the accounting measures is redundant, given the other and given the cash flow observation.)

The Language and Algebra of Valuation

Conceptually, then, we view accounting stock and flow measures as reporting, at each instance, some partition of the underlying state. Clearly, if the accounting system is indeed reporting some partition, the firm must be observing that partition. Otherwise it would not know enough to "do the accounting" in the desired fashion. For example, using a lower of cost or market rule presumes the firm is observing a reliable market price in the first place. Likewise, recording a problematic liability presumes the firm knows about that liability.

But what about the reverse question? The apparatus, the language of stocks and flows and the underlying algebra, has its roots in the classical theory of value and income measurement. We periodically hear concerns that the accounting model is inadequate or somehow deeply flawed.

For example, as of this writing we are told the accounting model, not to mention economic theory in general, simply could not cope with so-called "dot coms" (firms that display an unusual combination of possible future prospects and a near empty current balance sheet) and the inherent option nature of their fundamentals.

To bring this concern into focus, suppose the firm's fundamentals result in the state dependent cash flows of $CF(s) = [CF_0(s), CF_1(s), ..., CF_T(s)]$, a vector of time dated cash flows (to and from the owners, of course) that depends on state $s \in S$. Further assume the firm has access to, and wants to convey via the accounting system, whatever it observes from the following sequence of information sources:

$$\eta(s) = [\eta_0(s), \eta_1(s), ..., \eta_T(s)].$$

At period t, then, the firm learns the realizations of these two information sources; denote them by $c_t = CF_t(s)$ and $y_t = \eta_t(s)$, for $t = 0, 1, ..., T$. c_t, of course, is the cash flow at time t, which for convenience we assume is distributed to the owners. y_t, in turn, might be additional information dealing with product development activities, product market issues, etc.

The accounting system has access to, can observe and process, these additional pieces of information, as well as what was observed in prior periods. As of time t, then, write this string of observations as

$$h_t \equiv (c_0, y_0; c_1, y_1; ...; c_t, y_t).$$

We interpret h_t as the accounting system's historical documentation (or audit trail) as of time t. It is the full history of whatever the accounting system has archived.[13] Notice h_t is, in reality, a function of the state. It describes an ever re-fining partition of S, or a sequence of subpartitions of S. And conveniently, we have a recursive expression of $h_t = (h_{t-1}; c_t, y_t)$.

Now, since the accounting stock measure at time t can only use this history of realizations, it cannot rely on additional details beyond this history, we can re-express the stock measurement at time t as

$$A_t(h_t) = A_t(h_{t-1}; c_t, y_t). \qquad [4a]$$

That is, the valuation at time t depends on what the accounting system "knows" or processes at time t. And what it knows at time t is what it knew at time $t - 1$ coupled with the new observations (the cash flow and additional information realizations) in period t.

In parallel fashion, imagine an accounting income series that articulates with this accounting

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[13]A natural interpretation here is the accounting system maintains a library of observations. Examples are recent monthly statistics on a division's performance, the profile of the firm's physical assets, and the compensation records for its various employees. Naturally, we expect a great deal of aggregation as well. It is also important to understand h_t is not meant to catalogue everything the firm has learned or observed through time t. Hardly. It catalogues the subset of these observations that are allowed to enter or influence the accounting statements. For example, a major R&D breakthrough will be awfully important, but will not be immediately recognized in the accounting system.

stock series:

$$\hat{I}_t(h_{t-1}; c_t,y_t) = \hat{I}_t(h_t) = c_t + A_t(h_{t-1}; c_t,y_t) - A_{t-1}(h_{t-1}). \quad [6a]$$

(Notice how we carry the history forward, making certain the change in accounting value is consistent with the underlying history.)

Given duality between the stock and flow measures, it is sufficient to focus on one of these series. The accounting value series is more transparent in what follows, so that is our focus. Now, the underlying information structures are the partitions defined by $CF(s)$ and $\eta(s)$. Under what conditions, then, would this sequence of information structures be informationally equivalent to reporting cash flow and accounting value each period? Cash flow in period t, c_t, is reported each period under either regime. That part is easy.

The real question, then, is whether we can decode the accounting value calculation to infer the underlying additional information event conveyed by $\eta(s)$. Suppose the two regimes are informationally equivalent up to time t. Then when the time t report is observed we know the prior history h_{t-1}, current cash flow, c_t, and current accounting value, $A_t(h_{t-1}; c_t,y_t)$.

Do we therefore know, or can we infer, the underlying additional information signal, y_t? The answer is yes if the accounting valuation function at this point reports a distinct value for each possible realization of y_t. Stated differently, we can decode the observables at this point to learn y_t if the valuation function $A_t(h_{t-1}; c_t,y_t)$ is invertible in y_t for each possible partial history, consisting of h_{t-1} and c_t.

Return to Exhibit 6. Our use of a nonzero restructuring charge (i.e., d) when $\eta_1(s)$ reports s_1 is true accomplishes precisely this. If we observe the restructuring charge we know the information event is $\{s_1\}$, just as we know it is $\{s_2, s_3, s_4\}$ if we do not observe the charge. It ensures we can decode the accounting series, in the presence of the cash flow series, to discern the underlying information.

The point, now, is the language and algebra of classical valuation that the accounting system uses do not preclude it from being an information source. But how onerous is the invertibility condition? Somewhat casually, we merely must guarantee the valuation function has enough flexibility to respond to the underlying information. Then, what kind of restrictions did we place upon the accounting series? The answer is next to none. We only imposed a set of boundary conditions in our valuation series that requires $A_T(h_T) = 0$. Consequently it is important that $\eta_T(s)$ be null for the invertibility condition to hold. Otherwise, it is of no import.

We identified accruals early on as the centerpiece of the accounting product. Notice here that, if cash basis recognition is used, the accounting system only reports cash flow and has no ability to convey anything else. Moreover, if serious accrual accounting is engaged, any information conveyed beyond that conveyed by the cash flow itself is embedded in the accruals. Deciphering the accruals in $A_t(h_{t-1}; c_t,y_t)$ is precisely how the additional information is conveyed.[14] Thus, the

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[14]A number of more subtle issues also arise here. First, in our simple example, d nonzero and completely arbitrary d_1 and d_2 convey the noted information. In this sense, the measurement apparatus offers a wide variety of equivalent scales. And we see the same in practice. Various stock depreciation patterns are illustrative. Second, the invertibility argument has the flavor of accounting using a specific code, and the user must be able to decode the report. This decoding is most simple when no other

accounting system's ability to convey information is dependent upon its ability to distinguish and signal different elements of a partition. The variation of the accounting report across different states carries the information. It is not the value in one particular state that matters.

The conclusion should not be missed. The concern over the accounting system's ability, in principle, to convey information is without merit. The accounting system's use of the language and algebra of valuation is, from an information perspective, a simple choice of scale with which to report some underlying information base. This means we must look elsewhere to understand why we see some things, such as sales and depreciation routinely reported while other things, such as order books or growth prospects associated with successful R&D, not routinely reported.

Valuation versus Information Content Themes

One schooled in the valuation view of accounting measurement will, no doubt, find the construction in Exhibit 6 baffling, if not mis-directed. A valuation perspective implores us to conceptually view the accounting system as reporting stocks and flows of economic value. We could not, then, offer an allegedly well-functioning construction, as in Exhibit 6, where for any non-zero restructuring charge and any depreciation schedule we are perfectly content with the accounting. The valuation theme implores us to report value, without ambiguity. Of course there is always the issue of which currency to use, and how to peg it to some purchasing power base. But these are regarded as minor annoyances within the larger theme of reporting value, or fair value in today's vernacular. There is also the problematic and hardly minor issue of what we mean by value when markets are not perfect and complete.

The information content theme, on the other hand, identifies the substance of the accounting report with the information it conveys. Conceptually, information is a partition of the underlying state space at some moment in time.[15] The precise language used to convey this information is, while clearly important in the world of affairs, strictly speaking outside the theory. It is the underlying partition that matters. We often think of temperature as a source of information, and are accustomed to different temperature scales. The same holds for information sources in general. Varying d (as long as it is not zero) and the depreciation pattern in Exhibit 6 merely re-scale the measurement scale, so to speak. Celsius and Fahrenheit scales tell us the same thing, just as do various combinations of d, d_1 and d_2 in Exhibit 6.

We do, to be sure, rely on various conventions here. U.S. GAAP, for example, codifies many of the scaling conventions a firm might employ. And, for that matter, the International Accounting Standards Board (IASB) offers a competing codification. We also require the organization disclose its accounting policies (here, the state partition and scaling convention in place), and any changes therein. Knowledge of this is important when the information that is contained in the accounting report is interpreted and translated into a useful format. This is the invertibility

information sources are available, but the same argument extends to a setting of multiple information sources. Finally, many look for a simple linear processing of the (coded) report, say value is equal to the capitalized income stream. But this is, as we shall see, a rather simplistic view of the world of affairs, not to mention theory.

[15]Subsequently, when we introduce labor contracting issues, we will expand the domain of the information story to include states and "acts" taken by managers, or agents.

condition at work.

It is also important to acknowledge the valuation and information content themes numerically converge. In the long run, total accounting income will approach total economic income (plus rent), simply because the two calculations converge to the total cash flow over the long horizon.[16]

But the two perspectives remain fundamentally different. The valuation school emphasizes stocks and flows of values, yet in practice it steps away from problematic valuations. Examples are R&D findings, human capital, and national highway systems. These items are surely not valued in any typical accounting system. The information content school emphasizes information conveyed by the accounting system. The invertibility requirement discussed here is sufficiently general that the substantive issue becomes what information to convey. Surely, again taking our cue from practice, the answer is not all information. In practice, the accounting system steps away from problematic information. Examples are new product plans, order books, and market values of various assets. Moreover, the information content theme confronts us with an array of scaling options (taking us back to the d, d_1 and d_2 parameters in Exhibit 6 again).

For now, though, the important point is to understand what it means to treat the accounting system as an information channel. Viewed so, the accounting system uses the language and algebra of valuation to convey its information. And this structure, this tie to valuation mechanics, is not constraining. The comparative advantage of the accounting channel, then, is not to be found in its use of valuation algebra and language. But to explore this further we must turn to the users of the accounting product.

Summary

Accounting valuation was described in Chapter 4 as a procedure that maps a cash flow series (cash that flows from the firm to the owners), and interest rate assumption, into a sequence of values subject to boundary conditions that the initial valuation is the invested amount and the terminal valuation is zero. We now extend this description to include uncertainty: a procedure that maps a cash flow series (cash that flows from the firm to the owners), interest rate assumption and states, into a sequence of values (or signals) subject to the noted boundary conditions. This is the essence of [4] and [4a]. Likewise, our earlier, certainty assuming, description of accounting income measurement is now extended to a mapping (or decomposition) of a cash flow series, interest rate assumption, and states into period by period amounts (or signals) that sum to the total of the given cash flow series. This is the essence of [6] and [6a], presuming articulation. Thus, with the $CF(s)$ and interest rate specified, accounting valuation and income measurement provide mappings from states into signals, the very essence of an information source. Under uncertainty, then, accounting value is the result of applying a particular accounting procedure, accounting valuation mapping, to a specific setting, a specific signal realization; and accounting income is the result of applying a particular procedure, accounting income mapping, to a specific setting, a

[16]This statement is a bit tongue-in-cheek. Under perfect and complete markets, economic value and economic income are well defined. Adding uncertainty to the stew, our state descriptor, expands the notion of complete and perfect markets to include a full set of insurance markets. Ex ante, then, the firm's plans would have a unique, fully insured, valuation. But insurance markets are surely not so rich. (Can you, for example, purchase career insurance?) And less than complete markets implies economic value is not in general well defined.

specific signal realization. Further recall that two distinct mappings that induce the same state partition are informationally equivalent. They merely do so with different scales, different numerical assignments that convey the same underlying information.

Of course, accounting is a highly structured language, with the underlying stock and flow calculations tightly connected (through articulation). This raises the question of what limits are imposed by this reliance on the language and algebra of valuation. The answer is virtually none. The structure of accounting is not limited in its ability to convey information. This means we must look elsewhere to identify and understand what information is conveyed by the typical accounting system.

Selected References

The modeling of accounting as an explicit information source, one that must be decoded or inverted to discern the underlying state partition, is developed in Demski and Sappington [1990] and Antle, Demski and Ryan [1994]. Butterworth [1972] is an important precursor. In turn, this inversion theme is extended in Arya, Fellingham and Schroeder [2000] and Arya, Fellingham, Glover, Schroeder and Strang [2000] to focus on identifying the set of transactions the firm might have engaged in, given the financial reports. Beaver and Demski [1979] highlight the valuation versus information content themes, in a setting where the market structure is not too accommodating.

Key Terms

We have learned to conceptualize information as a partition of some set of states, and an information system as a mapping from states to signals. This mapping is merely the language that is used to convey the underlying partition. Turning to accounting, *accounting valuation*, or *accounting value measurement* provides an explicit mapping from states into signals, taking the realized cash flows and the interest rate and mapping them into *accounting values*. Equivalently *accounting income measurement* specifies a mapping of states into *accounting incomes*. Then particular accounting value and accounting income realizations become signal realizations from information systems. The mappings provide the language with which the underlying state partition is conveyed. Here *scaling* is important, as different mappings that induce the same state partition are informationally equivalent and thus convey the same information.

Problems and Exercises

1. What is the connection between information and uncertainty? Give several illustrations of this connection.

2. Return to the illustration in Exhibit 4, where reported income depends on the depreciation schedule (d_1 and d_2). We claim choice of depreciation schedule here is arbitrary, that regardless of the specific schedule chosen the same partition of S will be revealed, through time. Why is this so, and why do we refer to this as an issue of scaling?

3. This is a continuation of the immediately above exercise. Why, in the illustration in Exhibit

6, do we require the restructuring charge, d, be nonzero?

4. The illustration in Exhibit 6 is one in which accrual reporting is able to convey strictly more information than cash basis accounting. Even so, the precise accruals (i.e., d, d_1 and d_2) are far from unique. This illustrates an old Danish design adage: form follows function. Explain.

5. Consider the fuel gauge in your automobile. What do you know about its peculiarities? Is it "late" to report? For example, does it take a while to register full after filling the tank? Is it well calibrated? For example when it reports a reading of "half" does that mean the stock of gasoline equals one half the tank's capacity? Is it useful? What, then, does the fuel gauge purport to measure? Do you see an analogy to the information content perspective in accounting?

6. Return to the three product firm, and story, in Exhibits 2 through 6, where the firm observes the partition $\{\{s_1\},\{s_2\},\{s_3, s_4\}\}$ at time $t = 1$. Contrary to the example, however, now assume the firm can adjust its third period output, based on this information at time $t = 1$. Determine the firm's optimal production plan, assuming all the details of the setting, such as the prices and random structure of the first and third period selling prices remain. Display your findings in exhibits that parallel Exhibits 3 and 6.

7. This is a continuation of Exercise 5 in Chapter 4. Now assume the first and third period selling prices are uncertain. The first will be 220 ± 40 and the third will be 266.20 ± 50. The price possibilities are equally likely, and independent. The firm is risk neutral, and must commit to its production schedule at the start, prior to learning the prices. (This implies the optimal factor choices and output schedules remain as determined in the original exercise.) Determine the firm's cash flow prospects, as a function of the state, following the pattern in Exhibit 3. Then, for an arbitrary depreciation schedule, determine the firm's income in each period and state, following the pattern in Exhibit 5. What state partition is induced, at each point in time, by your method of calculating the firm's income?

8. Ralph is now dealing with a two period cash flow story, in which there are six equally likely states. The periodic cash flows, as a function of the state, are displayed in the table below. Notice the expected cash flow in the first period is 110, while its counterpart in the second period is 121. Naturally, the interest rate is 10%. Also notice the information source that will provide one of three signals. This information is privately observed by Ralph, just as the first period cash flow is also privately observed by Ralph.

	s_1	s_2	s_3	s_4	s_5	s_6
CF_0	- 200	- 200	- 200	- 200	- 200	- 200
CF_1	100	100	100	120	120	120

CF_2	101	101	141	141	141	101
signal y	y_1	y_2	y_2	y_3	y_2	y_2

a] What partition of the set of states is identified (for Ralph) at time $t = 1$ by the combination of observing CF_1 and the noted signal? (Notice prior observation of CF_0 is superfluous.)

b] Now suppose Ralph wants to convey all of this information by issuing a report at time $t = 1$, a report consisting of CF_1 and accounting income for the first period. Accounting income, of course, is linked to the accounting valuation series. This series has $A_0 = 200$, and $A_2 = 0$. Assume A_1 is defined by straight line depreciation, nothing else. Will the report convey all that Ralph knows? Be specific.

c] Repeat part b] for the case $A_1 = \mathrm{E}[CF_2/1.1 \mid CF_1]$.

d] Repeat part b] for the case $A_1 = E[CF_2/1.1 \mid y]$.

e] Repeat part b] for the case $A_1 = E[CF_2/1.1 \mid CF_1 \text{ and } y]$.

f] Repeat part b] for the case $A_1 = \text{minimum } \{120, E[CF_2 \mid CF_1 \text{ and } y]\}$.

g] Repeat part b] for the case $A_1 = \text{minimum } \{120, E[CF_2/1.1 \mid CF_1 \text{and } y]\}$.

h] Write a short paragraph summarizing your observations on the various ways to measure accounting value at time $t = 1$.

9. This is a continuation of Exercise 8 above. As before Ralph wants to convey all that is known at time $t = 1$ by issuing a report consisting of CF_1 and accounting income for the first period. In what sense are the measurement schemes explored above alternative scalings of what Ralph is seeking to disclose? Likewise, we know Fahrenheit and Celsius convey the same information, but with a different scale. Carefully contrast this notion of scaling with that you have identified here.

10. This is a continuation of Exercise 7 above. Now suppose there are two firms, identical in all respects. Initially assume the price realizations are independent, so one firm's price tells us nothing about the other's. Provide a state specification, common to both firms, and verify that each firm's accounting system is indeed a mapping from states into signals. What happens if the two firms face the same price realizations?

August 19, 2001, Joel