3

Classical Foundations

Accounting, we will learn, uses the language and algebra of valuation to convey information. Giving this curt phrase meaning requires care and study. The beginning point is "the language and algebra of valuation," the topic we now introduce. The difficulty is terms such as value and income have taken on near colloquial status, as illustrated by the FASB's and IASB's fascination with fair value, yet their substantive meaning is to be found in a setting of certainty, coupled with perfect and complete markets. This is the world of classical economics, and it is the source for our language, and algebra. It is inevitable, then, that our study begin with a reporting organization that exists in a classical environment, one with well functioning markets and foreknowledge.

Our study of accounting theory begins with a review of classical foundations, a setting where the important notions of economic value and economic income are well defined. Here we presume the reporting organization, or firm, operates in a world of certainty and perfect markets. Both assumptions are important. *Certainty* means there are no surprises; everyone in the economy shares an unshakeable confidence in their knowledge of what the future will bring. Of course that future depends on how resources are allocated, so the setting is not entirely predestined. *Perfect markets* means every resource is traded in a perfect market; everyone knows the price of every commodity and factor of production, and all trades take place in well organized, perfectly competitive markets where everyone acts as a price taker. (We are a bit casual here; we actually mean perfect and complete markets: there is a market in which every resource is traded, and that market is perfect.) These are daunting, unrealistic assumptions. But it is in this setting that the language and concepts of accounting are identifiable. Traditionally accounting is thought of as a valuation process and in the setting of this chapter such valuation is well defined. Absent perfect markets that is no longer the case.

It is also important to acknowledge this classical setting of certainty and perfect markets carries a rich and deep intellectual tradition, covering such items as the existence and efficiency of competitive equilibrium in an economy. By necessity, we are highly selective in the formalisms and insights we highlight and examine.

Initially we pose a simple cash flow setting, and review the notions of present value and economic income associated with that setting. Next, we step back, and identify the presumed cash flows as arising from profit maximizing behavior on the part of the reporting organization. This leads us into the relationship between economic cost and economic income, an essential step if we are to understand accounting.

Exogenous Cash Flows

Consider a setting where the firm faces the cash flow prospects detailed Exhibit 1. To avoid ambiguity we presume throughout the cash flow is denominated in dollars. Notice the cash flow is spread over three periods, with four distinct amounts. Further notice the sum of these cash flows is -25,000 + 4,950 + 9,680 + 16,637.50 = 6,267.50.

This cash flow sequence is, in fact, the one we derived at the end of Chapter 2, in Exhibit 6. But at this point, there is no derivation per se. We simply begin by assuming this cash flow sequence is in place. It is exogenous. As you suspect, we will return to the earlier derivation at a later point, but first things first. For the moment, we simply have a cash flow sequence, and to reinforce this stark story we display the sequence in the explicit time line format in Exhibit 1.

Now further suppose the periods are equally spaced, and interpret each period as a year. Let's further interpret the sequence of cash flows as the net cash flows between the firm and its owner or owners. We thus have a firm whose life history is described by an initial investment or cash flow from the owners of CF_0 = -25,000, followed by respective end-of-year cash flows distributed to the owners of CF_1 = 4,950, CF_2 = 9,680, and CF_3 = 16,637.50. It is important to remember in what follows that the cash flows are between the firm and its owners. Alternatively put, there is a claim to the firm's cash flows, and we will be economically valuing that claim.

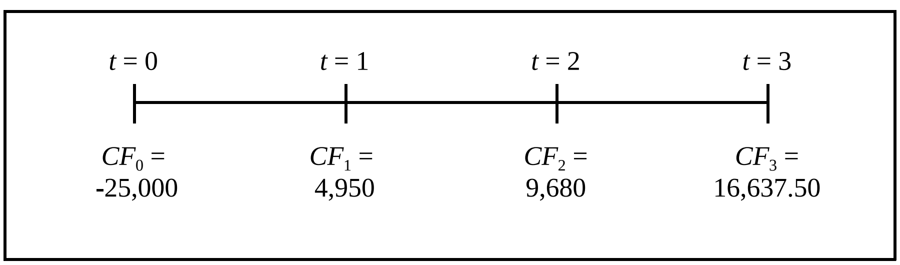

Exhibit 1: Cash Flows for t = 0, 1, 2 and 3

The firm lives for a short period of time, simply for our convenience. A more involved story would have the firm, among other things, growing and investing in a number of projects. This, however, would complicate the path ahead, without offering any substantive additional insight. So we stick with the most simple of stories.

Continuing, we further assume the interest rate is a constant r = 10% each period. Importantly, and emphatically, the interest rate is a market price. The market price in the current period of $(1 + r)^t$ dollars delivered t periods into the future is precisely one dollar.[1] For example,

---

[1]Note well, we are engaging in a partial equilibrium exploration. This particular set of prices, parameterized by the presumed (continued...)

the price in the current period of \$1.21 to be delivered in two periods is \$1.00. We can borrow \$1 today, by paying the lender \$1.21 two periods from today. Stated differently, Z dollars today and $Z(1 + r)^t$ dollars t periods from today are economically equivalent. Likewise, $Z(1 + r)^{-t}$ dollars today and Z dollars in t periods are economically equivalent.

present values

This economic equivalence is often expressed in terms of present value.[2] The present value of Z dollars in t periods, at interest rate r, is, recall, simply $Z(1 + r)^{-t}$. In this setting, present value is merely a statement of equivalence, based on market prices. With the presumed interest rate structure, holding Z dollars today gives one precisely the same command over resources as holding a claim to $Z(1 + r)^t$ dollars t periods from now. The one can be exchanged for the other. Perfect markets are awfully convenient.

Next we introduce some useful notation. Suppose we have a sequence of cash flows at times $t = 0, 1, ..., T$. Let CF_t denote the cash flow at time t. Also let CF denote the entire list, or vector of cash flows:[3]

$$CF = [CF_0, CF_1, CF_2, ..., CF_T].$$

Our example in Exhibit 1 has $T = 3$ and $CF = [-25{,}000, 4{,}950, 9{,}680, 16{,}637.50]$.

Given a cash flow vector, we now define the *continuation present value* at time t, based on interest rate r, as the present value of the remaining cash flows, calculated as of time t. Denoting this calculation by PV_t, we have:

$$PV_t = \sum_{j=t+1}^{T} CF_j (1+r)^{t-j}. \qquad [1]$$

To illustrate, the data in Exhibit 1 provide the following, using $r = 10\%$ of course:

$$PV_0 = 4{,}950(1.1)^{-1} + 9{,}680(1.1)^{-2} + 16{,}637.50(1.1)^{-3} = 25{,}000;$$
$$PV_1 = 9{,}680(1.1)^{-1} + 16{,}637.50(1.1)^{-2} = 22{,}550;$$
$$PV_2 = 16{,}637.50(1.1)^{-1} = 15{,}125; \text{ and}$$
$$PV_3 = 0.$$

---

[1](...continued)
constant interest rate, has not been derived from an equilibrium argument. We simply assume this is the interest rate. Various additional price assumptions will be introduced as the example deepens.

[2]Recall in Chapter 2 where we introduced the underlying details we took the notion of present value for granted. Here we emphasize its roots in an explicit market structure.

[3]Square brackets are used to denote such a listing, in vector format.

Notice the calculation in period t ignores the cash flow in period t. The focus is on the future cash flows as of time t.

Economically, the continuation present value is simply the market value of the remaining cash flows in vector CF, as of time t. To appreciate this, suppose we purchased the $t = 1, 2, 3$ sequence of cash flows in our example at time $t = 0$. Given the assumed interest rate, we should pay $4{,}950(1.1)^{-1} + 9{,}680(1.1)^{-2} + 16{,}637.50(1.1)^{-3} = 25{,}000$. This amount is, of course, the continuation present value at $t = 0$ that we calculated above. But suppose we paid 24,000. This means we have a money pump: we pay 24,000, but can immediately sell the claim to the cash flow sequence for 25,000. Do this until the sun sets, and possibly beyond.

Surely something is wrong. Our mistake is assuming in the first place we could buy something for 24,000 that could be immediately sold for 25,000. Intertemporal arbitrage is not possible in the perfect markets setting. So at time t the remaining future cash flows in vector CF must have a market value of PV_t.

Present value is market value and market value is present value in the setting in this chapter. More precisely, for cash flow sequence CF the market value at time t of the remaining cash flows is the continuation present value at that point, and the continuation present value at that point is the market value of the remaining cash flows at that point. Furthermore, this market value is unique and unambiguous. Consequently, the continuation present value is also called the *economic value*.

This valuation perspective turns out to be important in what follows, so we will embellish it with some notation. The firm's activities are summarized by cash flow series CF, and the interest rate is r. Valuation portrays this in terms of a sequence of economic values, here assumed to be the continuation present values in [1]. Fundamentally, then, we have a valuation function, call it V, that assigns to any CF series in the presence of some interest rate r a series of temporal values. Formally, we have:

$$V(CF,r) = [PV_0, PV_1, ..., PV_T] \qquad [2]$$

where PV_t is defined via expression [1]. Literally, then, the present value apparatus is a mapping or rule that assigns a sequence of continuation present values to any such cash flow vector and interest rate specification.[4]

economic income

[4]To develop this a bit further, a function is a rule that takes points from one set to points in a second set. The only requirements are every point in the first set must be carried to some point in the second set and no point in the first set can be carried to more than one point in the second set. (We often find this idea of a function written as f: $X \to Y$, which reads "the function f maps set X into set Y.") We have, you should notice, not been this formal. To tighten things up, then, we should specify the set of CF vectors and interest rates we have in mind, and the mapping would be to some subset of $T + 1$ dimensional Euclidian space.

Closely associated is the notion of economic income. To set the stage, recall the Exhibit 1 story were investors invested or "paid" 25,000 in exchange for the noted future cash flow series, one that has a continuation present value of $PV_0 = 25{,}000$. (After all, in a perfect market setting competition ensures no firm earns any rent.)

Nevertheless, we see the owners have paid 25,000, and eventually will receive payments totaling 4,950 + 9,680 + 16,637.50 = 31,267.50. How should we interpret the difference of 31,267,50 - 25,000 = 6,267.50? Presumably, we, as accountants, would call it "income." But this begs many questions, not to mention the issue of how much should be attributable to each of the three periods.

Economic income, though, is unambiguous in this setting. It is simply the increment in economic value over the period plus cash flow during the period. Cash flow, of course, is the cash that moves between the owners and the firm. Economic value, in turn, is our friend, the continuation present value (in [1]). Let I_t denote the economic income for period t. We have the following algebraic definition of *economic income*:

$$I_t = PV_t - PV_{t-1} + CF_t. \qquad [3]$$

Note well, economic income is simply the increment in value ($PV_t - PV_{t-1}$) plus cash flow of the period (CF_t). Stated differently, it is the increment in value plus resources received if $CF_t > 0$ or the increment in value less additional resources provided if $CF_t < 0$.[5] Economic value, in turn, is measured by present value, at interest rate r, in our setting.

Moreover, just as the present value apparatus gave us a mapping from cash flow vectors and interest rates into a sequence of continuation present values, expression [2], the economic income apparatus gives us a mapping from cash flow vectors and interest rates into a sequence of income numbers. Specifically, we have an income function I defined via

$$I(CF,r) = [I_1, ..., I_T] \qquad [4]$$

where, of course, I_t is defined in expression [3]. The valuation function, [2], defines a sequence of stock measures while the income function, expression [4], defines a sequence of flow measures.

In this world of stringent market conditions, then, we readily identify (economic) stock and flow measures associated with cash flow sequence CF and interest rate (i.e., market price) r. An equivalent way to visualize the flow measure is to think in terms of "net receipts" less economic depreciation in period t. Net receipts, of course, corresponds to CF_t, and *economic depreciation* is the change in value, $PV_{t-1} - PV_t$. So $I_t = PV_t - PV_{t-1} + CF_t = CF_t - (PV_{t-1} - PV_t)$.

Returning to our numerical example in Exhibit 1, we have the following calculations:

---

[5]By analogy, in a security price setting we would think in terms of change in value of the underlying security plus dividends received (or additional investment made). Further notice I_t depends on the underlying cash flow vector and the interest rate.

$$I_1 = PV_1 - PV_0 + CF_1 = 22{,}550 - 25{,}000 + 4{,}950 = 2{,}500;$$
$$I_2 = PV_2 - PV_1 + CF_2 = 15{,}125 - 22{,}550 + 9{,}680 = 2{,}255; \text{ and}$$
$$I_3 = PV_3 - PV_2 + CF_3 = 0 - 15{,}125 + 16{,}637.50 = 1{,}512.50.$$

These calculations are summarized in a more familiar income statement format, of revenue less expenses, in Exhibit 2. There we call the periodic cash inflows, the “net receipts,” “revenue.” And with an up-front investment of 25,000 and no other visible expenses, depreciation is the only expense, and here it is, of course, economic depreciation. Also notice the implicit asset valuation here. It is simply the continuation present value.

	$t = 1$	$t = 2$	$t = 3$	**total**
“net receipts” = “revenue” (CF_t)	4,950	9,680	16,637.50	31,267.50
“depreciation” = “economic depreciation” (PV_{t-1} - PV_t)	2,450	7,425	15,125	25,000
“income” (I_t)	2,500	2,255	1,512.50	6,267.50

Exhibit 2: Income Statement Format for Economic Income Calculations

The tight linkage between economic income and present value calculations allows us to demonstrate two implications of the definition of economic income. First, economic income in any period is simply the interest rate multiplied by the beginning of period value:

$$I_t = r \cdot PV_{t-1}. \qquad [5]$$

You should convince yourself this is indeed the case for our cash flow sequence (in Exhibit 1) and calculations (in Exhibit 2).

To see the logic behind this important linkage, begin with the fact that adjacent continuation present values are intimately linked:[6]

$$PV_t = (1 + r)PV_{t-1} - CF_t.$$

From here apply the definition of economic income:

$$I_t = PV_t - PV_{t-1} + CF_t = (1 + r)PV_{t-1} - CF_t - PV_{t-1} + CF_t = r \cdot PV_{t-1}.$$

[6] Intuitively, we take the continuation present value at time t - 1 and move it forward in time one period. CF_t, however, must be subtracted as the time t calculation takes place after the cash flow occurs in period t. More precisely, it is routine to verify $PV_{t-1} = (1 + r)^{-1}(PV_t + CF_t)$, which implies $(1 + r)PV_{t-1} = PV_t + CF_t$.

Second, the series of economic income calculations is "tidy" in the following sense:

$$\sum_{t=1}^{T} I_t = \sum_{t=1}^{T} CF_t - PV_0. \qquad [6]$$

That is, the total of the economic income assigned to the periods equals the total of the cash flows from $t = 1$ forward, less the initial continuation present value.[7] (This readily follows by using the definition of economic income and summing the terms from $t = 1$ to T.) Again, you should verify this in Exhibit 2.

Recall, now, with our cash flow series in Exhibit 1 we also have $CF_0 = -PV_0$: the invested amount is the negative of the present value of the future cash flows. The tidiness property now explains why the sum of the economic incomes equals the sum of the cash flows in our simple story.[8]

To this point, we have identified the economic income, period-by-period as well as in total, associated with the cash flow story in Exhibit 1. We might even embellish our calculations with accompanying balance sheets. But more important is the question of interpretation. What does it mean, say, to claim this firm's economic income was 2,255 in period $t = 2$? Fisher's [1906] stress on income as a flow of wealth or Hick's [1946] stress on how much can be consumed without diminishing the ability to consume in the future come to mind. But this is getting ahead of our story. It is time to retreat and dig into the background details of the story in Exhibit 1.

Endogenous Cash Flows

We now connect the cash flow sequence in Exhibit 1 to the richer story of a firm that purchases labor and capital in respective factor markets and uses its technology to produce products that are sold in the product markets. This exercise is important, as it sheds light on the income interpretation question, and also exposes an important connection between economic cost and economic income.

---

[7]Also recall our firm faces a short life. Alternatively, suppose T is unbounded large and the firm's growth rate is zero, it is neither growing nor declining. This implies the continuation present value is constant, and [5] provides the familiar case of value being the capitalized income stream.

[8]Suppose we paid less than PV_0, implying $PV_0 + CF_0 > 0$, for the claim to the cash flow sequence, a clear violation of our perfect market assumption. Now write the value up, at time $t = 0$, to PV_0, and label the difference, $PV_0 + CF_0$, economic rent. More broadly, then, the totality of economic income plus economic rent equals the sum of the cash flows, from $t = 0$ to $t = T$. We will return in due course to the issue of economic rent.

To do this, we return to the prototypical firm presented in Chapter 2. The firm uses three types of labor and one type of capital to produce and sell three different products.[9] q_1, q_2 and q_3 denote the non-negative quantities of the three products. L_1, L_2 and L_3 denote the non-negative quantities of the three labor inputs, and K denotes the non-negative quantity of physical capital. The production technology, recall, is given by the following three technology constraints:

$$q_1 \le \sqrt{KL_1} \quad [7a]$$
$$q_2 \le \sqrt{KL_2} \quad [7b]$$
$$q_3 \le \sqrt{KL_3} \quad [7c]$$

along with an upper bound on the physical capital:

$$K \le K^{\max} \quad [8]$$

timing details

Now add an important time dimension. Suppose the firm produces and sells product i in period i, implying we have a three period story. Physical capital is acquired and paid for at the beginning of the story (t = 0), and output is delivered at the end of the respective periods (t = 1, 2 and 3). Customers pay upon delivery. Labor for each product is paid at the end of the respective product's period. That is, rather than everything taking place in the same period, the firm now operates in spot markets stretched across three periods. The interest rate is denoted r.

Receipts and expenditures must now be identified by time. Let p_K denote the (time t = 0) price of physical capital. Similarly, let p_{L1} denote the (time t = 1) price of labor used in the first period, p_{L2} its counterpart in the second period, and p_{L3} its counterpart in the third period. Using parallel notation, let the (time t = 1) selling price of the first product be P_1; counterparts for the other two products are denoted P_2 and P_3.

Now, given these timing details, the firm's expenditures on factors and receipts from sales in the product markets are stretched out on the time line detailed in Exhibit 3.

From here, we assume the firm selects its output and factor combination to maximize its *economic profit*, where economic profit is defined as the present value, at time t = 0, of the cash flows (i.e., the net gain to the owners). Conveniently, we approach this in stages.

---

[9]We should also reflect on the economic meaning of a product. Naturally, the concept is expansive, covering goods and services. Moreover, the same good or service produced at different points in time will be treated as economically distinct products. So, for example, a specific type of refrigerator produced in year t is economically distinct from the same type of refrigerator produced in year t + 1.

	$t = 0$	$t = 1$	$t = 2$	$t = 3$
receipts		P_1q_1	P_2q_2	P_3q_3
expenditures	p_kK	$p_{L1}L_1$	$p_{L2}L_2$	$p_{L3}L_3$

Exhibit 3: Timing of the receipts and expenditures

First, for any output schedule, q_1, q_2 and q_3, the firm will select the efficient combination of factors. That is, among those combinations that support production of the noted output, it will select the one that minimizes the present value of factor expenditures. This is none other than our earlier definition of economic cost in Chapter 2.

Given the timing assumptions, the (time $t = 0$) present value of the firm's expenditures on factors is simply

$$p_KK + p_{L1}L_1(1 + r)^{-1} + p_{L2}L_2(1 + r)^{-2} + p_{L3}L_3(1 + r)^{-3}.$$

So the firm's best choice of factors to produce q_1, q_2 and q_3 repeats our earlier development of economic cost:

$$C(q_1,q_2,q_3) \equiv \underset{K,\, L_1,\, L_2,\, L_3 \,\geq\, 0}{\text{minimum}} \;\; p_KK + p_{L1}L_1(1 + r)^{-1} + p_{L2}L_2(1 + r)^{-2} + p_{L3}L_3(1 + r)^{-3} \quad [9]$$

$$\text{subject to [7a], [7b], [7c], and [8].}$$

The only difference is cost expression $C(q_1,q_2,q_3)$ is now stated in economically equivalent present value terms. Spot prices, so to speak, are converted to economically equivalent present value expenditures. The cost expression, in other words, is now measured in (economically equivalent) present value terms. This insight is important in what follows.

With the cost expression identified, the second stage selects the profit maximizing output schedule. Naturally, profit now takes on the convenient structure of the present value of receipts less the above determined cost.[10] Recall the timing convention that a unit of product i sells at time i for a price of P_i. The present value of the receipts from customers is

$$P_1q_1(1 + r)^{-1} + P_2q_2(1 + r)^{-2} + P_3q_3(1 + r)^{-3}.$$

So the profit maximizing choice is described by

$$\underset{q_1,\, q_2,\, q_3\, \geq\, 0}{\text{maximize}}\;\; P_1q_1(1 + r)^{-1} + P_2q_2(1 + r)^{-2} + P_3q_3(1 + r)^{-3} - C(q_1,q_2,q_3) \qquad [10]$$

The firm, in other words, makes its choices by maximizing the present value of the cash flows, or profit. Judiciously, we divide this exercise into stages of initially determining the optimal factor combination for arbitrary output choices, the economic cost expression in [9], followed by the optimal output combination, the profit maximization in [10].

a recycled illustration

To illustrate, suppose the selling prices are $P_1 = 132$, $P_2 = 193.60$, and $P_3 = 266.20$; and the labor prices are $p_{L1} = 110$, $p_{L2} = 121$, and $p_{L3} = 133.10$. The capital price is $p_K = 200$. In addition, the capital limit is $K^{\max} = 125$. The interest rate, of course, is $r = 10\%$.

Given the timing conventions, the (time $t = 0$) present value of expenditures on factors is

$$200K + 110L_1(1.1)^{-1} + 121L_2(1.1)^{-2} + 133.1L_3(1.1)^{-3} = 200K + 100L_1 + 100L_2 + 100L_3.$$

So the firm's economic cost is defined by the following specific version of [9]:

$$C(q_1,q_2,q_3) \equiv \underset{K,\, L_1,\, L_2,\, L_3\, \geq\, 0}{\text{minimum}}\;\; 200K + 100L_1 + 100L_2 + 100L_3$$
$$\text{subject to [7a], [7b], [7c], and [8].}$$

[10]The use of the phrase "profit" here is purposely colloquial. In a literal sense the firm seeks to maximize its economic rent (defined as the present value of the cash flows). With perfect markets, though, the solution entails zero rent. Further notice that the firm maximizing in this fashion is hardly gratuitous. With the perfect and complete markets, consumers and producers acting as price takers (in the sense they optimize, taking the prices as given) and firms maximizing their profits, the resulting equilibrium is efficient. No one can be made better off without making someone worse off. The firm's behavior, in this setting, is therefore well directed by the market prices and efficient. This is one of the celebrated theorems of welfare economics (i.e., efficiency of a competitive equilibrium). It is also called the Fisher separation theorem, reflecting the fact market prices "separate" the firm's decision making from the tastes of the consumers. Firms maximize their profits: and consumers, the firms' owners, then take their largest possible consumption budgets to the consumption markets.

The solution, provided we stay in the region where the output is not so large the K^{max} constraint is binding, should be familiar. It is the very expression we derived in Chapter 2:[11]

$$C(q_1,q_2,q_3) = 200\sqrt{2(q_1^2 + q_2^2 + q_3^2)}\,.$$

There we had a setting where output was produced simultaneously, with a capital price of 200 and labor price of 100 for each labor type. This is economically equivalent to the present story, where the output is produced sequentially. (The reason is we have the same technology, and we have rigged the factor prices so their time $t = 0$ equivalent prices are the prices we used in the original setting.) But we linger.

Turning to the output choice, the present value of the receipts from customers is

$$132q_1(1.1)^{-1} + 193.6q_2(1.1)^{-2} + 266.2q_3(1.1)^{-3} = 120q_1 + 160q_2 + 200q_3.$$

So the profit maximizing choice is described by the following specific version of [10]:

$$\underset{q_1,\, q_2,\, q_3\, \geq\, 0}{\text{maximize}}\;\; 120q_1 + 160q_2 + 200q_3 - C(q_1,q_2,q_3)$$

A solution has $q_1 = 75$, $q_2 = 100$ and $q_3 = 125$. Moreover, the underlying factor choices are $K = 125$ ($= K^{max}$), $L_1 = 45$, $L_2 = 80$, and $L_3 = 125$.[12]

Now tally the cash inflows and outflows. Details are displayed in Exhibit 4. In addition you should verify the present value of the inflows is 50,000, just as the present value of the outflows is 50,000. The firm's profit is precisely zero.

---

[11] Repeating the details, for any value of $K > 0$, the firm will acquire $L_i = q_i^2/K$ units of labor. So the expenditure calculation becomes:

$$200K + 100q_1^2/K + 100q_2^2/K + 100q_3^2/K = 200K + \theta/K,$$

where $\theta = 100q_1^2 + 100q_2^2 + 100q_3^2$. Setting the derivative equal to zero gives us the minimizing choice of K:

$$200 - \theta/K^2 = 0.$$

So $K^2 = \theta/200$, or $K = \sqrt{\theta/200}$. From here, the present value of the overall expenditure is $200K + \theta/K$. Working through the details gives us $C(q_1,q_2,q_3) = 2\sqrt{200\theta} = 200\sqrt{2(q_1^2 + q_2^2 + q_3^2)}$, the expression in the text, again presuming the K^{max} constraint is not binding. Additional details were developed in Chapter 2.

[12] Again, we are revisiting the earlier example in Chapter 2. There we verified this solution by differentiating the profit expression for each output quantity and setting that derivative to zero:

$$120 - \partial C(q_1,q_2,q_3)/\partial q_1 = 0;$$
$$160 - \partial C(q_1,q_2,q_3)/\partial q_2 = 0, \text{ and}$$
$$200 - \partial C(q_1,q_2,q_3)/\partial q_3 = 0.$$

Also from our earlier work the marginal cost expression for project i is $\partial C(q_1,q_2,q_3)/\partial q_i = 2p_{Li}q_i/\sqrt{\theta/p_K}$ if the K^{max} constraint is not binding and $2p_{Li}q_i/K^{max}$ otherwise. ($\theta = 100q_1^2 + 100q_2^2 + 100q_3^2$.) From here you can verify $q_1 = 75$, $q_2 = 100$ and $q_3 = 125$ satisfy these conditions. As noted earlier any output in the proportion of 3: 4: 5 that does not violate the K^{max} constraint satisfies these conditions, and we have merely taken from among these the largest possible output as our particular solution.

Importantly, we have come full circle and are back to the cash flow series originally assumed in Exhibit 1. But how is it we speak so casually about our firm in a perfectly competitive setting having zero profit, yet having strictly positive economic income, as calculated in Exhibit 2 (or for that matter a lifetime income of 6,267.5)?

	$t = 0$	$t = 1$	$t = 2$	$t = 3$
inflows		132(75) = 9,900	193.6(100) = 19,360	266.2(125) = 33,275
outflows	200(125) = 25,000	110(45) = 4,950	121(80) = 9,680	133.1(125) = 16,637.50
net, CF_t	-25,000	4,950	9,680	16,637.50

Exhibit 4: Cash Outflows, Inflows and Net Cash Flow

Back to Economic Income

The answer is in the timing. The best way to see this is to re-tell the story for the case where all customers pay at the end of period $t = 3$, while all factor payments are made at time $t = 0$. We also adjust the various prices so their present value at time $t = 0$ remains the same. So the (time $t = 0$) factor prices are $p_K = 200$ and $p_{L1} = p_{L2} = p_{L3} = 100$. Similarly, the (time $t = 3$) product prices are $P_1 = 132(1.1)^2 = 159.72$, $P_2 = 193.6(1.1) = 212.96$ and $P_3 = 266.20$. Note well: we have preserved the present value of each and every price.

Revisiting the cost curve and profit maximization exercises, therefore, leaves the original choice of outputs and factors undisturbed. After all, the respective present value expressions are equivalent. The cash flows, though, are a different matter. Now the only cash outflows occur at $t = 0$. We have

$$CF_0 = -\,(200(125) + 100(45) + 100(80) + 100(125)) = -\,50{,}000.$$

Likewise, the only cash inflows occur at $t = 3$:

$$CF_3 = 159.72(75) + 212.96(100) + 266.20(125) = 66{,}550.$$

See Exhibit 5.

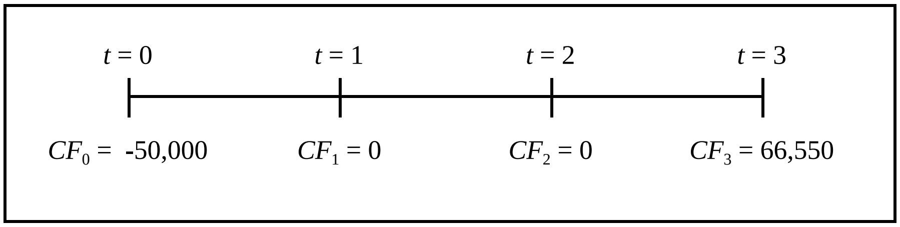

Exhibit 5: Equivalent Cash Flows for Exhibit 4 Story

Exhibits 4 and 5 are, of course, economically equivalent stories. The present value of the factor payments in Exhibit 4 is 50,000, just as the ($t = 3$) future value of all customer payments is 66,550. Moreover, the firm's profit remains at $66,550(1.1)^{-3} - 50,000 = 0$. Yet its income over the three periods has increased to 66,550 - 50,000 = 16,550. Is this magic, or what? We seem to be maximizing income!

But this is deceptive. The factor payments total 50,000, when converted to time $t = 0$ dollars. Clearly, the firm's economic cost is 50,000 stated in $t = 0$ dollars. But this cost tally is $50,000(1.1)^3 = 66,550$ when stated in time $t = 3$ dollars. If all factor purchases are paid at $t = 0$, the cost is 50,000 at that time. If all factors are paid at $t = 3$, the cost is 66,550 at that time. The latter is equivalent to borrowing 50,000 to purchase the factors up front, and paying the loan, plus interest, at $t = 3$. Indeed, in this loan scenario the firm's cash flow is precisely zero in each and every period, as it borrows everything up front (the 50,000 datum), then receives payments at time $t = 3$ totaling 66,550 and immediately pays off its loan (principal plus accumulated interest) for 66,550.

Now think back to the initial presentation of this four factor, three product firm in Chapter 2. There, production was timeless, and all cash transactions took place at the same point in time. If, then, all receipts and expenditures take place at the same instant of time, economic cost is the minimum expenditure on factors, at that point in time. But if production, factor acquisition, and output take place at various points in time, our usual formulation of economic cost prevails when we envision all factors as being paid for at time $t = 0$. Otherwise, we focus on the present value of all factor expenditures as of a particular point in time. Moving the time at which this present value is reckoned alters the magnitude of economic cost. If the calculation is centered on $t = 0$, the economic cost is 50,000, but if it is centered on $t = 3$ the economic cost is 66,550. The two are economically equivalent, of course.

For the record, we present the economic income calculations for the Exhibit 5 story in the same format as used in the original discussion. The continuation present value moves from 50,000 to 55,000, to 60,500. Be certain you understand the calculations.[13]

---

[13]Also notice, with our endogenous perspective, we continue with the format of "revenue" less expenses in the calculation, but notice the "revenue" is "net receipts" in this case. Looking at the underlying details, it consists of cash inflow from the customer less payment for the respective labor factor. Think of this as revenue less direct cost (in particular, direct labor). We (continued...)

	t = 1	**t = 2**	**t = 3**	**total**
"net receipts" = "revenue" (CF_t)	0	0	66,550	66,550
"depreciation" = "economic depreciation (PV_{t-1} - PV_t)	-5,000	-5,500	60,500	50,000
"income" (I_t)	5,000	5,500	6,050	16,550

Exhibit 6: Income Statement Format for Economic Income Calculations for Exhibit 5 Story

Notice the associated economic income calculations step the economic cost forward in time, to t = 3. Economic cost at time t = 3 (66,550), in this specialized case, is economic cost at time t = 0 (50,000) plus the sum of the economic income over the three periods. Economic income is a component of economic cost.

This is no accident. Return to the features of economic income developed in expressions [5] and [6]. In [6] we highlight the fact that total economic income equals the arithmetic sum of the cash flow series, given we begin with zero economic profit. In [5] we highlight the fact that economic income is merely the interest rate multiplied by the beginning of period continuation present value. It is a "charge" so to speak for monetary factors, or investment.

Recall that markets are perfect and complete here. So at time t, when the remaining portion of the cash flow sequence has a value of PV_t the continuation present value, it is actually possible to sell the claim to the remaining portion for precisely PV_t. Implicitly, then, the claimant is investing a monetary amount, totaling PV_t, at this time. It is as if *monetary capital*, in the amount PV_t is invested. The market price of a one period use of this amount is none other than $r{\cdot}PV_t$. After all, given perfect markets, this is what could be earned by investing this amount elsewhere.

The story in Exhibit 5 calls for acquiring all factors up front, at a total expenditure of 50,000. Three periods hence production is completed and receipts totaling 66,550 are then in hand. The firm has therefore used labor, physical capital *and monetary capital* in the process. The economic cost of the monetary capital is what we earlier termed economic income. Viewed from the firm's perspective, economic income arises only when transactions are spread out on the time line, and it is a component of economic cost. Then, the income is equal to the rate of interest multiplied by the invested capital. This is what [5] says!

To be sure, the story in Exhibits 5 and 6 is unusual in that expenditures and receipts are completely separate. The Exhibit 4 story, the cash flow sequence with which we began this exploration, mixes receipts and expenditures on the time line. This means fewer monetary

---

[13](...continued)
continue with the net, the "revenue" format because that is how the exercise began, with exogenous cash flows. In Chapter 4, when we introduce the accountant's view of this story, we will have revenue measured as receipts from customers, and expenses separately tallied for the labor and capital factors.

resources are required, the cost of monetary resources is therefore less, and overall economic income is less. Pure and simple, economic income is the period's cost of monetary resources. It is a component of economic cost. This is most vivid in the calculations of economic income in Exhibit 6.

At this risk of beating a dead horse, this fact can be readily visualized by explicitly including the cost of the monetary factors in the periodic economic flow calculations. Using the original (Exhibit 4) cash flow series, we modify the original income calculation (in Exhibit 2) to include an explicit "charge" for the cost of the monetary factor employed. See Exhibit 7.

You should recognize the bottom line here, I_t - $r{\cdot}PV_{t\text{-}1}$, as residual income: income less a monetary capital charge. Others call it abnormal earnings or economic value added. It is precisely zero in each and every period here because the firm is facing zero economic profit. What would residual income look like for the story in Exhibit 6?

Economic profit in our world of certainty and perfect markets is the present value of receipts less the present value of expenditures. Economic income is the cost of monetary resources committed to the organization during the period in question. It is a component of economic cost that shows up when we reckon economic cost at a time other than t = 0.[14]

	t = 1	t = 2	t = 3
"net receipts" = "revenue" (CF_t)	4,950	9,680	16,637.50
"depreciation" = "economic depreciation" ($PV_{t\text{-}1}$ - PV_t)	2,450	7,425	15,125
"income" (I_t)	2,500	2,255	1,512.50
"monetary capital charge" ($rPV_{t\text{-}1}$)	2,500	2,255	1,512.50
"residual income" (I_t - $rPV_{t\text{-}1}$)	0	0	0

Exhibit 7: Revised Income Calculation Including Monetary Capital Charge

Does this reflect the familiar notion of how much can be consumed during the period while maintaining one's position? Yes is the short answer. The economic income in a period is the cost of monetary resources used during the period. It is akin to a rental charge. Consuming no more than the rental charge leaves the underlying asset undiminished.[15] It reflects the market rate for use of monetary assets, just as the labor wage rate reflects the market rate for use of labor

[14]The economic equivalence of the stories in Exhibit 6 and 7 is an illustration of the celebrated Modigliani-Miller Theorem: financial structure does not matter in a world of perfect markets.

[15]This, in turn, leads us to the notion of capital maintenance.

services. The market price of labor used is a component of economic cost just as is the market price of monetary resources used.[16]

Product "Profitability" and Rents

We conclude this visit to the world of certainty and perfect markets with a brief look at two remaining issues, so-called product "profitability" and the distinction among economic profit, economic income and economic rent.

product "profitability"

Return to our original cash flow sequence (Exhibit 4) and associated economic income calculations (Exhibit 2). Also recall the underlying story here is that a specific product is produced and sold during a specific period, and the labor uniquely associated with each product is acquired at the start of the respective period. So we have product revenues separated by period, and we also have labor directly identified with each product. Moreover, economic depreciation appears to be identified by product as well. So, in this admittedly unusual circumstance is economic income in period *t* attributable to the product produced and sold that period? Certainly any reasonable accounting procedure would identify revenue, labor cost, and depreciation for each period, and invite an interpretation that the income calculated that period was the income earned on that particular product line.

This movement from an income calculation to an assessment of product "profitability" is, in a word, fallacious. The economic cost curve is not separable here. The economic income we found is a charge for monetary resources which are not dedicated to a single product. Therefore, we cannot unambiguously speak of how much of the organization's total income is due to any one product. We can, of course, ask about *marginal* revenue and *marginal* cost of any product, given some output schedule (and product and factor price specification). But ascribing a portion of total (economic) income to any particular product presumes separability, separability in the sense we can unambiguously treat each product as though it had no connection to other products. In short, we would have to assume there was no reason in the first place for the firm to produce a variety of products. There is simply no connection between economic income and the product "profitability" assessments often associated with accounting based calculations.[17] Indeed, economic income measures the cost of monetary factors employed during the period, and there is no reason to presume those factors were employed exclusively for the product actually sold during the period in question.

---

[16]Again reflecting on the fact financing does not matter in this world, notice our organization could make its economic income arbitrarily large by issuing debt early on, investing the proceeds, and not making interest or principal payments until late in the time line. This would not affect its economic profit, but would surely increase its reliance on monetary factors.

[17]Assigning total income in a period to the various products, then, is a troubling perspective. The same comment applies to assigning economic profit to the various products. On the other hand, it makes perfect sense to compare, say, economic profit with a product in place with economic profit when that product is not in the organization's portfolio.

economic rent

This interpretation of economic income should be carefully distinguished from the notion of economic rent. We have judiciously confined our stories to ones in which the firm's economic profit (the present value of its cash flows at inception, recall) was precisely zero. Suppose we now give in to temptation ever so slightly and admit our firm has strictly positive economic profit (or a strictly positive NPV project). Without belaboring the details, suppose we slightly lower the initial investment in the Exhibit 1 story, from 25,000 to 24,000.

With this single change, the continuation present values remain exactly as before, PV_0 = 25,000, PV_1 = 22,550, etc. The firm also has an economic profit of 1,000, as it just paid 24,000 for something worth 25,000.[18] Applying the economic income definition, in [3], we have the same income numbers as before. Moreover, the three incomes sum to 6,267.50 (as before), but the cash flows sum to 7,267.50 (versus 6,267.50 in the original story).

The troublesome economic profit of 1,000 must be forced into the rendering. Under certainty this is straightforward. We know at time $t = 0$ what the remaining cash flows will be. We know at that point we have exchanged 24,000 for something with a continuation present value of 25,000. The difference, the gain, is called an *economic rent*. It would be recorded at time $t = 0$ here, so to speak. The firm's asset, that is, would be immediately written up to 25,000. So, in this case, the firm has economic income totaling 6,267.50 over its lifetime, coupled with economic rent of 1,000 at its inception.

Economic rent, then, arises when the firm devises a production plan whose economic value is strictly positive, whose continuation present value at inception exceeds the associated investment (i.e., $PV_0 + CF_0 > 0$). Economic rent is synonymous with strictly positive economic profit. This notion is, of course, awfully important; but introducing it in a world of perfect markets is a little tongue in cheek. Competition would simply not allow for economic rent in such a setting.[19]

By analogy, though, and looking ahead, the accounting report reflects accounting revenues less accounting expenses. It is a residual amount reflecting, to a degree, a combination of economic income and economic rent.

Summary

Accounting, as we shall emphasize, uses the language and algebra of valuation. Here, in a setting of certainty coupled with well functioning markets, valuation is a present value exercise. For any conceivable project or firm, certainty means its life can be described in terms of a sequence of cash flows, cash that flows between the firm and its owners. Valuation then enters

---

[18] Recall [10]; economic profit is the present value of customer payments less the present value of factor expenditures.

[19] Equally clear, relying on something less than perfect (and complete) markets would allow for rent, but render the very meaning of value ambiguous. Market guides in such a setting, in the form of prices in well functioning markets, would be absent. This is why we stress the importance of market structure in developing the notions of economic value and economic income. Be patient, our study is in the early stages.

via the continuation present value of the remaining cash flows, as of each and every point in time. More broadly, valuation is simply a mapping from cash flow possibilities and interest rates into an implied sequence of temporal valuations.

From here we readily identify the associated notion of economic income, the change in value plus the cash flow delivered to the owners at that time. Indeed, economic income is simply the noted interest rate multiplied by the beginning of period value. We thus wind up with a pair of measures, the economic stock and the economic flow so to speak. The stock measure is, of course, the value measure. It is the then economic equivalent of the remaining cash flow series. The flow measure, the economic income, is the implicit cost of resources tied up during the period. We stress its interpretation as a component of the firm’s cost, an interpretation that becomes most vivid when we concentrate on the firm’s residual income, its income adjusted for this capital charge.

Finally, while it makes perfect sense to identify, measure and discuss the firm’s economic income in this setting, asking the seemingly related question of the “profitability” of each of the firm’s products is a different matter. The difficulty is economic forces compel the firm to jointly produce a variety of products. So the firm’s cost is not separable. We cannot divide or apportion its cost among the various products. We thus cannot well identify the profit associated with each of its products. Economic income, the cost of the monetary factor in the period in question, applies to all of the firm’s products.

Appendix: Inventory “Valuation” and Non-Separability

The non-separability theme is further illustrated by extending the setting to one where the firm holds inventory as well as the capital asset. Traditionally, of course, we think of the firm’s assets as individually valued, and the total of those individual values as reflecting the total value of the assets. Yet non-separability surfaces here as well.

To see this, stay with the setup in the (recycled) illustration in Exhibit 4, but change the story in one respect: the three products are identical, except for time, and the firm must supply 75 units in the first period, 100 in the second and 125 in the third. So, if the firm produces according to this delivery schedule it will behave precisely as portrayed in Exhibit 4.

Now for some fun. Might the firm smooth its production? If so, it would build inventory in the short run. The cost of producing (q_1,q_2,q_3) in the three periods is simply our friend above, $C(q_1,q_2,q_3)$. This much is clear. But what quantities should be produced? The least costly way to meet these requirements is given by the following, where $\underline{\underline{C}}(Q_1,Q_2, Q_3)$ is the cost, in present value terms, of supplying Q_1 units in the first period, Q_2 units in the second period and Q_3 in the third.

$$\underline{\underline{C}}(Q_1,Q_2,Q_3) \equiv \underset{q_1,q_2,q_3 \geq 0}{\text{minimum}} C(q_1,q_2,q_3) \qquad [11]$$

$$\text{subject to: } q_1 \geq Q_1\text{; and} \qquad [12a]$$

$$q_2 + q_1 - Q_1 \geq Q_2 \qquad [12b]$$

$$q_3 + q_2 + q_1 - Q_1 - Q_2 \geq Q_3. \tag{12c}$$

The first constraint, [12a], requires whatever is produced in the first period, q_1, be at least as large as the quantity required to be available in the first period, Q_1. Any amount above current requirements, any positive $q_1 - Q_1$, will be held as inventory and be available for meeting second period requirements. Thus, the balance requirement in the second period, constraint [12b], requires second period production, q_2, plus any inventory accumulated in the first period, $q_1 - Q_1$, be at least as large as the quantity required to be available in the second period, Q_2. [12c] follows equivalently.[20]

The key here is to understand the economic forces that invite accumulation of inventory. The technology allows physical capital to be shared across time periods. It also invites "well balanced" production, as this allows the most efficient combination of labor and capital in each period. Well-balanced, in turn, means we have exhausted any economic possibility of shifting production from the third to the first period. Implicitly, then, if the balance constraints, [12a] and [12b], do not get in our way, we will arrange production across periods so that the marginal cost of production is equalized across the periods. Otherwise inter-temporal shifting is possible, by moving some production from the high to the low marginal cost period.[21]

In our particular story it turns out the optimal production schedule is (q_1,q_2,q_3) = (100,100,100). This relies on an initial capital choice of K = 122.474 and leaves us with an inventory of 25 at the end of period 1. The capital choice is less than in the original story in Exhibit 4, and reflects the fact inventory is now being used to substitute for capital. This, in turn, leads to a non-separability between capital and inventory.

	$t = 0$	$t = 1$	$t = 2$	$t = 3$
inflows		132(75) = 9,900	193.6(100) = 19,360	266.2(125) = 33,275
outflows	200(122.474) = 24,495	110(81.650) = 8,981	121(81.650) = 9,880	133.1(81.650) = 10,868
net, CF_t	-24,495	919	9,480	22,407

Exhibit 8: Cash Flows, Given $q_1 = q_2 = q_3 = 100$

[20]Now glance back at the last exercise in Chapter 2.

[21]A more complicated story would also acknowledge explicit costs associated with inventory per se, as illustrated by obsolescence, taxes, and shrinkage. In our streamlined story the only cost associated with inventory is the cost of the associated monetary capital, and this is properly treated in the underlying present value calculations.

Remaining details are summarized in Exhibit 8. At time $t = 1$, the continuation present value is $PV_1 = 9{,}480(1.1)^{-1} + 22{,}407(1.1)^{-2} = 27{,}136$. Moreover, the firm's assets consist of its capital stock and the inventory. The question is how to divide the total among the two assets.

One way to proceed is to compare this continuation present value with what it would be were the firm unable to hold inventory at time 1.[22] If, then, the firm could not hold inventory at time 1 it would find it optimal to produce 75 units in the first period, followed by 112.5 in each of the remaining periods. Details are summarized in Exhibit 9, where we retain the same capital choice as in Exhibit 8.

	$t = 0$	$t = 1$	$t = 2$	$t = 3$
inflows		132(75) = 9,900	193.6(100) = 19,360	266.2(125) = 33,275
outflows	200(122.474) = 24,495	110(45.928) = 5,052	121(103.338) = 12,504	133.1(103.338)= 13,754
net, CF_t	-24,495	4,848	6,856	19,521

Exhibit 9: Cash Flows, Given no Inventory at t =1 ($q_1 = 75$, $q_2 = q_3 = 112.5$), and $K = 122.474$

Here the $t = 1$ continuation present value is $PV_1 = 6{,}856(1.1)^{-1} + 19{,}521(1.1)^{-2} = 22{,}366$.[23] So, with no inventory capital is the only asset, and capital by itself has a value of 22,366 at $t = 1$. Similarly, the remaining cash flow series is worth an additional 27,136 - 22,366 = 4,770 as a result of being able carry inventory.[24] In other words, if the firm retained the same capital stock but could hold no inventory at $t = 1$, its value at that point would increase by 4,770. Sounds like the inventory is worth 4,770.

With this success in hand, let's reverse the experiment and ask what the continuation present value would be were inventory the only asset on hand at $t = 1$. This requires some additional assumptions, as production requires strictly positive amounts of capital and labor. Suppose, then, the firm must abandon its capital at the end of the first period, and at the start of

[22]Notice we are treading on the perfect and complete market assumption here. There is no market for the inventory, and for that matter the firm's customer has agreed to a specific delivery schedule that guarantees the firm strictly positive rent. Moreover, alternative approaches to valuing the inventory, such as net realizable value, will lead to the same conundrum.

[23]Capital is set at $K = 122.474$ so our comparison rests on the same physical capital. The efficient choice, however, is $K = 124.373$.

[24]This comparison presumes the firm behaves optimally, which implies, for example, it adjusts its capital choice.

the second period will acquire new capital, capital that will last for two periods. Further suppose this new capital is paid for at time $t = 2$, at a price of $200(1.1)^2 = 242$ per unit. Holding the production plan constant, so only the capital is varied, leads to the details summarized in Exhibit 10.

	$t = 0$	$t = 1$	$t = 2$	$t = 3$
inflows		132(75) = 9,900	193.6(100) = 19,360	266.2(125) = 33,275
outflows	200(70.711) = 14,142	110(141.421) = 15,556	242(100) + 121(100) = 36,300	133.1(100) = 13,310
net, CF_t	-14,142	5,656	-16,940	19,965

Exhibit 10: Cash Flows, Given no Capital at $t = 1$ ($q_1 = q_2 = q_3 = 100$)

Here the continuation present value at time 1 is $PV_1 = -16{,}940(1.1)^{-1} + 19{,}965(1.1)^{-2} = 1{,}103$. That is, if the firm is unable to carry capital between the first two periods, its time 1 value will be 1,103. So its time 1 value declines from 27,136 to 1,103 if inventory is kept constant, at 25 units, but its capital drops to zero. This suggests the time 1 capital's value is 27,136 - 1,103 = 26,033.

If, now, these calculations made sense the time 1 value of 27,136 (via Exhibit 8) would be the amount contributed by the inventory (implied via Exhibit 9), or 4,770, plus the amount contributed by the capital (implied via Exhibit 10), or 26,033. But 4,770 + 26,033 > 27,136. We simply cannot isolate the component of total time 1 value due to each of the assets.

This reflects the non-separability of the firm's technology. It does not allow us to unambiguously identify how much of total value is attributable to its capital and how much to its inventory. The two assets interact. Thus we cannot, even in this simple, deterministic setting, talk unambiguously about the value of a specific asset.

Selected References

The importance of perfect markets in classical valuation is well articulated by Beaver [1998], as well as by the typical micro economics or finance textbook. Hirshlefier [1970] provides a deeper treatment, including the separation between production and consumption choices and the irrelevance of financial structure. In turn, use of this setting to develop the notion of income, and the pitfalls that ensue when we move beyond this setting, leads to the works of Fisher, Lindahl, Hicks, Kaldor, etc. An excellent primer is provided by Parker, Harcourt and Whittington [1986].

Key Terms

Certainty is foreknowledge, a setting in which there is no ambiguity as to what will transpire. In our stylized setting, the firm's activities lead to a cash flow series, or vector, and this cash flow series is regarded as fact, as guaranteed to transpire. We then imagine the claim to this cash flow series as being traded in a perfectly competitive market, at each and every point in time. *Perfectly competitive market* means there are no transactions costs of any sort, everyone knows the equilibrium price (the price at which supply equals demand), and everyone acts as a price-taker. In turn, markets are *complete* if each and every conceivable trade is, indeed, available in a market. We then stylize the market price of the claim to the firm's remaining cash flow series, as of time t, as the *continuation present value* at time t, equation [1], where we assume a constant interest rate. *Economic value* is market value, in a setting of perfect and complete markets. It is the price at which supply equals demand in the presumed market setting. Given our stylization, it is given by the continuation present value. *Economic income* is change in economic value, adjusted for cash flow, equation [3]. Given complete and perfect markets, it is the cost of monetary factors employed, the interest rate multiplied by the beginning of period continuation present value. The change in economic value is called *economic depreciation*. *Economic profit*, in this setting, is the present value of the firm's cash flow, the present value of the customers' payments less the present value of the payments to those who provide the factors of production. *Residual income* is economic income less the cost of the monetary capital employed. *Economic rent* is present when the firm's profit is strictly positive.

Problems and Exercises

1. What role is played by perfect markets in this chapter? What role is played by certainty?

2. Define economic income, economic profit and economic rent. In what sense do your definitions rely on perfect markets?

3. Given the perfect markets assumption, is it correct to write the firm's cash flow vector as $CF = [-PV_0, CF_1, CF_2, ..., CF_T]$? Explain.

4. Examine expressions [3] and [5] more closely. Does it make sense to think of economic income as "all you can consume this period without affecting your ability to consume in the future" or as a "flow of wealth?"

5. Consider a two period setting where the cash flows are given by $CF = [-1{,}000, x, 1{,}210 - 1.1x]$ and the interest rate is $r = 10\%$. Notice for any value of x we have $PV_0 = 1{,}000 = x(1.1)^{-1} + (1{,}210 - 1.1x)(1.1)^{-2}$. Plot PV_0 (= 1,000), PV_2 (= 0) and PV_1 for $0 \leq x \leq 500$.

Does your plot identify a valuation function *V*, as in expression [2], that maps cash flow vectors into sequences of value, for the given interest rate? Explain.

6. Return to the setting in Exhibit 1, but now suppose the firm retains all cash, and distributes the total in a single liquidating dividend at time $t = 3$. Cash on hand is invested, and earns at the rate $r = 10\%$. (After all, markets are perfect). Determine the firm's economic value and economic income sequences. Comment on the pattern that emerges. (Hint: the cash balance at $t = 2$ will be 9,680 + 4,950(1.1).)

7. Ralph's Enterprise displays the following annual cash flow series: CF = [-2,000, 700, 600, 1,155] over its life of $T = 3$ periods. The cash flow at times $t = 1$, 2 and 3 are paid out in a dividend just as the books are closed, so the end of period cash balance is always zero. Determine the value of Ralph's Enterprise at each and every period, as well as the economic income each period. Finally, how would these calculations differ if the dividend were declared and paid one instant after the books are closed?

8. Return to the setting in Exhibit 4, but now assume labor is paid at the beginning of the respective periods. First determine the three wage rates so we have the economically equivalent story. Then determine the economic income for each of the periods. Comment on your results.

9. This is a continuation of Exercise 14 in Chapter 2. Now assume the timing details in Exhibit 4, where capital is acquired immediately, product *i* is delivered and paid for at the end of the *i*th period, and labor for product *i* is also paid at the end of the *i*th period. The interest rate is $r = 10\%$. Product *i* customers therefore pay $200(1.1)^i$ per unit, at time *i*; and labor source *i* is paid $100(1.1)^i$ per unit, again at time *i*. Determine the firm's cash flow vector, sequence of economic valuations, and economic income for each period.

July 8, 2001, Joel