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The Reporting Organization

Our study begins with some organization for whom conveying something through its accounting system is a substantive issue. This "reporting organization," or firm, will always be present, as it is essential to ground our study of accounting in a setting where the particular accounting issue arises endogenously. The trick we use is to carry along a simple model of a firm, simple enough to be tractable but flexible enough so the firm will find it rational to engage in behavior, such as acquisition of a long lived asset or contracting with a manager, that creates an accounting issue. Indeed, we will come to appreciate the fact that understanding why the firm is engaging in such behavior is the key to resolving the accounting issue associated with that behavior.

This firm, you will learn, uses factors of production and its technology, to produce goods and services, goods and services that are, in turn, sold to customers. But patience is also in order, as we must understand this simple firm before we move on to use it in our study of accounting.

We now introduce an explicit model of the reporting organization. Two themes are important here. First, what we mean by the reporting organization is purposely left vague. Accounting thought often refers to the reporting organization as the "entity." Digging deeper we find, at times, an emphasis on personal stake (the proprietary view, where the organization is inherently identified with its owners), economic distinction (the entity view, where the organization is distinct from its owners), or a defined set of activities (the fund view, where assets and obligations are grouped together). Clearly, though, accounting practice includes financial reporting, divisional performance measurement, product line profitability assessment, not-for-profit reporting, municipal reporting and national income reporting. So we purposely leave open the question of what type of organization we are considering."[1]

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[1]The FASB, with its emphasis on corporate and not-for-profit financial reporting uses the term entity in a nearly colloquial manner: Paragraph 24 of *Concepts Statement No. 6* (FASB [1985]) reads as follows: "All elements are defined in relation to a particular entity, which may be a business enterprise, an educational or charitable organization, a natural person, or the like. An item that qualifies under the definitions is a particular entity's asset, liability, revenue, expense, and so forth. An entity may comprise two or more affiliated entities and does not necessarily correspond to what is often described as a "legal entity." The definitions may also refer to "other entity," "other entities," or "entities other than the enterprise," which may include individuals, business enterprises, not-for-profit organizations, and the like. For example, employees, suppliers, customers or beneficiaries, lenders, stockholders, donors, and governments are all "other entities" to a particular entity. A subsidiary company that is part of the same entity as its parent company in consolidated financial statements is an "other entity" in the

Second, we typically, and traditionally, approach an accounting issue, such as how to depreciate a long lived asset, by presuming the asset is in place and then asking how best to report, or measure, the important consequences associated with that asset. From our standpoint, though, the beginning point should be a setting where the reporting organization finds it rational, if you will, to acquire the long lived asset in the first place. So, when we examine inventory reporting, we want to do so in a setting where the organization finds it rational to hold inventory. When we examine derivatives, we want to do so in a setting where the organization finds it useful to hold derivatives. When we confront the interactions inherent in a multi-product firm, we want to do so in a setting where the organization finds it useful to produce multiple products. Taken together, this compels us to begin with a model of the reporting organization.

This insistence on a reporting organization that finds it rational to engage in behavior that creates the accounting issue in the first place is not an academic nicety. A firm that produces multiple products is, presumably, driven by an economy of scope; but an economy of scope means we cannot fully separate the income and assets associated with each of the products. A firm that holds inventory does so for a reason. To examine accounting for that inventory we have to worry about why the firm is holding that inventory and why reporting, somehow, on that holding of inventory might be useful. Similarly, a firm that uses derivatives does so for a reason. To examine accounting for those derivatives we have to worry about why the firm is using derivatives and why reporting, somehow, on that use of derivatives might be useful.

Having said that, we still face the problem of placing some structure on the activities and behavior of the prototypical organization whose accounting practice we will study. For this purpose we focus on an organization that acquires factors of production and, using some specified technology, transforms those factors into goods and services. To give the story context, we interpret it as a profit seeking firm, though this is done for expositional reasons, and you are encouraged to think in the broadest possible terms.

To build intuition, and to ease the pain of assimilation, we begin with a single product setting. We then move on to the important multiproduct setting. Keep in mind we want a stylized, minimalist description of the reporting organization's activities and behavior, one rich enough to help train our intuition, to capture important first-order effects, but not so complicated as to be unworkable, not to mention annoying and counterproductive.

The Single Product Case

Our organization, or firm, then, combines factors to produce and sell some product or service. Initially, there is only one product. Let q (≥ 0) denote the quantity produced (and sold). Production requires an appropriate combination of factors be available. Two factors are sufficient at this point, we call them "capital," in quantity K (≥ 0), and "labor," in quantity L (≥ 0).

The appropriate combination of factors is specified by the (exogenous) technology. We simply assume it is given by the following restriction:

$$q \leq \sqrt{KL}. \qquad [1]$$

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separate financial statements of its parent."

The idea is we require $K > 0$ and $L > 0$ to produce output q (> 0). But capital and labor are substitutes. For example, both $K = 1$ and $L = 4$ and $K = 4$ and $L = 1$ can be used to produce $q = 2$ units. Indeed, any combination such that $KL = q^2$ can be used to produce output quantity q ($K > 0$ and $L > 0$, of course).

Naturally, the factor prices enter at this stage. If, for example, capital is unusually expensive, labor will be substituted for capital, and vice versa.[2] We do, however, assume the technology is limited in the sense no more capital than K^{max} units can be employed:

$$K \le K^{max}. \qquad [2]$$

This upper bound limits the physical size of the capacity. The technology simply does not allow the firm to become arbitrarily large.

Beyond this, the capital and labor mix depends on the factor prices. For simplicity we assume the factors are acquired in perfect markets. Let p_K denote the price per unit of capital, and p_L the (wage or) price per unit of labor. The total expenditure on factors, then, is simply $p_K K + p_L L$. If, now, q units are to be produced, the K and L are chosen to minimize this total expenditure, subject to feasibility. This gives us the cost of producing q units, defined as follows:

$$C(q) \equiv \underset{K,\, L\, \ge\, 0}{\text{minimum}} \;\; p_K K + p_L L \qquad [3]$$

$$\text{subject to [1] and [2].}$$

In other words, the firm selects its capital (K) and labor (L) to minimize the total expenditure on factors (K and L here), subject to being able to produce the desired output. [3] is often referred to as *economic cost* to distinguish it from an accounting construct called accounting cost.

an illustration

To illustrate, suppose the factor prices are given by $p_K = 200$ and $p_L = 100$. So capital is twice as expensive as labor. Further suppose the capital limit is $K^{max} = 125$, and $q = 150$ units are to be produced. Solving program [3], using these specifications, gives us optimal choices of $K = 106.0660$ and $L = 212.1320$, along with a cost of $C(150) = 42{,}426.41$.

Varying q, and solving for the optimal K and L choices along the way, allows us to determine the firm's cost curve.[3] Exhibit 1 plots $C(q)$ for $0 \le q \le 300$. Notice it is linear, with

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[2]This is a specific version of what economists call a Cobb-Douglas technology. More generally, for two factors, the restriction is written $q \le K^{\alpha}L^{\beta}$, where the exponents are non-negative. (Our specific version, which is adequate for our purpose, rests on $\alpha = \beta = .5$.) Now, constant returns means a proportionate increase in factors leads to a proportionate increase in output; and $\alpha + \beta = 1$ implies constant returns to scale. Likewise, $\alpha + \beta < 1$ implies decreasing returns to scale, and $\alpha + \beta > 1$ implies increasing returns to scale. Importantly, constant returns implies a linear cost curve, a cost curve with constant marginal cost.

[3]Notice the solution, for any q, depends on the factor prices. So, technically, we should denote the cost by $C(q$; factor prices). This dependency on the factor prices should be understood, but will not be a formal part of our notational assault on your senses.

a slope of about 283 (282.8427 to be more precise), up to about q = 175 units (176.7767 to be more precise), and then increases at an increasing rate.

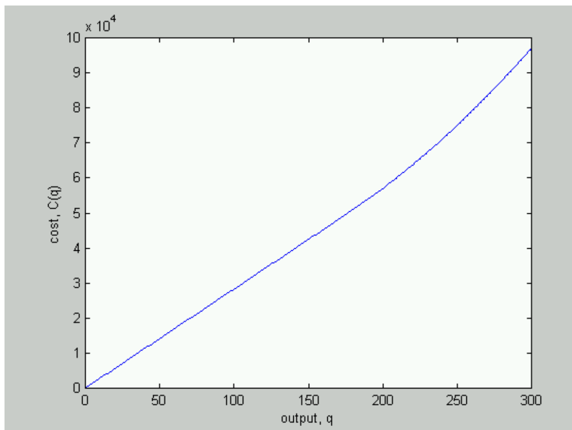

Exhibit 1: $C(q)$ for Numerical Illustration

This is more evident in Exhibit 2, where we plot the corresponding marginal cost.[4] (*Marginal cost* is the rate of change of economic cost with respect to the product.) Notice the marginal cost is constant, reflective of the underlying linear cost function, up to about q = 175; beyond that it is increasing.

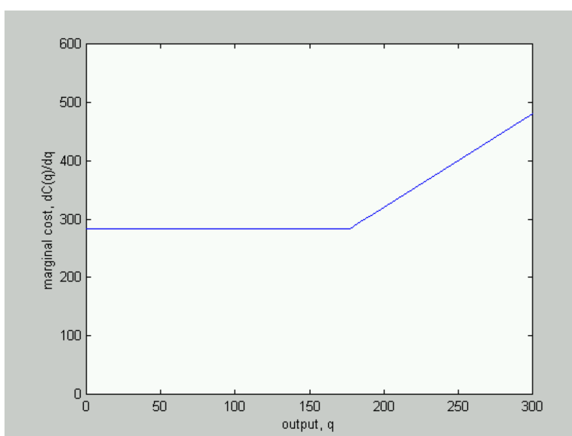

Exhibit 2: Marginal Cost for Numerical Illustration

[4]We often think of marginal cost as well approximated by the incremental cost, at some particular point, of producing one additional unit. Technically, however, it is the slope of the cost curve at a particular point. Literally, the marginal cost when output quantity q^* is being produced is the slope of $C(q)$ at the point $q = q^*$, it is the derivative of the cost function evaluated at point $q = q^*$.

The culprit here is the capacity limitation of $K^{max} = 125$. Given the factor prices of $p_K = 200 > p_L = 100$, the firm uses more labor than capital, but naturally increases both as output expands. (In fact, the firm uses twice as much labor, L, as capital, K, in this case.) When K^{max} is reached, however, it can no longer use the optimal balance between the two factors. Hence, beyond that point production becomes inefficient, marginal cost increases, and total cost increases at an increasing rate.

Additional details are developed in the Appendix.

a short run cost curve

From here we encounter the related notion of a short run cost curve. The idea is straightforward. Suppose one or more factors cannot be altered. Then the firm has limited freedom in finding the best mix of factors. In our simple two factor story, we are in serious trouble if both factors are fixed. So, staying with our numerical story, suppose the firm initially acquires $K' = 75$ units of capital. In the short run this capital choice cannot be altered. Only labor can be altered, and the technology constraint forces $L = q^2/75$ if q units are to be produced. Clearly, then, with $K = K'$ frozen in place, producing q units will require labor (via [1]) of $L = q^2/K' = q^2/75$. So the firm's short run cost curve is simply

$$C^{SR}(q) = p_K K' + p_L q^2/K' = 200(75) + (100/75)q^2.$$

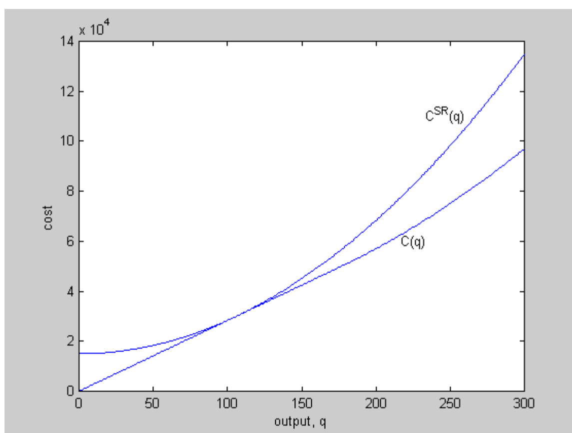

Exhibit 3: Short Run and Long Run Curves for Numerical Illustration

In Exhibit 3 we superimpose this particular short run curve on the underlying long run curve, $C(q)$.[5]

[5]Notice the subtle terminology. Long run is the case where all factors can be varied. A particular (and emphatically, not the) short run is one where some subset of factors is fixed at some specified point. Also, the K^{max} constraint is a statement about

The short run versus long run distinction surfaces in a variety of contexts. For example, if we are trying to value inventory at replacement cost, it is essential to understand which if any factors are fixed at the point of the calculation. Similarly, if we are using a derivative instrument to hedge some type of exposure, it is likewise essential to understand which if any factors are fixed were the firm called upon to deal with that exposure.

The Multiproduct Case

We next turn our attention to the important multiproduct case. The firm is now assumed to produce (and sell, of course) three products, with respective non-negative quantities denoted q_1, q_2 and q_3. The products will share a common capital base, K in our earlier story; but they will have dedicated labor inputs. Let L_1, L_2 and L_3 denote the non-negative quantities of the three labor inputs, where it is understood labor quantity L_i is used in production of the ith product.

The original technology constraint, [1], is now replaced by the following family of constraints:

$$q_1 \le \sqrt{KL_1} \qquad [1a]$$
$$q_2 \le \sqrt{KL_2} \qquad [1b]$$
$$q_3 \le \sqrt{KL_3} \qquad [1c]$$

Again, capital and labor are substitutes. In addition, capital is shared among the products, suggesting an economy of scope: it is less costly to produce the three products within the same firm. Beyond this we retain our earlier upper bound on the amount of capital that can be employed, constraint [2].

The firm operates in perfectly competitive factor markets. The price of labor type i is p_{Li} per unit and the price of capital, as before, is p_K per unit.

The firm's cost curve is located in, hopefully, familiar fashion. Paraphrasing our earlier work, we have the following:

$$C(q_1,q_2,q_3) \equiv \underset{K,\, L_1,\, L_2,\, L_3\, \ge\, 0}{\text{minimum}} \;\; p_K K + p_{L1}L_1 + p_{L2}L_2 + p_{L3}L_3 \qquad [4]$$
$$\text{subject to [1a], [1b], [1c], and [2].}$$

To illustrate, we expand our earlier setting to three products, and assume the labor prices are given by $p_{Li} = 100$ and also continue to assume $p_K = 200$. Again, $K^{max} = 125$. Now suppose $q_1 = 150$, while $q_2 = q_3 = 0$. We readily find $C(150,0,0) = 42{,}426.41$. You should recognize this as the same cost we had for producing 150 units in the single product story. The reason is simple. If we only produce one of the three products, the technology reduces to the original technology, and we have, conveniently, assumed the same factor prices.

Also notice the built in symmetry. With the labor prices identical, producing 150 of any one of the three products coupled with zero of the other two implies a cost of 42,426.41. Contrast

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the long run capabilities of the technology. The technology in the long run simply cannot use capital in excess of K^{max}.

this with the case where $q_1 = q_2 = 150$ and $q_3 = 0$. Solving [4], we find $K = K^{\max} = 125$, $L_1 = L_2 = 180$ ($L_3 = 0$, of course), and a cost of $C(150,150,0) = 61{,}000 < 2(42{,}426.41) = 84{,}852.82$. It is much less costly to simultaneously produce the products. This reflects the technological assumption that the products can share the capital.

This is further evident when we analytically construct the firm's cost curve. If the $K^{\max}$ constraint is not binding, we readily find [6]

$$C(q_1, q_2, q_3) = 2\sqrt{p_K(p_{L1}q_1^2 + p_{L2}q_2^2 + p_{L3}q_3^2)}. \qquad [5]$$

If two of the products are set to zero, we are back to our original, single product setting.

separability concerns

We should also note the fact this cost curve is not separable. We cannot separate it into three components, one for each of the three products. We cannot write the cost curve in the form $C(q_1,q_2,q_3) = F(q_1) + G(q_2) + H(q_3)$. While seemingly one more arcane point, it is important to understand the multiproduct firm does not, in general, exhibit a *separable* cost curve, and this lack of separability means we cannot unambiguously speak of the "cost of product *i*." In fact, this may be the very reason for having a multiproduct firm instead of having a series of single product firms. Accounting costing procedures, however, lure us into thinking otherwise, but this is nothing other than a false impression created by approximation techniques used by the typical costing system.

One way to visualize this is to focus on marginal cost. In Exhibit 4 we plot the marginal cost of the first product, assuming $q_2 = q_3 = 0$, as well as for the case where $q_2 = q_3 = 100$.[7] The first case, of course, has no presence of the second and third products, and thus reduces to our single product story (and the marginal cost plot in Exhibit 2). The second has a substantive presence of the other two products, which results in $K = 100$ at the initial point of $q_1 = 0$. So we have, for low values of q_1, a dramatically lower marginal cost of the first product, simply because the large amount of capital, shared among the products, implies very little labor is required to produce small amounts of the first product.

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[6] Deriving this expression is a simple extension of material developed in the Appendix. For any value of $K > 0$, producing q_i units of product i will lead the organization to acquire precisely $L_i = q_i^2/K$ units of labor. So the expenditure calculation becomes:

$$p_K K + p_{L1}q_1^2/K + p_{L2}q_2^2/K + p_{L3}q_3^2/K = p_K K + \theta/K,$$

where $\theta = p_{L1}q_1^2 + p_{L2}q_2^2 + p_{L2}q_3^2$. Setting the derivative equal to zero allows us to identify the minimizing choice of K:

$$p_K - \theta/K^2 = 0.$$

So $K^2 = \theta/p_K$, or $K = \sqrt{\theta/p_K}$. But then the overall expenditure is $p_K K + \theta/K = p_K\sqrt{\theta/p_K} + \theta/\sqrt{\theta/p_K} = 2\sqrt{\theta p_K}$. So we have $C(q_1,q_2,q_3) = 2\sqrt{\theta p_K}$ (where, again, $\theta = p_{L1}q_1^2 + p_{L2}q_2^2 + p_{L2}q_3^2$), as long as the chosen K does not violate the capacity constraint, [2]. In turn, [2] is not violated by unconstrained choice of K as long as $K = \sqrt{\theta/p_K} \le K^{\max}$, or $\theta/p_K \le (K^{\max})^2$. Outside this region, we resort to use of the maximum feasible capital and have a cost expression of $p_K K^{\max} + p_{L1}q_1^2/K^{\max} + p_{L2}q_2^2/K^{\max} + p_{L3}q_3^2/K^{\max}$.

[7] Given cost curve $C(q_1,q_2,q_3)$, the marginal cost of the first product at some particular point is simply the partial derivative of the cost curve, $\partial C(q_1,q_2,q_3)/\partial q_1$, evaluated at the point in question. Carrying over notation from the prior note, if the $K^{\max}$ constraint is not binding, we find $\partial C(q_1,q_2,q_3)/\partial q_1 = 2p_{L1}q_1/\sqrt{\theta/p_K}$. Conversely, if the $K^{\max}$ constraint is binding, we have $\partial C(q_1,q_2,q_3)/\partial q_1 = 2p_{L1}q_1/K^{\max}$.

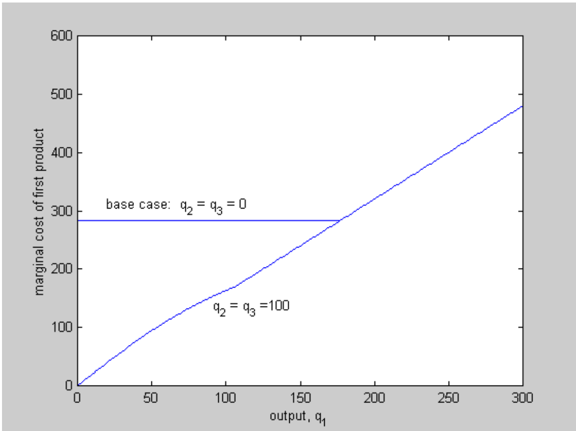

Exhibit 4: Marginal Cost of First Product for Three Product Case

A second way to visualize this is to focus on the shape of the total cost curve. In Exhibit 5 we plot $C(q_1,q_2,q_3)$ for two different cases: one where $q_3 = 0$ and the other where $q_3 = 100$. Can you explain the qualitative shape of the two surfaces?

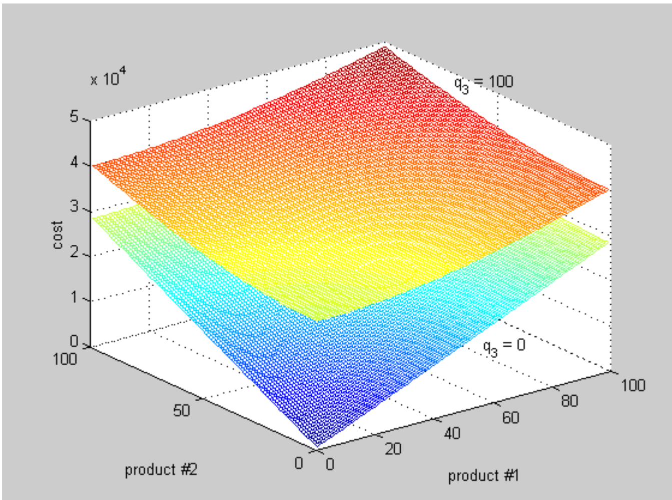

Exhibit 5: Total Cost Surface

We could, with sufficient patience, continue on and examine short run cost surfaces for our multiproduct firm.

marginal versus average cost

But the setting is a little more subtle than merely adding a few extra products to the usual story. Return to our running example, and stay in the region where the output is not so large the K^{max} constraint is binding. This is expression [5] above, but now using the specific factor prices of $p_K = 200$ and $p_{Li} = 100$. Collecting terms gives us the following:

$$C(q_1, q_2, q_3) = 200\sqrt{2(q_1^2 + q_2^2 + q_3^2)}.$$

Now use your intuition. We know capital is shared across the products, and also that capital and labor are substitutes. Suppose we are producing a large number of the second and third products, but a small number of the first. The marginal cost of the first product will then be unusually low, because considerable capital will be in place to support production of the last two, and thus a small amount of extra labor is all that is necessary to produce slightly more of the first product. Intuitively, the marginal cost of a product depends on how many of each product is being produced. Remember, capital and labor are substitutes here, and capital is shared across the products.

This is, in fact, evident when we explicitly calculate the marginal cost. Product i's marginal cost is the (partial) derivative of the cost expression with respect to the ith product quantity. That is,

$$\partial C(q_1, q_2, q_3)/\partial q_i = 400 q_i / \sqrt{2(q_1^2 + q_2^2 + q_3^2)}.$$

If, then, q_2 and q_3 are large while q_1 is small, the denominator is large. For the first product, though, the numerator is small, so the marginal cost of the first product is small, at that particular point. Exhibit 4 is illustrative.

The message, we hope, is clear. If the firm benefits from an economy of scope, if the firm finds it economical to produce a variety of products, we should expect the marginal cost of a product to depend on how many of the other products are being produced. We cannot, that is, view a product in isolation. The cost curve is simply not separable. This is the nature of the multiproduct firm.

We speak, then, without hesitation about a particular product's marginal cost in the multi-product setting. The additional complication is that, in general, we expect a product's marginal cost to depend on the output of the other products.

Average cost, though, is another story. The idea is deceptive, of course: divide the total cost associated with the product by the number of units of that product. This presumes we know, unequivocally, how much of the total cost is due to each of the products. But glance back at our expression for the firm's cost. There is simply no way to separate that total into components that depend uniquely on each of the products, and that sum to the total cost in question. The cost curve is not separable. This means average cost is not defined in this case. The simple, intuitive idea of average cost developed in the world of a one product firm does not, in general, extend to the multiproduct firm. Marginal cost is a centerpiece concept in the world of multiproduct firms, but average cost is, in general, an oxymoron in that setting.[8]

The difficulty, again, is we have no way of separating the total cost. In accounting, of course, we employ cost allocation procedures to provide such separation. But we must emphasize this is an accounting procedure, a procedure quite capable of distorting the underlying reality. Average cost simply does not exist in the typical multiproduct firm![9]

It is important to dwell on these last two Exhibits. In accounting our tendency is to treat groups of activities as more or less independent. We view accounting income of a period as more or less independent of other periods, and we view the cost of a particular product as more or less independent of the other products. Indeed, in the product costing arena we almost always work with a linear approximation to the firm's cost curve. We should expect, however, that a firm lasts for many periods and produces many products because grouping these activities is efficient. If so, if there is an economy of scope, we should not expect the activities to be independent. We should expect the marginal cost of one product to depend on the other products, and so on. Indeed, we should expect the marginal cost of some product at time *t* will depend on products the firm anticipates producing in future periods. Enough!

Variations on a Theme

This three product story, the basis of [5], provides a platform for our exploration of accounting. To offer a glimpse of its versatility, we now briefly sketch three variations that will turn out to be useful in our work.[10]

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[8]With enough work and notation, we could extend this theme to a multiperiod setting where the firm had multiple products as well, and assets. And we would wind up having difficulty fully separating the assets, so each product was associated with a unique asset base.

[9]Once K^{max} is binding, the cost curve is $p_K K^{max} + p_{L1} q_1^2/K^{max} + p_{L2} q_2^2/K^{max} + p_{L3} q_3^2/K^{max} = 200(125) + 100(q_1^2 + q_2^2 + q_3^2)/125$. Again the concept of average cost is problematic, as we must address how much of the 200(125) expenditure on capital should be associated with each of the products.

[10]Another variation, a fixed proportions story, provides an opportunity to deepen our understanding of the particular (square root) technology assumption. So suppose the three products are produced in the fixed proportions of α: β: γ. For example, the three products might be produced in the proportions 3: 4: 5. This means any output profile can be written in the convenient form $q_1 = \alpha z$, $q_2 = \beta z$ and $q_3 = \gamma z$, for some number z. To illustrate, in the 3: 4: 5 case we might have $q_1 = 3(10) = 30$, $q_2 = 4(10) = 40$ and $q_3 = 5(10) = 50$.

Now force this pattern into the cost curve. If K^{max} is not binding, but the output is produced in fixed proportions, we can simplify our original expression for the cost surface in the following manner:

$$C(q_1,q_2,q_3) = 2\sqrt{p_K(p_{L1}q_1^2 + p_{L2}q_2^2 + p_{L3}q_3^2)} = 2\sqrt{p_K(p_{L1}(\alpha z)^2 + p_{L2}(\beta z)^2 + p_{L3}(\gamma z)^2)}$$
$$= 2z\sqrt{p_K(p_{L1}\alpha^2 + p_{L2}\beta^2 + p_{L3}\gamma^2)}.$$

This is simply a constant, multiplied by the scale factor, z. In our running example, based on $p_K = 200$ and $p_{Li} = 100$, and assuming proportions of 3: 4: 5, we have a cost surface expression of $C(q_1, q_2, q_3) = 1{,}000z$. Cost is linear in the scale factor, z. Doubling output, while maintaining the noted proportions, doubles the cost. Now think back to the single product story. There, when K^{max} was not binding we had a linear cost curve. The same arises here, in the fixed proportions case. Intuitively, if output is always produced in fixed proportions, we de facto have a single product story, where by product we mean the scale

discounted cash flow

The first variation is a discounted cash flow story. To set the stage, stay with the running example. But further suppose the three products are sold in perfectly competitive markets at respective prices of 120, 160 and 200 per unit. The firm seeks to maximize its profit, consisting of sales revenue less cost. That is, it wants to

$$\underset{q_1,\, q_2,\, q_3 \geq 0}{\text{maximize}} \;\; 120q_1 + 160q_2 + 200q_3 - C(q_1,q_2,q_3)$$

A solution has $q_1 = 75$, $q_2 = 100$ and $q_3 = 125$. This provides a total revenue of 50,000, and carries a total cost of 50,000. Merely breaking even should come as no surprise, we presume perfectly competitive markets and any strictly positive profit prospects would be competed away. For later reference, the cost total is based on factor choices of $K = 125$ (= K^{max}), $L_1 = 45$, $L_2 = 80$, and $L_3 = 125$.[11]

Now alter the story in the following manner. Capital is acquired at time $t = 0$, and production immediately commences. The first product is sold (and customers pay) at the end of the first period, where the first product's labor factors are also paid. The second product is sold at the end of the second period, where that product's labor factors are also paid. A parallel pattern applies to the third product. The interest rate is $r = 10\%$.

Further suppose the respective selling prices are $120(1.1) = 132$, $160(1.1)^2 = 193.60$, and $200(1.1)^3 = 266.20$. The respective labor prices follow the same pattern: $100(1.1) = 110$, $100(1.1)^2 = 121$, and $100(1.1)^3 = 133.10$. But now when the firm seeks to maximize the present value of receipts less expenditures, the prices all discount to precisely what we had before. For example, the present value of the time $t = 3$ selling price is $266.2(1.1)^{-3} = 200$. Our earlier solution thus reappears in full glory. (Of course this exact replication depends on the judicious price assumptions, such as a spot labor price growing at the rate of interest.)

But we now have a story where the firm makes an up-front investment, and then harvests that investment in later periods. This is evident in the tally of period-by-period cash flows, detailed in Exhibit 6. You should verify the present value of the cash flow series calculated as of time $t = 0$ is precisely 0. In present value terms, the firm breaks even. Such is the power of perfect competition.

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factor, z.

[11]This is a well chosen razor's edge case where the firm's behavior is identified by equating marginal revenue with marginal cost for each product, and where, at such a solution, it earns zero rent, allowing us to interpret the story as one of perfect competition. To verify our solution, differentiate the profit expression for each q_i and set the results equal to zero (That is, equate marginal revenue with marginal cost, for each product.):

$$120 - \partial C(q_1,q_2,q_3)/\partial q_1 = 0;$$
$$160 - \partial C(q_1,q_2,q_3)/\partial q_2 = 0, \text{ and}$$
$$200 - \partial C(q_1,q_2,q_3)/\partial q_3 = 0.$$

Now recall our earlier work on the marginal cost expressions. For product i, we have $\partial C(q_1,q_2,q_3)/\partial q_i = 2p_{Li}q_i/\sqrt{\theta/p_K}$ if the K^{max} constraint is not binding and $2p_{Li}q_i/K^{max}$ otherwise. (Recall $\theta = p_{L1}q_1^2 + p_{L2}q_2^2 + p_{L2}q_3^2$.) It is easy to verify $q_1 = 75$, $q_2 = 100$ and $q_3 = 125$ satisfy these conditions, and put us right at the break point where any additional production encounters the K^{max} constraint. Indeed, any output in the proportion of 3: 4: 5 that does not violate the K^{max} constraint satisfies these conditions, and we have merely taken from among these the largest possible output as our particular solution.

	$t = 0$	**$t = 1$**	**$t = 2$**	**$t = 3$**
time t expenditures	200(125) = 25,000	110(45) = 4,950	121(80) = 9,680	133.1(125) = 16,637.50
time t receipts		132(75) = 9,900	193.6(100) = 19,360	266.2(125) = 33,275
net cash flow	-25,000	4,950	9,680	16,637.50

Exhibit 6: Cash Flow Calculations for Time Line Story, $q_1 = 75$, $q_2 = 100$, $q_3 = 125$

We will, in fact, make extensive use of this variation, and this example, in the next few chapters. To whet your appetite, notice the firm now expends a total of 25,000 + 4,950 + 9,680 + 16,637.50 = 56,267.50; and it receives revenues totaling 9,900 + 19,360 + 33,275 = 62,535. So it earns a profit, over the three periods, of 6,267.50. Typically, now, we ask how much of this profit is properly associated with each period Here, that boils down to asking how we should depreciate the initial investment of 25,000. We defer the answer, but make the important point that the cash flows in Exhibit 6 arise from rational behavior by the firm. We begin, in other words, with a firm that finds it rational to engage in the behavior that creates our accounting concern.

inventory

The second variation on our theme involves inventory. Stay with the time line sketched in the section above, where capital is acquired at time $t = 0$, the first product is delivered at time $t = 1$, etc. Also retain the same factor prices and K^{max} setting. But now suppose we are dealing with the same product, say some subcomponent, that must be delivered at times $t = 1$, 2 and 3. The customer requires delivery of 75 units at time $t = 1$, 100 at time $t = 2$, and 125 at time $t = 3$.

One approach is to produce according to the customer's time schedule, thus setting $q_1 = 75$, $q_2 = 100$ and $q_3 = 125$. We know from our work immediately above that this policy will cost the firm (in present value terms) a total of $C(75,100,125) = 50,000$.

A second approach is to smooth production by setting $q_1 = q_2 = q_3 = 100$. Here the firm would build inventory in the first period, and eventually deplete that inventory in the third period. This will cost the firm (again in present value terms) a total of $C(100,100,100) = 48,990 < 50,000$. It is less costly for the firm to smooth production, and use inventory to absorb the differences between its output schedule and the customer's required delivery schedule. The reason is, with shared capital and unchanging labor prices, it is less costly to produce a given total by equalizing the production across periods.[12] The firm rationally holds inventory at the interim stage; and this provides us a setting in which to examine proper accounting treatment of that inventory.

[12]In fact, producing 100 each period is the most efficient arrangement here. You can verify that this schedule equalizes the marginal cost of production, across the three periods. Absent equalization, we would gain by shifting, at the margin, from the higher to the lower marginal cost setting. This does, however, presume the only holding cost associated with inventory is the monetary cost of the investment in that inventory.

managerial activity

The final variation on our theme concerns managerial activity. Eventually we will examine the use of accounting measures in evaluating the performance of the firm's management. Again, the approach we will follow rests on an endogenous perspective. Here we require it be rational for the firm to hire the services of a manager, and to subsequently worry about the services actually supplied, to the extent evaluating the performance of the manager is an important task.

The initial part of this, rational use of a manager, is accomplished by expanding the list of factors from capital and labor, to capital (K), labor (L), and a manager (M). The specific technology setup simply expands what we have used to this point.

To avoid clutter, return to the single product setting. Introducing the manager now alters the original technology constraint in [1] to [1′]:

$$q \leq M\sqrt{KL}. \qquad [1']$$

The idea is the three factors are substitutes. More important, the new factor, M, can make the other two more, or less, productive. (No cynical comments, please.) For any positive K and L, the initial story was one in which output totaling $\sqrt{KL}$ could be produced. In the expanded setup, output totaling $M\sqrt{KL}$ can be produced. Implicitly, we used $M = 1$ in our earlier work.

The latter part, rational concern for evaluating the manager, is handled by presuming the managerial factor is available in an imperfect market, one where contracts for explicit services are not enforceable because the service actually supplied cannot be verified. For example, we can see whether the manager produced a new product plan, but not the diligence, care, and strategic vision that went into producing that plan. Performance evaluation now enters the fray, as an information device aimed at this lack of verifiability. For example, a more talented or focused manager is likely to provide a higher M, and a higher M makes the other factors more productive. Implicitly, then, the productivity of the other factors informs about the manager's talent and behavior. This establishes a connection between the cost or profit incurred, and the manager's performance.

Several additional details must be added to the story to bring all of this alive, details that will be supplied in Chapter 11. For the moment it is sufficient to see the flexibility of our model of the reporting organization.

Summary

Our study begins with a reporting organization, an organization in generic form that combines factors of production with its technology to produce goods and services. In its most extensive form, we identify three factors: capital (K), labor (L) and management (M). The discounted cash flow story stresses the investment aspects, the inventory story stresses substitution issues, and the managerial activity story stresses managerial behavior. We will move in and out of these variations throughout our study. The goal at each twist and turn is to exhibit the accounting issue in a natural setting, with minimal distraction.

Appendix: An Analytic Aside For The Single Product Case

It is possible to shed more light on the cost curve exercise in the text. It turns out that when the capital constraint is not binding, when the firm is free to select its mix of K and L, it does so in the following fashion:[13]

$$K/L = p_L/p_K.$$

Intuitively, the higher the relative price of K, the more L is employed, and vice versa. In our specific story, we have $p_L/p_K = 100/200$, implying $L = 2K$, as noted above.

Now, with this insight, and using the specific prices in our illustration, we use the technology constraint [1] to tease out the factor choices:

$$q = \sqrt{KL} = \sqrt{K(2K)} = K\sqrt{2}\text{; or}$$
$$K = q/\sqrt{2} \text{ and } L = 2K = q\sqrt{2}.$$

Now, how large can q be before this delicate balancing of $L = 2K$ runs afoul of K^{max}? Well, $K \le K^{max}$ implies $K = q/\sqrt{2} \le K^{max}$; or $q \le K^{max}\sqrt{2} = 125\sqrt{2} = 176.7767$. Beyond this point, the best our firm can do is set $K = K^{max} = 125$, and disproportionately use more and more labor, in effect solving $L = q^2/K^{max} = q^2/125$.

Collecting these details, when q is not too large and the firm is free to balance its mix of K and L, it sets $L = 2K$ (which implies $K = q/\sqrt{2}$ and $L = q\sqrt{2}$). Beyond this point the firm sets $K = K^{max}$. So the cost curve is

$$C(q) = 200K + 100L = 200(q/\sqrt{2}) + 100q\sqrt{2} = 200\sqrt{2}q = 282.84q,$$

if $q \le K^{max}\sqrt{2} = 176.7767$; and

$$C(q) = 200K^{max} + 100q^2/K^{max} = 200(125) + (100/125)q^2 = 25{,}000 + .8q^2$$

[13] To derive this, square both sides of [1]: $q^2 \le KL$. So, for any tentative choice of K (> 0 of course), the L choice must satisfy $q^2/K \le L$. Naturally, we don't waste resources, so we match this tentative K with just the "right amount" of L: $q^2/K = L$. Now substitute this expression for the L into the overall expenditure calculation:

$$p_K K + p_L L = p_K K + p_L q^2/K.$$

This expression, for any output q, depends only on K. Setting the derivative equal to zero allows us to identify the minimizing choice of K:

$$p_K - p_L q^2/K^2 = 0.$$

This implies $K^2 = (p_L/p_K)q^2$, or $K = q\sqrt{p_L/p_K}$. But then $L = q^2/K = q^2/(q\sqrt{p_L/p_K}) = q\sqrt{p_K/p_L}$. We thus have $K/L = p_L/p_K$, as claimed.

Now, for the technically inclined, we have reduced this to a choice of K, and then used the standard technique of setting the derivative to zero to find the optimal K (presuming, of course, the K^{max} condition is not violated). We should also check the second order condition here, to convince ourselves we have located a minimum. (For example, setting the derivative to zero identifies the minimum point of the function if that function is convex, or "U-shaped.") We will not mention this issue again, but assure the reader that throughout our work the optimization problems will remain well behaved, so these more technical considerations will not be an issue.

otherwise. Notice the marginal cost, defined as the derivative of $C(q)$ with respect to q, is 282.84 in the first region, and $1.6q$ in the second. From here you should be able to replicate Exhibits 1 and 2.

Indeed, we can go further and derive the general structure of the cost curve here:[14]

$$C(q) = 2q\sqrt{p_K p_L} \text{ if } q \le K^{\max}\sqrt{p_K/p_L}\text{; and}$$
$$= p_K K^{\max} + p_L q^2/K^{\max} \text{ otherwise.}$$

Notice in the first region, where $q \le K^{\max}\sqrt{p_K/p_L}$, the quantity is not too large, the cost curve is linear; it is a constant multiplied by quantity q. Beyond this point, quantity is so large it leads to capital and labor choices that run afoul of the maximum capital size.

In this way we rationalize the firm's cost curve as an expenditure minimizing choice of factor combinations, an efficient approach to production. From here we determine the firm's marginal cost curve, as in Exhibit 2. Its average cost curve, $C(q)/q$, provided $q > 0$, also falls into place. It is important to remember the underlying concept is efficient choice of factors. This is the essence of the minimization in [3].

References

As our firm is described in terms of selecting the best mix of factors for some specific purpose, we are dealing with economic cost. This is, in fact, an extensively developed line of thought. Your micro economics textbook will contain the basics, though with an emphasis on a single product firm. Chambers [1988] and Fare and Primont [1995] provides extensive treatments of the subject. Demski and Feltham [1976] and Demski [1994] provide connections to accounting. Clark [1923] is an important historical reference.

Key Terms

Economic cost is the minimum expenditure on factors that will allow the firm to produce whatever is to be produced. *Marginal cost* is the rate of change of economic cost with respect to the product in question, the derivative of economic cost with respect to the product in question. *Incremental cost* is the change in (economic) cost as output is changed by some specified amount. An *economy of scope* occurs when the economic cost of producing two or more products in the same firm is less than the economic cost of producing the products in separate firms. The firm's cost structure is not separable if it cannot be expressed as the sum of single product cost functions, if the marginal cost of one product depends on how many of the other products are being produced.

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[14]Recall in the prior note we derived factor choice expressions of $K = q\sqrt{p_L/p_K}$ and $L = q\sqrt{p_K/p_L}$. This provides an overall cost expression of

$$C(q) = p_K K + p_L L = p_K q\sqrt{p_L/p_K} + p_L q\sqrt{p_K/p_L} = 2q\sqrt{p_K p_L}.$$

Of course, this all assumes the chosen K satisfies $K \le K^{\max}$. If not, we set $K = K^{\max}$, and thus have

$$C(q) = p_K K^{\max} + p_L q^2/K^{\max}.$$

Finally, $K \le K^{\max}$ requires $K = q\sqrt{p_L/p_K} \le K^{\max}$ or $q \le K^{\max}\sqrt{p_K/p_L}$.

Exercises

1. In what sense are capital and labor substitutes in the technology assumed in relation [1]? Give examples of factors that are substitutes.

2. When the story is expanded to include three products, relations [4] and [5], why does the cost curve turn out to be non-separable? What is the importance of this lack of separability?

3. Verify the plot in Exhibit 1, using the same prices and K^{max} of course, by solving for the optimal K and L choices for $q \in \{0, 25, 50, 75, 100, ..., 300\}$, and then plotting total expenditure on K and L versus output. Then repeat the exercise, but using $K^{max} = 50$. Explain the difference between your two plots.

4. Exhibit 2 provides a plot of the firm's marginal cost, as output varies. Using the same prices and K^{max} as in the Exhibit, plot the incremental cost of one additional unit, defined as $C(q+1) - C(q)$, for $q \in \{0, 25, 50, 75, 100, ..., 300\}$. Contrast your plot with that in Exhibit 2.

5. Suppose, in our single product firm, capital is priced at $p_K = 200$ while labor is priced at $p_L = 200$ per unit. Let $K^{max} = 125$. Solve for the firm's optimal factor choices and cost for outputs of $q \in \{0, 25, 50, 75, 100, ..., 300\}$. Plot the implied cost curve. Carefully explain its relationship to the cost curve plotted in Exhibit 1.

6. Using the prices in Exercise 5 above, suppose capital is fixed at 75 units. Determine the firm's short run cost curve. Also replicate the short run and long run cost curve display in Exhibit 3. Carefully explain the difference between this display and that in Exhibit 3.

7. Return to the setting in Exhibit 4 where three products are present, along with $p_K = 200$ and $p_{L1} = p_{L2} = p_{L3} = 100$, and $K^{max} = 125$. Suppose the first product sells for $P_1 = 320$ per unit, while the second and third sell for zero. Determine the firm's optimal output, and associated profit. Provide an intuitive explanation.

8. This is a continuation of Exercise 7. Now change the second product's selling price from 0 to 50. Again determine the optimal output, and associated profit. Provide an intuitive explanation. What would the firm's optimal output be if the first and third product sold for zero, while the second sold for 50?

9. This is a continuation of Exercise 8. Now change the respective selling prices to 320, 160 and 200. Determine the firm's optimal output and profit, and provide an explanation for the difference between this output schedule and that in the text (where the selling prices are 120, 160 and 200).

10. Ralph's Firm (RF) uses three inputs to produce output. Let q denote the quantity of output and x_i the quantity of input i, $i = 1, 2$, or 3. Respective factor prices are 100 per unit, 1 per unit, and 4 per unit. Output $q \geq 0$ requires the following inputs:

$$x_1 \geq q; \text{ and}$$
$$\sqrt{x_2 x_3} \geq q[125 - 10q + q^2] \equiv f(q).$$

Determine RF's economic cost curve for $q \in \{0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$. Plot total and average economic cost, using your data points.

11. This is an extension of Ralph's Firm (RF) above. Here, we adopt a specific short run setting and assume factor x_3 is set at the value of $x_3 = 250$. Notice this is the x_3 amount that would have been chosen under $q = 5$ in the original problem. With this factor "fixed" in this manner, determine Ralph's short run total cost and average cost, for the original output series. Comment on the relationship with the original total cost and average cost constructions.

12. We now find Ralph managing a two product firm. The technology mixes capital and labor, to produce two products. Capital is shared, while labor is specific to each of the two products. Capital and labor are also substitutes. The technology is given by relations [1a], [1b] and [2] in the text. $K^{max} = 15$; $p_K = 100$ per unit, $p_{L1} = 50$ per unit; and $p_{L2} = 75$ per unit. (So we are dealing with a two product version of the three product illustration in the text.) Determine Ralph's cost, $C(q_1, q_2)$ for all combinations of $q_1, q_2 \in \{5, 10, 15, 20, 25\}$. Plot your data to provide a graphical depiction of Ralph's cost surface. What patterns emerge?

Finally, suppose Ralph produces $q_1 = 15$ and $q_2 = 20$ units. What total cost would an accounting system report? What would the accounting system claim each unit of the first product cost? What would it claim each unit of the second product cost?

13. Redo Exhibit 6 for the case where all factor payments take place at the end of the third period, while customer payments occur as originally assumed. Carefully explain your findings.

14. Suppose in the three product setting (especially expressions [4] and [5]) the firm faces factor prices of $p_K = 300$ and $p_{L1} = p_{L2} = p_{L3} = 100$, along with $K^{max} = 225$. First, determine the firm's cost curve for the special case of $q_2 = q_3 = 0$. Second, suppose each product sells for 200 per unit. Determine the firm's optimal output and factor choices, assuming $K = K^{max}$. Does the firm earn any rent at this point? Explain.

15. This is a variation on the discounted cash flow story in Exhibit 6. For convenience, only two periods are present (so, in a sense, $q_3 = 0$). Capital is purchased at time $t = 0$, at a price of $p_K = 500$. First period labor, which is paid at $t = 1$, carries a spot price of $p_{L1} =$

100 and second period labor, which is paid at $t = 2$, carries a spot price of $p_{L2} = 110$. As usual, the interest rate is $r = 10\%$. $K^{max} = 1{,}500$. Output in each of the two periods is really the same product, and the firm can produce to inventory in the first period. The firm has only one customer, and must supply 100 units in the first period and 700 units in the second period. Determine and interpret the firm's optimal production schedule and factor choices. Repeat the exercise assuming $p_{L2} = 200$. Explain the differences between your two solutions.

July 14, 2001, Joel