Appendix: A Closer Look At Scaling

In this Appendix we provide a closer examination of the scaling phenomenon. To keep things simple, we streamline the story to where the firm experiences but a single cash inflow, and assume this eventual cash inflow can take one of but two values. Denote it $D \in \{0,1\}$, with 50-50 odds. (So, in usual notation, $\pi(D = 1) = \pi(D = 0) = .50$.) Also let the interest rate be zero. The latter allows us to keep discounting calculations from cluttering the analysis. It also implies, presuming risk neutrality, the initial value of the firm is simply the expected value of D, or (conveniently)

$$E[D] = 1 \cdot \pi(D = 1) + 0 \cdot \pi(D = 0) = \pi(D = 1) = .50.$$

This is also its cost.

a sequence of information releases

We also want to introduce a sequence of informative signals, or information sources. The easiest way to do this is to stretch out the time line, and assume the cash inflow of D occurs at some distant point, say at time $T = 8$. So the cash flows are $CF_0 = -.50$, $CF_8 = D$, and zero otherwise.

Turning to the information story, simplicity is again in order. Let y denote a report. Each report will be "good news" ($y = g$) or "bad news" ($y = b$), according to the following probability structure: $\pi(y = g \mid D = 1) = \pi(y = b \mid D = 0) = \beta$. When multiple sources are present, they all operate in this fashion and are conditionally independent. β, then, is simply the probability any such source is "correct." (Of course, $\beta > .5$.) Examine Exhibit 11, where we portray an informative observation, denoted y_t, each period. Additional details are given in Exhibit 12, where we display the joint probabilities, $\pi(D \text{ and } y)$, for $t = 3$ periods. A full information history here, then, is the time $t = 3$ history of $h_3 = (y_1, y_2, y_3)$. We also use $\beta = .9$. Notice that over the three periods we have eight possible information histories.[21]

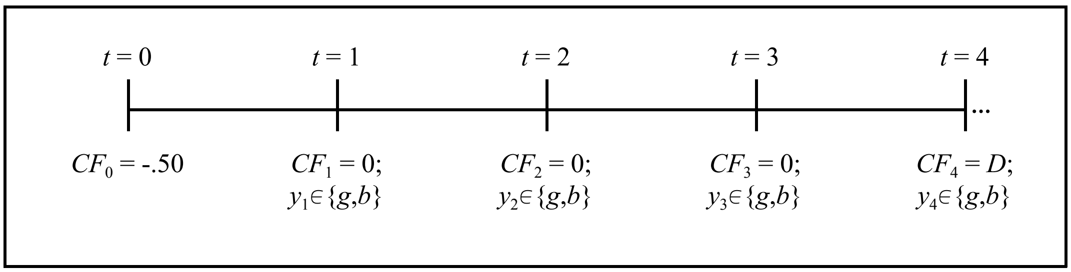

Exhibit 11: Information Sequence for Single Project with Uncertain Cash Inflow

With this in place, what is the expected value of D, the market value of the claim, or asset, conditional on all available information at time t? In mechanical terms, having observed history h_t, we calculate this as

$$P(h_t) = E[D \mid h_t] = 1 \cdot \pi(D{=}1 \mid h_t) + 0 \cdot \pi(D{=}0 \mid h_t) = \pi(D{=}1 \mid h_t).^{22}$$

[21] Some additional features of the stochastic setup should be noted. If $t = 1$ and $y_1 = g$, we have $\pi(D = 1 \mid y_1 = g) = \beta$. And if $t = 2$ and $y_t = (g,b)$ or (b,g), we have $\pi(D = 1 \mid y_t) = .50$. In addition, if you recall your study of statistics you will recognize the informative signal is a binary random variable. To illustrate, suppose $D = 1$; now, what is the probability of precisely $\hat{g}$ observations of signal g over t periods? This probability is given by

$$\binom{t}{\hat{g}} \beta^{\hat{g}}(1-\beta)^{t-\hat{g}},$$

the number of $\hat{g}$ successes in t trials (given $D = 1$).

[22] The conditional probability, in turn, follows from Bayes' Rule: $\pi(D = 1 \mid h_t) = \pi(D = 1 \text{ and } h_t)/\pi(h_t)$.

$h_3 = (y_1,y_2,y_3)$	$D = 1$	$D = 0$	$\pi(h_3)$
(*g*,*g*,*g*)	$.5\beta^3 = .3645$	$.5(1-\beta)^3 = .0005$	.3650
(*g*,*g*,*b*)	$.5\beta^2(1-\beta) = .0405$	$.5\beta(1-\beta)^2 = .0045$	.0450
(*g*,*b*,*g*)	$.5\beta^2(1-\beta) = .0405$	$.5\beta(1-\beta)^2 = .0045$	.0450
(*g*,*b*,*b*)	$.5\beta(1-\beta)^2 = .0045$	$.5\beta^2(1-\beta) = .0405$	.0450
(*b*,*g*,*g*)	$.5\beta^2(1-\beta) = .0405$	$.5\beta(1-\beta)^2 = .0045$	.0450
(*b*,*g*,*b*)	$.5\beta(1-\beta)^2 = .0045$	$.5\beta^2(1-\beta) = .0405$	.0450
(*b*,*b*,*g*)	$.5\beta(1-\beta)^2 = .0045$	$.5\beta^2(1-\beta) = .0405$	.0450
(*b*,*b*,*b*)	$.5(1-\beta)^3 = .0005$	$.5\beta^3 = .3645$	.3650
$\pi(D)$	.5000	.5000	

Exhibit 12: $\pi(D \text{ and } h_t)$ for $t = 3$ and $\beta = .9$

We tally these revised assessments for the above information story in Exhibit 13, where we present distinct calculations for $t = 1$, 2 and 3. For example, using the data in Exhibit 12 we calculate:

$$E[D\,|\,h_2 = (g,g)] = \pi(D = 1\,|\,h_2 = (g,g)) = \frac{.3645 + .0405}{.3645 + .0405 + .0005 + .0045} = \frac{.4050}{.4100}$$

$$= .9878$$

$h_3 = (y_1,y_2,y_3)$	$P(h_t) = E[D\,|\,h_t]$		
	$t = 1$	$t = 2$	$t = 3$
(*g*,*g*,*g*)	.9000	.9878	.9986
(*g*,*g*,*b*)	.9000	.9878	.9000
(*g*,*b*,*g*)	.9000	.5000	.9000
(*g*,*b*,*b*)	.9000	.5000	.1000
(*b*,*g*,*g*)	.1000	.5000	.9000
(*b*,*g*,*b*)	.1000	.5000	.1000
(*b*,*b*,*g*)	.1000	.0122	.1000
(*b*,*b*,*b*)	.1000	.0122	.0014

Exhibit 13: $E[D\,|\,h_t] = \pi(D = 1\,|\,h_t)$ for $t = 1$, 2 and 3

Now consider the change in price, in market value, as we move through time. At time $t = 1$, prior to observing the information, the change in market value is a random variable: $\Delta_1 = E[D|y_1] - .50$. At time $t = 2$ it is the random variable $\Delta_2 = E[D|(h_1,y_2)] - E[D|h_1]$, and so on. Details are provided in Exhibit 14.

$h_3 = (y_1,y_2,y_3)$	Δ_1	Δ_2	Δ_3	$\pi(h_3)$
(g,g,g)	.4000	.0878	.0108	.3650
(g,g,b)	.4000	.0878	-.0878	.0450
(g,b,g)	.4000	-.4000	.4000	.0450
(g,b,b)	.4000	-.4000	-.4000	.0450
(b,g,g)	-.4000	.4000	.4000	.0450
(b,g,b)	-.4000	.4000	-.4000	.0450
(b,b,g)	-.4000	-.0878	.0878	.0450
(b,b,b)	-.4000	-.0878	-.0108	.3650

Exhibit 14: Change in Market Value Calculations (Δ_t) for $t = 1$, 2 and 3

Notice the expected value of the change in market value is zero: $E[\Delta_1] = E[\Delta_2] = E[\Delta_3] = 0$.[23] Moreover, at any point in time we face a fair game with respect to the next information release. That is, the expected value of the time t expected value of D, given we know history realization h_{t-1}, is $E[E[D|h_{t-1}, y_t]|h_{t-1}] = E[D|h_{t-1}]$ for any history of signals h_{t-1}. Volatility, though, will systematically decline because the assumed probability structure implieseach additional piece of information is, on average, less informative. This is reflected in Exhibit 15, where we provide the mean absolute value and variance of Δ_t, for $t = 1$, 2 and 3. Be aware that this particular pattern of declining information content is driven by the binary random variable (and *iid*) characterization of the information sources. Nevertheless, it leads to important insight into the impact of multiple sources of information.

time t	$E\|\Delta_t\|$	$VAR(\Delta_t)$
1	.4000	.1600
2	.1440	.0351
3	.0878	.0296

Exhibit 15: Summary Effects of Information Release

an accounting source

[23]Recall we have neutralized risk, by assuming risk neutrality. So changes in risk are moot. We also have neutralized time, by assuming a zero interest rate. So change in market value, in price, as we step through time is due solely to the arrival of new information and its impact on the expected value of D.

Now suppose one, but only one, of these information sources is the accounting system. In particular, across the first four information releases assume one, but only one, comes from the accounting system. Let τ denote the index in the sequence where the accounting source is operative. If, then, $\tau = 3$, the information at times $t = 1$, 2 and 4 comes from non-accounting sources, while the information at time $t = 3$ comes from the accounting source. This provides a simple, intuitive way of mixing accounting and non-accounting sources of information. We have a sequence of informative observations, and one in the sequence is the accounting source.

Accounting, though, does not report some abstract signal. It conveys its information through the application of accounting procedures to underlying events. We examine two such procedures. Both, naturally, begin by recording historical cost (at time $t = 0$ of .50), and both report the realization of D at time $T = 8$. The intermediate point of time τ is where they differ. One method is a fair value method, in which accounting and economic value are equated at the time of the accounting release. So for any history of non-accounting information up to the time of the accounting report, $h_{\tau\text{-}1}$, and period τ signal realization, y_τ, the accounting system will record a time τ accounting value of $A_\tau = E[D|(h_{\tau\text{-}1},y_\tau)]$ and associated accounting income of $\hat{I}_\tau = E[D|(h_{\tau\text{-}1},y_\tau)]$ - .50. The asset, so to speak, is marked to market; and any change in accounting value is reflected in the accounting income measure.

The second method is a myopic, or highly restricted, version of fair value. For any history of non-accounting information, $h_{\tau\text{-}1}$, it simply records a time τ accounting value of $A_\tau = E[D|y_\tau]$ and associated accounting income of $I_\tau = E[D\,|\,y_\tau]$ - .50. This procedure ignores, or does not recognize, the earlier non-accounting information, summarized in history $h_{\tau\text{-}1}$. It uses the same valuation formula as in the market value calculation, but with a restrictive recognition rule.

Nevertheless, both methods reveal the time τ realization of y_τ. The fair value method, for example, reports an income of $E[D\,|\,(h_{\tau\text{-}1},y_\tau)]$ - .50. Prior to the report, other sources have reported, in cumulative terms, the history of $h_{\tau\text{-}1}$, and the market value of the asset just prior to the accounting report is known to be $E[D\,|\,h_{\tau\text{-}1}]$. Decoding the accounting report, then, we see that reported accounting income is surprisingly large (given $h_{\tau\text{-}1}$) if and only if the reported accounting value is surprisingly large; and this value is surprisingly large if and only if $y_\tau = g$. So, a "positive surprise" conveys the fact $y_t = g$, just as a "negative surprise" conveys the fact $y_t = b$.

Similarly, at time τ the myopic method reports an income of $E[D|y_\tau]$ - .50. This is positive if and only if $y_\tau = g$. The two methods use different measurement scales, but are informationally equivalent, in the presence of the non-accounting information. Understanding this is critical to what follows.[24]

Now suppose the accounting report takes place at time $\tau = 3$. What can we say about the relationship between the accounting and economic measures? We concentrate on the accounting income measure. Recall, for the fair value system, reporting at time $\tau = 3$ results in an income measure of $\hat{I}_3 = E[D|(h_2,y_3)]$ - .50. In turn, the expected value of the income measure just before its release depends on the information releases up to that point, so we write it as $E[\hat{I}_3\,|\,h_2]$. We

[24]In Exhibits 16 and 17 we identify the accounting income measures for a time $\tau = 3$ report, for each possible realization of the sequence of information variables. For the myopic system, Exhibit 17, notice positive income is equivalent to $y_3 = g$. Similarly, for the fair value system, Exhibit 16, notice that, for each possible history of signals, the accounting report differs between $y_3 = g$ and $y_3 = b$.

tabulate the possibilities in Exhibit 16. (The last column, Δ_3 is taken from Exhibit 14.)

$h_3 = (y_1,y_2y_3)$	**Accounting Income:** $\hat{I}_3 = E[D \mid h_3] - .5$	**Expected Income:** $E[\hat{I}_3 \mid h_2]$	**Difference (Surprise)**	**Price Change at** $t = 3$: Δ_3
(*g*,*g*,*g*)	.4986	.4878	.0108	.0108
(*g*,*g*,*b*)	.4000	.4878	-.0878	-.0878
(*g*,*b*,*g*)	.4000	.0000	.4000	.4000
(*g*,*b*,*b*)	-.4000	.0000	-.4000	-.4000
(*b*,*g*,*g*)	.4000	.0000	.4000	.4000
(*b*,*g*,*b*)	-.4000	.0000	-.4000	-.4000
(*b*,*b*,*g*)	-.4000	-.4878	.0878	.0878
(*b*,*b*,*b*)	-.4986	-.4878	-.0108	-.0108

Exhibit 16: Accounting Income vs. Market Value Change for Fair Value System

Study the last two columns: the change in market value at the time of the accounting report coincides with the surprise in the accounting income measure. This is intuitive. With the fair value system, the accounting stock measure is identical to its market counterpart at the time of the report (time τ). The market measure, though, receives some information at an earlier time. We control for this by focusing on the surprise in accounting income, the income less what we expected that income to be, based on all available information just prior to the accounting report.

Contrast this with the myopic system. Since the two systems are informationally equivalent, given the non-accounting information, the change in market value at the time of the accounting report will be the same. The accounting income number, though, is no longer so closely aligned to the change in market value. Details are presented in Exhibit 17.[25]

[25]To verify these calculations, notice the income report is .4 if $y_3 = g$; so the key is to identify $\pi(y_3 = g \mid h_2)$. Go back to Exhibit 12. Suppose $h_2 = (g,g)$. We then have $\pi(y_3 = g \mid h_2 = (g,g)) = .365/.410$. Thus, the expected value of the forthcoming income report, having observed $h_2 = (g,g)$ is .4(.365/.410) - .4(.045/.410) = .3122.

$h_3 = (y_1, y_2, y_3)$	**Accounting Income:** $\hat{I}_3 = E[D \mid y_3]$ - .5	**Expected Income:** $E[\hat{I}_3 \mid h_2]$	**Difference (Surprise)**	**Price Change at** $t = 3$: Δ_3
(g,g,g)	.4000	.3122	.0878	.0108
(g,g,b)	- .4000	.3122	- .7122	- .0878
(g,b,g)	.4000	.0000	.4000	.4000
(g,b,b)	- .4000	.0000	- .4000	- .4000
(b,g,g)	.4000	.0000	.4000	.4000
(b,g,b)	- .4000	.0000	- .4000	- .4000
(b,b,g)	.4000	- .3122	.7122	.0878
(b,b,b)	- .4000	- .3122	- .0878	- .0108

Exhibit 17: Accounting Income vs. Market Value Change for Myopic System

We clearly have a positive relationship between the accounting and market measures. Accounting income is positive only when the change in market value is positive; and controlling for earlier information preserves the connection between positive surprise and positive market value change. But the connection is hardly one-to-one.[26] Intuitively, this must be the case. The change in market value reflects the totality of all available information, while the accounting calculation is restricted in its access to this information. The two systems bring the same new information forward, but wrapped, so to speak, in different measurement scales.

more scaling

Another view of this scaling phenomenon surfaces when we mimic the Ball and Brown experiment (Exhibit 8): assume at time $t = 0$ we privately learn what the accounting report at time $\tau = 3$ will be. This report can be either "good news" or "bad news." For the myopic method, this boils down to knowing in advance whether $y_\tau = g$ or b. Suppose we do this for a large, large number of assets of this type.

[26]Even this degree of connection is critically dependent on our information setup. Observing a report of g is always good news here; information complementarities are well under control.

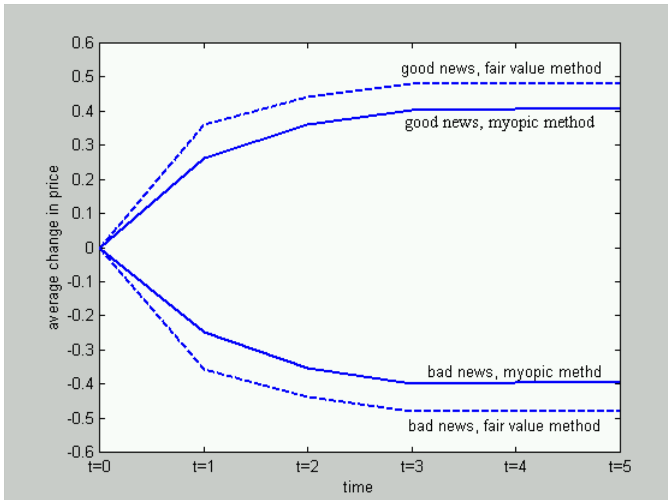

Exhibit 18: Market Value Calculations Presuming Advance Knowledge of Time $\tau = 3$ Report

In Exhibit 18 we plot, again using $\beta = .9$, the average change in market value of the asset (Δ_t recall), conditional on knowing in advance whether the time τ report was good or bad news. The upper solid line in the graph is the case where we know $y_3 = g$. Notice it begins at zero, and systematically increases to .4. Beyond that point we have no information, so the expected value of the price is simply $E[D \mid y_3 = g] = .9$, implying a change in market value of .9 - .5 = .4. The lower solid line is the companion case of advance knowledge that $y_3 = \text{b}$.[27]

Indeed, we have chosen the parameters in the simulation so the calculations lead to an appearance qualitatively similar to that in the Ball and Brown experiment. (We have the luxury, however, of being able to precisely identify the market's expectation of the forthcoming accounting release.) Further notice in Exhibit 18 that the average change in market value at the time of the accounting release (the value at $t = 3$ versus at $t = 2$) is positive for the upper graph and negative for the lower graph, though hardly one to one. Accounting value and market value tend to move together, but less than perfectly, just as we saw in Exhibit 17.

Now consider the fair value procedure. It reports income in period τ of $\hat{I}_3 = E[D \mid (h_{\tau-1}, y_\tau)]$ - .5. The upper dashed line in Exhibit 18 reports the corresponding expected value of the change

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[27]To remove some of the mystery here, consider the upper solid line in Exhibit 18, the case where we know $y_3 = g$. Using the calculations in Exhibits 12 and 13, the expected value of the asset's market value at time $t = 1$, when we know $y_3 = g$, is simply

$$E[P(h_1) \mid y_3 = g] = \sum_{h_1} E[D \mid h_1]\ \pi(h_1 \mid y_3 = g)$$
$$= [.9(.365) + .9(.045) + .1(.045) + .1(.045)]/[.365 + .045 + .045 + .045] = .756.$$

So the expected value of the change in value, as of time $t = 1$, is simply .756 - .5 = .256.

in market price, conditional on knowing $E[D|(h_{\tau-1},y_\tau)]$ - .5 ≥ 0. It is strictly above its counterpart for the myopic accounting procedure. The reason is this procedure employs strictly more historical information in conveying its report at time $\tau = 3$. So advance knowledge of that report implies we are conditioning on strictly more information. The good news filter, so to speak, is a stronger filter here. Of course, this means the companion bad news graph, the lower dashed line where the expectation is conditional on knowing that $E[D|(h_{\tau-1},y_\tau)]$ - .5 < 0, is strictly below the myopic graph. Moreover, we now tighten the connection between the accounting and market measures.

What happens, now, when we move the release point earlier or later? This is sketched in Exhibits 19 and 20. The pattern should be taking on a familiar appearance. In particular, the slope of each graph in the region between times τ - 1 and τ is flatter the larger is τ. With a later report time, a larger τ, more information has been released in front of the accounting report. And in our particular setup each additional piece of information, each additional observation, is less informative. Diminishing returns to information, so to speak, are built into the simulation. A large τ, then, corresponds here to an accounting system whose information is largely overshadowed by the reports of earlier reporting sources.[28]

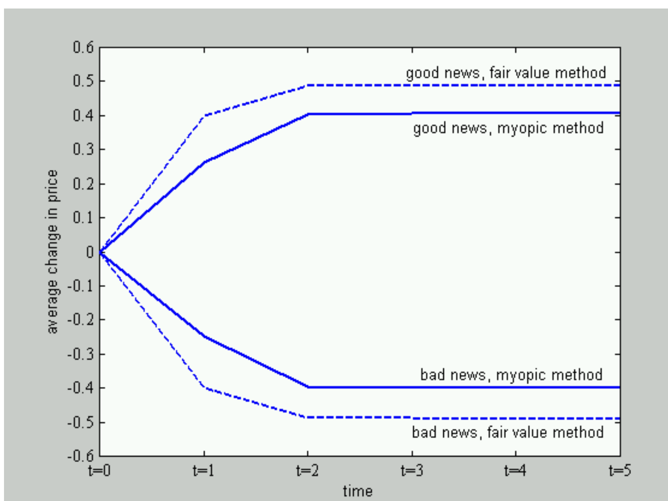

Exhibit 19: Market Value Calculations Presuming Advance Knowledge of Time $\tau = 2$ Report

[28]What would these Exhibits look like if we instead had the accounting report at time τ merely report the information provided by the τ - 1 earlier reports? This corresponds to the case where we use the delayed accounting report to check on the veracity of the earlier reports. Assuming, then, these earlier reports are, in equilibrium, correctly reporting, the accounting report would appear to be completely redundant.

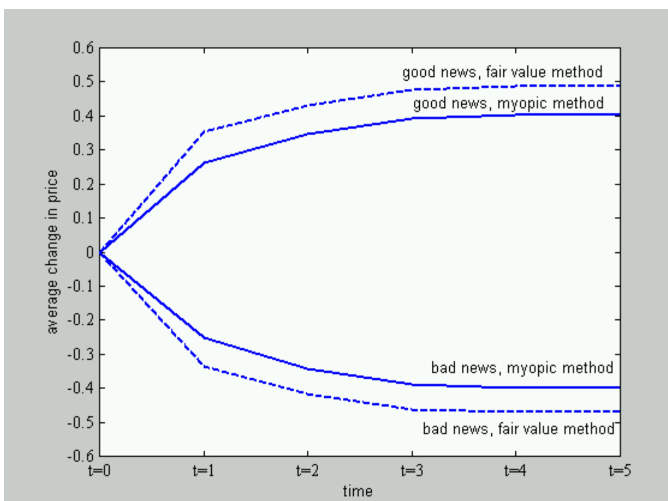

Exhibit 20: Market Value Calculations Presuming Advance Knowledge of Time $\tau = 4$ Report