Bayesian Ralph

Ralph's production planning is facilitated by assessing general economic conditions. He judges it is equally likely the economy is favorable or unfavorable. Based on experience, Ralph believes demand for his products (gauged by revenues) is related to economic conditions.

Part A

In particular, revenues are normally distributed with mean 150 and standard deviation 10 when economic conditions are favorable and revenues are normally distributed with mean 100 and standard deviation 10 when economic conditions are unfavorable. That is, conditional density functions are

$$f\left(y \mid fav\right) = \frac{1}{\sqrt{2\pi}10} \exp\left[-\frac{1}{2}\frac{\left(y-150\right)^2}{100}\right]$$

and

$$f\left(y \mid unfav\right) = \frac{1}{\sqrt{2\pi}10} \exp\left[-\frac{1}{2}\frac{\left(y-100\right)^2}{100}\right]$$

Suggested:

Determine the posterior or updated probability the economy is in favorable condition given Ralph observes revenues equal to 127.

Note: Bayesian revision (as always) involves the ratio of the joint distribution of the signal and state to the marginal distribution of the signal. That is,

$$\Pr\left(fav \mid y=127\right) = \frac{f\left(y=127 \mid fav\right)\Pr\left(fav\right)}{f\left(y=127 \mid fav\right)\Pr\left(fav\right) + f\left(y=127 \mid unfav\right)\Pr\left(unfav\right)}$$

where the joint distribution of revenues and states is given by the following joint density function

$$f\left(y,s\right) = \begin{array}{ll} \frac{1}{2}\frac{1}{\sqrt{2\pi}10}\exp\left[-\frac{1}{2}\frac{(y-150)^2}{100}\right], & s=fav \\ \frac{1}{2}\frac{1}{\sqrt{2\pi}10}\exp\left[-\frac{1}{2}\frac{(y-100)^2}{100}\right], & s=unfav \end{array}$$

Part B

Ralph reconsiders the above analysis on recognizing that his two primary product lines have a different relation with the state of the economy — one is procyclical and the other is countercyclical. In particular, revenues from the two lines are jointly normally distributed with means 120 and 30 when economic conditions are favorable and 65 and 35 given unfavorable economic conditions. The complementary nature of the information is reflected in the variance-covariance relation conditional on the state s.

$$\begin{aligned} V &= \begin{bmatrix} Var[y_1 \mid s] & Cov[y_1, y_2 \mid s] \\ Cov[y_1, y_2 \mid s] & Var[y_2 \mid s] \end{bmatrix} \\ &= \begin{bmatrix} 121 & -21 \\ -21 & 25 \end{bmatrix} \end{aligned}$$

Hence, the joint distribution of revenues conditional on the economy is given by the density function

$$f(y_1, y_2 \mid s) = \frac{1}{2\pi |V|^{\frac{1}{2}}} \exp\left[-\frac{1}{2}(y - \mu_s)^T V^{-1} (y - \mu_s)\right]$$

where $|V|$ is the matrix determinant of V and $(y - \mu_s) = \begin{bmatrix} y_1 - \mu_{s1} \\ y_2 - \mu_{s2} \end{bmatrix}$ are the mean deviations of revenues (the information signals). (note: you might find the operators mdeterm(), minverse(), and mmult() in excel instructive.)

Suggested:

Determine the posterior or updated probability the economy is in favorable condition given Ralph observes revenues for the first product line equal to 94 and revenues for the second product line equal to 33.

Note: Bayesian revision again involves the ratio of the joint distribution of the signals and state to the marginal distribution of the signal. That is,

$$\Pr(fav \mid y_1 = 94, y_2 = 33) =$$

$$\frac{f(y_1 = 94, y_2 = 33 \mid fav)\Pr(fav)}{f(y_1 = 94, y_2 = 33 \mid fav)\Pr(fav) + f(y_1 = 94, y_2 = 33 \mid unfav)\Pr(unfav)}$$

where the joint distribution of revenues and states is represented by the density function

$$f(y, s) = \begin{array}{ll} \frac{1}{2}\frac{1}{2\pi|V|^{\frac{1}{2}}} \exp\left[-\frac{1}{2}\left(y - \mu_{fav}\right)^T V^{-1}\left(y - \mu_{fav}\right)\right], & s = fav \\ \frac{1}{2}\frac{1}{2\pi|V|^{\frac{1}{2}}} \exp\left[-\frac{1}{2}\left(y - \mu_{unfav}\right)^T V^{-1}\left(y - \mu_{unfav}\right)\right], & s = unfav \end{array}$$

Part C

Suppose Ralph assigns revenues (y) conditional on the state of the economy a uniform distribution where conditional density functions are

$$f(y \mid fav) = \tfrac{1}{60}, \quad 120 < y < 180$$

and

$$f(y \mid unfav) = \tfrac{1}{60}, \quad 70 < y < 130$$

Suggested:

If revenues are observed to be $y = 127$, what is the probability the state of the economy is favorable?

Suppose $f(y \mid unfav) = \tfrac{1}{30}, \quad 100 < y < 130$ and revenues are again observed to be $y = 127$, what is the probability the state of the economy is favorable?

Suppose $f(y \mid unfav) = \tfrac{1}{25}, \quad 100 < y < 125$ and revenues are observed to be $y = 127$, what is the probability the state of the economy is favorable?

Why is uniformly distributed information so stark?