Income and Efficiency in Incomplete Markets

by

Anil Arya[†]

John Fellingham[†]

Jonathan Glover[††]

Doug Schroeder[†]

Richard Young[†]

April 1996

[†] Ohio State University

[††] Carnegie Mellon University

Income and Efficiency in Incomplete Markets

1. Introduction

Aggregation is central to accounting. Vast quantities of data are reduced to a handful of numbers. Attempts are made to capture the entire financial activity of an entity during a period in one number - income. A legitimate question to ask is how much useful information can be conveyed by income. The answer must depend on the context. What questions are we using the income number to resolve?

In this paper we examine a simple but suggestive setting in which the income number arises naturally (and directly) from competitive markets. While no explicit assumptions are made about individual firm's objectives, it turns out that equilibrium is consistent with the maximization of a number which is equal to the difference between the value of commodities sold and the value of resources purchased.[1] This number, which we call income, is instructive about the central economic questions of equilibrium and productive efficiency in the following way.

1. If any firm's income is not maximized the economy is not in equilibrium.[2]
2. Given equilibrium prices, if each firm's income is maximized production is efficient.

Forging links between an income number, equilibrium and efficiency is important in establishing that income measurement is an interesting and useful activity. For our

---

[1] This differs from the approach in Beaver and Demski (1995), who assume firms maximize net present value of cash flows.

[2] Our model is essentially timeless -- there is no interest rate. Also, we do not model uncertainty, so not only is expected income zero, but so is realized income.

purposes, we will regard income calculation as the accountant's primary role in the economy.

When there are unrestricted trading opportunities (complete markets), we show that the income number is relatively easy to calculate for all producing entities. Resources are valued at their market prices. A more interesting case is when trading opportunities are restricted (incomplete markets), so that some resources do not have a market price.

It turns out that in the incomplete markets setting there exists a method of income measurement that retains income's illuminating properties about equilibrium and efficiency. The key feature of income calculation is that firm specific "personal prices" are used for non-tradable resources.[3] The personal prices have the following characteristics.

1. They may differ across firms for physically identical resources.
2. They are determined by economy wide forces; they can not be identified by restricting attention within the firm.

The need to resort to personal prices in constructing an income number linked to equilibrium and productive efficiency implies that the accountant's job is difficult as well as important. Accounting judgment is brought to bear in income determination, as

[3] The use of personal prices in this paper follows the development in Beaver and Demski (1995).

personal prices are not the result of trades.[4] In other words, mark-to-market accounting is not a viable option.

We make several simplifications, two of which we should mention at the outset.

1. Linear technology
2. Inelastic supply of resources.

These simplifications allow a linear programming formulation and characterization of general equilibrium. For the complete market setting these are convenient, but by no means necessary, assumptions.[5] Where the two simplifying assumptions come in very handy, and we rely on them heavily, is in the incomplete market analysis. Here the linear programming structure allows us to: (1) formally characterize what we mean by an incomplete market, (2) derive the equilibrium solution in exactly the same way as in the complete market situation and (3) develop a link between equilibrium, efficiency and income maximization.

Linear programming has been used to address fundamental economic questions, such as characteristics of a competitive equilibrium when trading is unrestricted.[6] In this regard the Duality theorem is remarkably powerful and insightful. It seems natural to extend the analysis to incomplete markets, wherein accounting issues are salient.

The paper proceeds as follows. Section two introduces the general model used to characterize an equilibrium. The notions of income and efficiency are also discussed.

---

[4] Asset valuation where markets do not function perfectly is the subject of long-standing debate in accounting (e.g., Chambers, 1966; Edwards and Bell 1961; Paton 1973; Sterling 1970).

[5] For example, see Debreu (1959) for a more general development.

[6] The complete market analysis in this paper closely follows the development in Dorfman, Samuelson and Solow (1987).

Section three describes income measurement in the complete markets case. Section four takes care of the incomplete markets case. Section five concludes the paper.

2. Model and Definitions

We restrict attention to linear production technologies captured in a transformation matrix A with m rows and n columns. m is the number of factor resources and n the number of commodity outputs in an economy. For concreteness consider the following A, which we will use as a recurring example.

$$A = \begin{bmatrix} 1 & 1 \\ 0.5 & 2 \\ 4 & 0.25 \end{bmatrix}$$

There are two commodities, x_1 and x_2, and three resources, r_1, r_2 and r_3. An entry in the matrix specifies how much of the input resource is required to produce the output commodity. For example, to produce one unit of x_1 requires one unit of r_1, one-half unit of r_2, and four units of r_3. In the example we fix the amount of resources available.[7] In vector notation

$$r = \begin{bmatrix} 4 \\ 6 \\ 10 \end{bmatrix},$$

meaning we have four units of resource one, and so forth.

To characterize an equilibrium we need to specify a vector of commodity prices, p, a vector of resource prices, v, and a production schedule specified in a vector of produced commodities, x.

We now define equilibrium.

Definition: {p, v, x} is a *competitive equilibrium* if:

1. $A.x \leq r$

   These conditions simply state that the resources used by the commodities can not exceed the resources available.

2. $A^T.v \geq p$

   These conditions state that the cost of producing a unit of commodity should be no less than the selling price of the commodity. This is just what we would ask of competitive market prices. If the price of the commodity strictly exceeds the cost of producing it, equilibrium forces would act to bid up the prices of the resources.

3. Prices in equilibrium are consistent with the demand functions. For exposition purposes we will consider linear demand functions.

   $D.x = p$, where D is a square matrix of dimension n.[8]

   For our example $D = \begin{bmatrix} -1 & 1 \\ 10 & -1 \end{bmatrix}$.

4. Define M_i as the i-th row of a matrix M (so M_i^T is the i-th row of M^T and the i-th column of M). Then:

   (i) If $A_i x < r_i$, then $v_i = 0$.

   (ii) If $A_i^T.v > p_i$, then $x_i = 0$.

   Condition (i) states that if resource r_i is not entirely used up, resource price v_i is equal to zero. That is, we do not require that all resources are used, but if they are not all used, they are free. This is like sands in the Sahara. Condition (ii) states that if the cost of commodity x_i is greater than its price, none of the commodity is produced in equilibrium. That is, we do not require that the costs to make a commodity are equal

---

[7] We can expand the analysis to elastic supply of resources, but no further insights are extracted, and the inelastic supply case is more streamlined.

[8] Linear demand functions are not necessary for establishing relations among income, equilibrium prices, and efficiency. However, they are useful in calculating equilibria, and in our setting yield unique equilibrium prices.

to their benefit, but if they are not equal, it must be that the commodity is not produced in equilibrium. This is like silk diapers.

5. Equilibrium prices and quantities are non-negative, with at least one price or quantity positive. The purpose of the strictly positive condition is to eliminate the trivial and uninteresting case where nothing is produced.

Notice that the definition of equilibrium says nothing about firms and their objective functions. In particular, no assumption is made that firms maximize income (or how income is measured).

<u>The Concept of Income</u>

At this point we appeal to the beautiful theory of linear programming in order to demonstrate that the concept of income arises directly from our assumptions about equilibrium conditions in the economy.

We first restate the equilibrium conditions.

$A.x \le r$	$A^T.v \ge p$
$x \ge 0$	$v \ge 0$
where for resource i	where for commodity i
if $A_i x < r_i$, $v_i = 0$.	if $A_i^T.v > p_i$, $x_i = 0$

The fact that the entries in A and the resource levels r are non-negative implies that both sets of inequalities are feasible: we can always choose small enough x_i's so that $A.x \le r$ and we can always choose large enough v_i's so that $A^T.v \ge p$. This set of conditions implies the following property of an equilibrium.

Observation 1: The quantity $p^T.x - v^T.r$ is maximized by every competitive equilibrium.

Proof: Since $v \geq 0$, premultiplying condition (1) by v^T provides: $v^T.A.x \leq v^T.r$. Since $x \geq 0$, premultiplying condition (2) by x^T provides: $x^T.A^T.v \geq x^T.p$. Transposing both sides yields: $v^T.A.x \geq p^T.x$. Thus, $v^T.r \geq v^T.A.x \geq p^T.x$, so $p^T.x - v^T.r \leq 0$. In addition, we know that $v_i A_i.x = v_i r_i$ and $A_i^T.v.x_i = p_i x_i$ for the following reasons. First, if $A_i.x = r_i$ and $A^T.v = p_i$ the result follows immediately for any v_i and x_i. Second, if $A_i.x < r_i$ equilibrium implies $v_i = 0$ and if $A_i^T.v > p_i$, equilibrium implies $x_i = 0$, and again the result follows immediately. Since this is true for all x_i and v_i, we have $v^T.A.x = v^T.r$ and $v^T.A.x = p^T.x$, we now have $p^T.x - v^T.r = 0$. And since we have previously proved $p^T.x - v^T.r \leq 0$ in equilibrium, $p^T.x - v^T.r$ has attained its maximum when it is equal to zero. ■

The above observation implies equilibrium behavior is consistent with maximization of a number, justifying the following definition of income. (The result is produced without assuming anything about individual firm behavior.)

Definition: *Income* is the difference between the revenues generated by the commodities produced in the economy, $p^T.x$, and the cost of resources used by the economy in production, $v^T.r$.

The Concept of Efficiency

We now have developed the concept of income and defined it within our model. The next step is to identify income's relation to productive efficiency, which we now define.

Definition: A productive allocation is *efficient* if there does not exist any other physically possible allocation which has more of at least one commodity and no less of any commodity. Notationally, let $x \geq 0$ satisfy $A.x \leq r$. The commodity vector x is efficient if there does not exist $x' \geq 0$ such that $A.x' \leq r$ and $x_i' \geq x_i$ for all i, and $x_j' > x_j$ for some j.

The following observation states the aggregation feature of the income calculation.

Observation 2: In equilibrium, income is maximized and economy-wide production is efficient.

Proof. We have already shown in the proof of Observation 1 that income (as defined above) is maximized in equilibrium. Consider the following two linear programs.

Program 1:		*Program 2:*	
$\underset{x}{\text{maximize}}$	$p^T.x$	$\underset{v}{\text{minimize}}$	$v^T.r$
subject to:	$A.x \leq r$	subject to:	$A^T.v \geq p$
	$x \geq 0$		$v \geq 0$

The two programs have the property that one is the dual of the other. We know an equilibrium p, v, and x exists and satisfies the constraints of both programs, since it satisfies (1), (2), and (5). Furthermore, as indicated in the proof of Observation 1, equilibrium implies zero income, i.e., the objective functions are equal. Using the Duality theorem of linear programming, it follows that in equilibrium x and v optimize both programs.[9] Since x is optimal in Program 1 and commodity prices are non-negative, it must be efficient. After all, if there existed another commodity vector, x', that is feasible and which dominates x, then x would not have been a solution in the first place. ■

As Dorfman et al. (1987, p. 370) eloquently state, "hidden in every competitive general equilibrium system is a maximum problem for value of output and a minimum problem for factor returns." These are the primal and dual programs presented in the proof of Proposition 1.[10]

We use the numerical example introduced previously to illustrate Observations 1 and 2.

Complete markets equilibrium

$x_1 = 0.9836$	$v_1 = 0$	$p_1 = 1.770$
$x_2 = 2.754$	$v_2 = 3.541$	$p_2 = 7.082$
	$v_3 = 0$	

The objective function of each of the programs is $79{,}056 / 61^2 \approx 21.25$.

One can verify that the prices and quantities satisfy the equilibrium conditions (1) through (5). In general, one can solve for an equilibrium by trying a set of prices and solving for x and v using Programs 1 and 2, and then checking to see if they are consistent with the demand functions. The procedure can be carried out systematically by choosing a set of binding constraints in Program 1 and checking whether all the other conditions are met. In this example, if one chose the first two constraints, one would obtain 4/3 and 8/3 for the commodity quantities (both positive). The demand equations

[9] The Duality Theorem of Linear Programming states, among other things, that if x and v are feasible in the two programs, and if the objective functions are equal, then x and v are optimal in their respective programs. See Luenberger (1989, page 89).

[10] The shadow price on the primal constraints are the resource prices, and the shadow price on the dual constraint is the commodity amount. If any of the constraints are non-binding, its corresponding shadow price is zero, just as one would desire in an equilibrium (property (4)). In fact, at the optimal solution the following is true:

would then dictate prices of 4/3 and 32/3 (both positive). But equilibrium condition (4) would imply the price of the first resource is negative. Further investigation would reveal that in this example no equilibrium is found when *any* two of the constraints in the first program bind. One would then proceed to test cases where a single constraint is binding. It turns out that the equilibrium is such that the second inequality alone is binding. The equilibrium thus satisfies equations (a) through (e).

$$0.5\ x_1 + 2\ x_2 = 6 \qquad \text{(a)}$$
$$-\ x_1 + \ x_2 = p_1 \qquad \text{(b)}$$
$$10\ x_1 - \ x_2 = p_2 \qquad \text{(c)}$$
$$0.5\ v_2 = p_1 \qquad \text{(d)}$$
$$2\ v_2 = p_2 \qquad \text{(e)}$$

The constraints (d) and (e) simply tell us the ratio of prices is 4, so we can solve for the commodities by using (a) and $(-\ x_1 + x_2)/(10x_1 - x_2) = 4$. The solution is $x_1 = 60/61 \approx 0.9836$ and $x_2 = 168/61 \approx 2.754$. Next substitute into (b) and (c) to obtain the commodity prices: $p_1 = 108/61 \approx 1.770$ and $p_2 = 432/61 \approx 7.082$.[11] (One should verify that all the inequalities are also satisfied.)

$(A_i x - r_i)v_i = 0$ and $(A_i^T .v - p_i)x_i = 0$. These are referred to as complementary slackness conditions.

11 One may avoid the trial-and-error approach by solving the following program (for a non-trivial solution).

$$\begin{array}{lll} \max\limits_{p,\,x,\,v} & p^T.x - v^T.r & \\ \text{s.t.} & A.x & \le r \\ & A^T.v & \ge p \\ & D.x & = p \\ & x & \ge 0 \\ & v & \ge 0 \\ & p & \ge 0 \end{array}$$

3. Complete Markets Accounting

The previous section laid sufficient groundwork to allow us to talk about accounting issues. Specifically, the income number can be viewed as economy-wide revenues (the objective function in the Program 1) minus economy-wide costs (the objective function in Program 2). If all production in the economy is accomplished by one firm, the income statement is as follows.

Income Statement: Complete markets - Economy

Revenues:		
x_1	(0.9836)(1.77) =	\$ 1.74
x_2	(2.754)(7.082) =	19.50
Expenses:		
r_1		0
r_2	(.5)(.9836)(3.541)+ (2)(2.754)(3.541) =	21.24
r_3		0
Income		\$ 0

Income calculation aggregates important economy-wide information about equilibrium and efficiency into a scalar, the income number. In this paper we will consider this aggregation activity as the accountant's job. We will discuss how difficult the job is under two settings: (1) when there are two firms and trading is unrestricted, and (2) when there are two firms but trading is restricted.

<u>Two firms</u>

It is useful to explore multiple firms (traders) so that we can compare complete and incomplete markets.[12] It turns out that the zero-income-property is not lost if production is conducted by more than one firm. To continue with the numerical example, suppose the two commodities are produced by two different firms. Firm A is endowed with the technology to produce commodity 1; product 1 technology consists of the first column of the A matrix. Similarly Firm B produces commodity 2. Furthermore, endow the two firms with resources as follows.

Endowments

Endowments	**Firm A**	**Firm B**
resource 1	2	2
resource 2	3	3
resource 3	3	7

Notice that the total of each of the resources is the same as before. And, most importantly, the equilibrium prices remain as in the previous section: 0.9836 units of commodity 1 are produced (by Firm A) and 2.754 units of commodity 2 are produced (by Firm B). The resource requirements for this production schedule are as follows.

[12] Of course, in this setting there are no transactions costs, so it is hard to rationalize why firms would exist (and even what they are). What we *do* need here is a set of trading entities.

Resource requirements -- Complete markets

Resource requirements	Firm A	Firm B	Total
resource 1	0.984	2.754	3.738
resource 2	0.492	5.508	6.000
resource 3	3.934	0.689	4.623

Notice that Firm B requires more of resources 1 and 2 than are available from its endowment. And Firm A has excess amounts of resource 1 and 2. The production plan is still achievable because Firm A is permitted to trade some of its unused resources to B. Also, Firm A requires more of resource 3 than is available from its endowment; its production is accomplished by trading with Firm B. Trades of the resources will occur at the following equilibrium prices: $v_1 = 0$, $v_2 = 3.54$, $v_3 = 0$. Resource 2 is the only resource with a positive price, as it is the only one whose supply is limited by the equilibrium production schedule. The entire 6 units available is used in production.

As is well-known, income measurement and valuation are inextricably linked (Beaver and Demski, 1995). Hence, calculation of individual firm income follows directly once the resource prices are determined.

Income statements: Complete markets - Individual firms

	Firm A		Firm B	
Revenues:	(.9836)(1.77) =	\$ 1.74	(2.754)(7.08) =	\$ 19.50
Expenses:				
resource 1		0		0
resource 2	(.4918)(3.541) =	1.74	(5.508)(3.541) =	19.50
resource 3		0		0
Income		\$ 0		\$ 0

At equilibrium firm income remains at zero for all firms, as documented in the following proposition.

Proposition 1: In a complete market setting each firm's income is zero in equilibrium and is determined using market prices for commodities and resources.

Proof: From the complementary slackness condition, we know that if any commodity, say x_i, is produced, then $A_i^T.v = p_i$. That is, the cost of resources used to produce the commodity is equal to the revenue generated by the commodity. Each commodity yields zero income in equilibrium, so irrespective of which commodities are produced by a firm, its income is zero. ■

In complete markets, aggregate supply of each resource, not each firm's initial endowment, is the determinant of equilibrium. That is, the p and v are the same as though a single firm were undertaking all production. In the complete markets setting, the economy operates as follows. Equilibrium forces determine p, v, and x. Individual firms simply trade resources at resource price v (determined at the market level) to reach equilibrium production.

Income measurement is instructive in that it is directly related to the efficiency of the production plan. This is stated in the next Corollary.

Corollary 1: Given equilibrium prices, if each firm's income is zero the production schedule is efficient.

Proof. Technology restrictions ensure that x satisfies the primal constraints; p and v being equilibrium prices ensure that the dual program is optimized. Each firm's income being zero implies economy-wide income is zero, which implies $p^T.x = v^T.r$. The Duality theorem then implies that x optimizes Program 1, i.e., x is efficient. ■

Even at the individual firm level, the income number retains its ability to aggregate information about the economy. The accountant's job is still not a difficult one, as income measurement follows directly from the observation of resource and commodity prices. The prices are easily determined as transactions occur at that price. "Mark-to-market" is a viable and effective valuation rule. However, the situation becomes more complex when markets are incomplete. The next section explores this issue.

4. Incomplete Market Accounting

In this section we confront accounting in incomplete markets. We model market incompleteness by restricting the trading opportunities. In particular, for the numerical example we prohibit trading in resource 3. Now each firm must design its production schedule with its use of resource 3 restricted to its initial endowments.

The way we capture the additional restriction is to reformulate the transformation matrix, A. Additional rows are added, and the resources are labeled r_1, r_2, r_{3A}, and r_{3B}. Given the restriction on trading in resource 3, we treat it differently for each of the firms, despite the resource being physically identical. The distinction between r_{3A} and r_{3B} is not the physical composition but rather the market restriction on trading. The linear formulation proves convenient since all definitions we have provided earlier (and the technique suggested for computing the equilibrium) remain valid except that the reformulated A matrix is used.

$$A = \begin{bmatrix} 1 & 1 \\ .5 & 2 \\ 4 & 0 \\ 0 & .25 \end{bmatrix} \qquad r = \begin{bmatrix} 4 \\ 6 \\ 3 \\ 7 \end{bmatrix}$$

The equilibrium in the incomplete market setting is as follows. [13]

Incomplete markets equilibrium

$x_1 = 0.7500$	$p_1 = 2.063$	$v_1 = 0$
$x_2 = 2.813$	$p_2 = 4.688$	$v_2 = 2.344$
		$v_{3A} = 0.2227$
		$v_{3B} = 0$

The objective function for each of the programs is 14.73.

[13] The following equalities are satisfied by an equilibrium.

$$0.5\, x_1 + 2x_2 = 6$$
$$4\, x_1 = 3$$
$$p_1 = - x_1 + x_2$$
$$p_2 = 10x_1 - x_2$$
$$0.5\, v_2 + 4v_{3A} = p_1$$
$$2\, v_2 = p_2$$

Resource requirements -- Incomplete markets

Resource requirements	Firm A	Firm B	Total
resource 1	0.75	2.813	3.563
resource 2	0.375	5.625	6.000
resource 3	3.000	0.703	3.703

Income measurement is now problematic since there has been no transaction involving resource 3, hence, a market price is unavailable. Nonetheless, when the individual shadow prices v_{3A} and v_{3B} are used in the income calculation, the illuminating property of income is preserved in the sense that zero income accompanies equilibrium. The use of shadow prices in income calculation parallels that in Scapens (1978) and Beaver and Demski (1995), who refer to them as "personal prices." [14] For the example the income statements follow.

Income statements: Incomplete markets

	Firm A		Firm B	
Revenues:	(.75)(2.063) =	$ 1.55	(2.813)(4.688) =	$ 13.18
Expenses:				
resource 1		0		0
resource 2	(.375)(2.344) =	0.88	(5.625)(2.344) =	13.18
resource 3	(3)(.2227)	0.67		0
Income		$ 0.00		$ 0.00

[14] Personal prices are used in the economics literature on efficiency in incomplete markets, e.g., Geanakopolos, Magill, Quinzii, and Dreze (1990).

We next state a proposition and a corollary analogous to those presented in the complete market setting. (The proofs are similar to those in Proposition 1 and Corollary 1.)

Proposition 2: In an incomplete market setting each firm's income is zero in equilibrium and is determined using *market* prices for commodities and all traded resources and *personal* prices for the untraded resources.

Corollary 2: Given equilibrium prices, if each firm's income is zero the production schedule is (constrained) efficient.

The definition of constrained efficiency is similar to the definition of efficiency given earlier. The only difference is that one uses the reformulated A matrix, which is more restrictive than the original A. We have productive efficiency, subject to the constraint that no trading in resource 3 is allowed.[15] Constrained efficiency manifests itself in a reduced value for the Program 1 objective function, 14.73, relative to unconstrained efficiency, with a Program 1 objective function value of 24.25.

Corollary 2 implies that income is still adequate to determine the efficiency of a firm's production schedule. However, as Proposition 2 points out, the difficulty is that the prices required for income calculation are no longer readily available. In fact, they are quite difficult to determine, since they are different for different firms even when the underlying resource is physically identical. This is what we mean by personal prices. Mark-to-market is no longer a viable valuation technique.

---

[15] Productive efficiency is not generally obtained in incomplete markets. When markets are incomplete and risk sharing is an issue, equilibrium and efficiency may not go hand-in-hand (Geanakopolos et al., 1990).

To estimate the personal prices of untraded resources requires examining the entire economy including the allocations and prices of the traded assets. Notice that in the numerical example the prices of all the commodities changed, as well as the price of resource 2, even though there were no restrictions placed on trading in those markets. This implies that the valuation problem can not be decomposed into a preliminary analysis of unrestricted markets concatenated with an analysis of the restricted markets. The entire economy must be considered simultaneously.

5. Conclusion

This paper demonstrates that aggregating information in the form of an income number is illuminating about the state of the economy. Income, by itself, reflects whether or not the economy is in equilibrium, and using a standard result in welfare economics, whether or not the allocation of resources is productively efficient. These results hold even when the market structure is not complete, that is, when some assets can not be traded.

However, what is different across complete and incomplete markets is the ease with which such a number can be computed. The accountant's job as an income calculator is significantly more difficult when markets are incomplete, as there are no market prices on which to rely. Instead the accountant must rely on personal prices. The estimation of personal prices requires a deep understanding of the equilibrium forces at work in the economy.

References

BEAVER, W., AND J. DEMSKI. "Income Measurement and Valuation." Working paper, University of Florida 1990.

CHAMBERS, R. *Accounting, Evaluation and Economic Behavior.* Prentice Hall, 1966.

DEBREU, G. *Theory of Value*. New Haven, CT: Yale University Press, 1959.

DORFMAN, R., P. SAMUELSON, AND R. SOLOW. *Linear Programming and Economic Analysis*. New York: Dover Publications, 1987.

EDWARDS, E., AND P. BELL. *The Theory and Measurement of Business Income*. University of California Press, 1961.

GEANAKOPLOS, J., M. MAGILL, M. QUINZII AND J. DREZE. "Generic Inefficiency of Stock Market Equilibrium when Markets are Incomplete." *Journal of Mathematical Economics* 19 (1990).

LUENBERGER, D. Linear and Nonlinear Programming. Reading, MA: Addison-Wesley Publishing Company, 1989.

PATON, W. *Accounting Theory.* Accounting Studies Press, Ltd., 1922.

SCAPENS, R. "A Neoclassical Measure of Profit." *The Accounting Review* 53 (April 1978).

STERLING, R. *Theory of the Measurement of Enterprise Income*. University of Kansas Press, 1970.