

Multiple Sources of Information, Valuation, and Accounting Earnings

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1. Introduction

Accounting researchers have long debated how to model the relation between earnings and security price. In this paper, we add a scaling dimension to this debate. The scale of an information system is the ordering scheme it uses to represent relations. The language of valuation (e.g., assets, equities, and income) provides accounting's scale. This scaling is restricted, however, by recognition rules that keep some information (e.g., order books) outside the accounting system. Restrictive accounting recognition rules imply that the inferences made using earnings must be conditioned on available other, nonaccounting information. As obvious and innocuous as this seems, the coupling of valuation scaling and restrictive accounting recognition rules implies earnings and price are monotonically (or linearly, a fortiori) related only under very strong conditions. In this paper, we describe these conditions. We also discuss more generally the problems of inference using financial statement numbers based on a valuation scale and restrictive accounting recognition rules.

Several other papers have modeled the impact of nonaccounting information on the relation between earnings and price.¹ These models typically

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1. The literature has long recognized that information content is a joint product of available information signals. Anderson (1975), for example, points out that with informational interactions, assigning information content to specific signals involves allocation problems analogous to those involving joint costs. Ronen (1979) points out a disclosure effect may depend on additional information that becomes available only after the disclosure in question. Gonedes (1978) highlights this theme when he defines two information sources as complements if they have information content jointly, but not separately, and as substitutes if neither has incremental information content conditional on knowing

begin with a setting that ensures a linear relation. For example, Feltham and Ohlson (1993) assume a linear dynamic system relating dividends, accounting variables, and nonaccounting information. Their model generates insights by imposing financial economic relations (e.g., price as discounted expected dividends) and accounting relations (e.g., clean surplus), but these insights are naturally limited to those possible in a linear model. In contrast, our information variables arrive scaled in a manner that does not admit a linear or even monotone relation between earnings and price. For such a linear relation to obtain, it would be necessary to rescale earnings to reflect all available information. In other words, it would be necessary to mimic the financial analyst who adjusts the earnings number to "undo" the effect of restrictive accounting recognition rules.²

The paper is organized as follows. In Section 2, we present a simple valuation context and some basic definitions. In Section 3, we define information content and provide sufficient conditions for a monotone relation between an abstract information signal and market values. In Section 4, we introduce our model of accounting valuation, based on restrictive recognition rules. In Section 5, we explore the relation between accounting and market values by providing some revealing examples, by presenting sufficient conditions for a monotone relation between earnings and price, and by presenting necessary and sufficient conditions for a linear relation between earnings and price. (As will become clear, price is measured by unexpected change in price and earnings by unexpected earnings.) The appendixes provide proofs and a discussion of alternative specifications of the accounting system.

Two caveats should be acknowledged at the outset. First, the model focuses on the relation between the market value and the information releases of a single entity. There is no sense of a population, of sampling, or

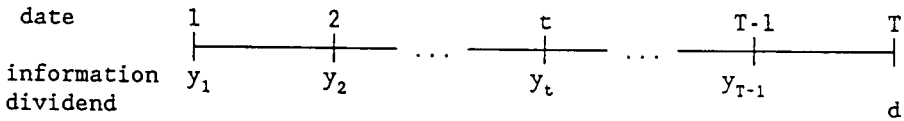
the other's report. These are extreme cases. Loosely speaking, complementarity means the joint information content differs from a simple combination of their marginal effects.

Similarly, Lundholm (1988) shows that two sources with correlated errors can become informational complements. In his model, this is manifested by an unfavorable conditional stock price reaction to an unconditionally favorable signal. Too much "good news" in a later signal can be evidence that previous information contained errors on the optimistic side. And Holthausen and Verrecchia (1988) show the variance of stock price changes corresponding to a sequence of disclosures is a function of the intertemporal and cross-sectional correlation among asset returns and the noise factors in information releases. These correlations capture complementarity and substitutability relationships.

Complementarity and substitutability relationships are important here, as well. The difference is we convey the accounting information with an explicit accounting format, one that clouds the price-earnings relationship from the point of view of an external observer.

2. Ohlson and Shroff (1992) examine the question of levels versus differences and stress, among other things, the possibility of a nonlinear relation between unexpected earnings and returns. We, too, focus on a nonlinearity, but in our case it is driven explicitly by the accounting recognition rules. It is information beyond that in the accounting system coupled with the restrictive recognition rules that determine the price-earnings relation in our model.

FIGURE 1



of measurement error. Therefore, many issues of interest in empirical work (e.g., power of tests) are outside the model. Second, the model focuses on risk-neutral valuation. This confines the study to the relation between the information variables and the first moment of the value distribution. Other moments are not considered.³

2. A Simple Model of Market Valuation with a Sequence of Information Signals

We now introduce the model.

2.1 Basic Structure

To keep periodic dividends from providing an additional source of information and unduly complicating the model, we assume a firm will pay a single, liquidating dividend at a known terminal time T , with $T \geq 2$. The dividend is denoted $d \in D$, where D is the set of possible dividends. Information arrives at each date prior to the dividend realization, that is, $t \in \{1, 2, \dots, T - 1\}$. The information realization at t is denoted $y_t \in Y_t$, where Y_t is the set of possible information realizations at time t .⁴ Although no information as such arrives at time T , it will ease notation to assume signal $y_T = d$ is observed at time T . (So $Y_T = D$.) We denote the entire vector of information realizations by $y \equiv (y_1, \dots, y_T) \in Y \equiv Y_1 \times \dots \times Y_T$, and the vector (or history) of information realizations through date t by $\bar{y}_t \equiv (y_1, \dots, y_t) \in \bar{Y}_t \equiv Y_1 \times \dots \times Y_t$.

The information history, \bar{y}_t , is common knowledge; over time, more is known about the dividend. Let $\pi(y, d)$ denote the joint probability of the information and terminal dividends. Bayesian revision yields the updated probability of the vector of information and dividend realizations at date t ,

3. Ball, Kothari, and Watts (1993) control for risk changes, in effect allowing the information to speak to risk and expected return assessments. They document, at the portfolio level, an ordinal price-earnings relationship. A similar ordinal relationship will appear in our model.

4. For convenience, all random variables have finite support, that is, D and Y are finite and are real numbers.

$\pi(y, d\bar{y}_t)$.⁵ The probability structure π is also common knowledge. Figure 1 depicts the sequence of events.

2.2 Market Valuation

The market value of the firm at date t is the present value of the expected dividend conditional on the information currently known by the market. A constant interest rate, $r \geq 0$, is assumed. At date t , the conditional expected value of the dividend is $\bar{d}_t(\bar{y}_t) \equiv \sum_d d\pi(d|\bar{y}_t)$. The market value of the firm at date t is

$$V_t(\bar{y}_t) \equiv \bar{d}_t(\bar{y}_t) (1 + r)^{t-T}. \quad (\text{A1})$$

The pricing function V_t represents an informationally efficient market in the sense beliefs are promptly revised subject to Bayes' rule. The expected rate of return is the interest rate r . The dividend expectation process $\bar{d}_t(\bar{y}_t)$ is a martingale with respect to history \bar{y}_t .

2.3 Unexpected Capital Gain

Empirical event studies usually work with an unexpected return measure of market reaction. In our setting of a single firm under risk neutral pricing, unexpected capital gain is an intuitive, although obviously derivative, focus.

Definition: Unexpected capital gain from dates τ to t , $\tau < t$, is $M_{t,\tau}(\bar{y}_t) \equiv V_t(\bar{y}_t) - (1 + r)^{t-\tau} V_\tau(\bar{y}_\tau)$.

Here \bar{y}_τ refers to the first τ elements of \bar{y}_t .

Three issues now arise. Can we verify usefulness of an information release by examining market reaction in the period of its release? Can we use a linear model for this purpose? Does accounting structure enter into these concerns in any substantive way?

3. Information Content

In this section, we define information content of date t information. The definition relates disclosures to market values, but does not pin down the period in which a disclosure might affect value. We show this by providing an example of complementarity across information signals.

5. Throughout, conditional probabilities are denoted in this standard fashion.

FIGURE 2

		(y_1, y_2)			
		(0, 0)	(0, 1)	(1, 0)	(1, 1)
d	0	0.25	0	0	0.25
	1	0	0.25	0.25	0

Following Gonedes (1978), we say that date t information has information content if it affects market value at any date. Denote the history of information realizations at date t with the realization at date $\tau < t$ removed by $\bar{y}_t^\tau \equiv \bar{y}_t \setminus y_\tau$. This is the history that would prevail had the time τ source been shut down.

Definition: The date τ information source has information content if there exists a current or future time $t \geq \tau$, and a possible sequence of events \bar{y}_t such that $V_t(\bar{y}_t \setminus y_\tau)$ is not equal to $V_t(\bar{y}_t)$.

The definition allows information arrival at one date to affect value at any future date; complementarities are possible. Disclosure with information content must have the potential to affect market value, sooner or later. But later is a distinct possibility. For example, disclosure of a potentially onerous legal action may be largely noise until the Supreme Court rules on a related case. Changes in orders received may be largely uninterpretable until changes in competitor orders are disclosed.

Gonedes' case of strict complementarity illustrates the extreme possibility that none of the effects of a disclosure are felt in the period of disclosure.⁶ To illustrate, assume $T = 3$, so we have two information sources. Each is binary, with $y_1, y_2 \in \{0,1\}$. The dividend is also a binary random variable, $d \in \{0,1\}$. The joint probability matrix is given in Figure 2. For example, $\pi(y_1 = 1, y_2 = 0, d = 1) = \pi(y_1 = 0, y_2 = 1, d = 1) = 0.25$. The expected value of d does not change on receipt of y_1 or on receipt of y_2 if y_1 is not received. Yet the expected value of d does change at date 2 if both signals are received. The date 1 disclosure has information content, but none of its effects are reflected in the market value until date 2. Intu-

6. Strict complementarity is not logically restricted to two information sources. We could have n information sources, each of which does not change dividend expectations without the other $n - 1$ signals.

itively, the only role of the date 1 information is to "condition" the date 2 information.⁷

3.1 Restrictions on the Probability Structure: Additive, Independent Cash Flows

It is, of course, possible to identify restrictions on the probability structure π that are sufficient to ensure at least some price reaction to an informative disclosure takes place in the period of the disclosure. Because these issues are important in our study of the price-earnings relation, we provide two such restrictions. The first is an additive, independent cash flow structure that rules out any possible complementarities; it is intuitive but strong. The second relies on a version of the monotone likelihood ratio property, and allows some complementarities. Although it ensures some price reaction takes place at the date of disclosure, it admits the possibility that later reactions also occur due to interaction effects.

Definition: π is an additive, independent cash flow setting if (1) the dividend is the future value of the information realizations, $d = \sum_t y_t(1 + r)^{T-t}$, and (2) the information signals are independent, $\pi(y_1, \dots, y_{T-1}) = \pi(y_1) \dots \pi(y_{T-1})$.

In the additive, independent cash flow case, the unexpected capital gain from $t - 1$ to t is particularly simple:⁸

$$M_{t,t-1}(\bar{y}_t) = y_t - E_{\pi}(y_t | \bar{y}_{t-1}) = y_t - E_{\pi}(y_t).$$

The change in market value at a date is simply the difference between the information realization and its unconditional expectation, so all consequences of a disclosure occur in the period of the disclosure. This rests on the independence assumption.

3.2 Restrictions on the Probability Structure: Conditional Monotone Likelihood Ratios

Instead of ruling out any complementarities, weaker conditions would ensure some, not all, reactions take place in the short window. This can be

7. This example might relate to a situation in which, at an early date, we report whether customers for a firm's newly designed product have been located. This would not lead to a revision of dividend expectations if the expected incremental profit from serving the customers is zero. At a later date, however, we report information about the profitability of satisfying these customers' demands. Only then might we have a revision of dividend expectations.

8. The notation convention of $E_{\pi}(x|y)$ refers to the expected value of random variable x , using probability structure π , conditioned on event y .

accomplished by imposing some ordering on information, so that (almost) every disclosure results in some immediate "good" or "bad" news. A variation of the monotone likelihood ratio property accomplishes this goal.⁹

Definition: π possesses the conditional monotone likelihood ratio property (CMLRP) if the following inequality holds for all $t < T$ and $\bar{y}_{t-1} \in \bar{Y}_{t-1}$: $\pi(y'_t|d', \bar{y}_{t-1})\pi(y_t|d, \bar{y}_{t-1}) > \pi(y_t|d', \bar{y}_{t-1})\pi(y'_t|d, \bar{y}_{t-1})$, for all $y'_t > y_t$ and $d' > d$ consistent with the history \bar{y}_{t-1} .

Notice the following, more suggestive, ratio condition if all the probabilities are strictly positive:

$$\frac{\pi(y'_t|d', \bar{y}_{t-1})}{\pi(y'_t|d, \bar{y}_{t-1})} > \frac{\pi(y_t|d', \bar{y}_{t-1})}{\pi(y_t|d, \bar{y}_{t-1})}.$$

CMLRP implies the distribution of the dividend conditional on history $\{\bar{y}_{t-1}, y'_t\}$ first-order stochastically dominates the distribution of the dividend conditional on history $\{\bar{y}_{t-1}, y_t\}$. This holds for all $y'_t > y_t$ consistent with the history \bar{y}_{t-1} and allows us to view y'_t as "better" news than y_t , given \bar{y}_{t-1} . For example, higher earnings is generally viewed as better news than lower earnings. Notice, given the strict way in which it has been defined, CMLRP allows at most one signal at a given date to be associated with a zero unexpected capital gain; this ensures a particularly focused picture of information content. The additive, independent cash flow case possesses CMLRP. The strict complementarity case illustrated in Figure 2 does not.¹⁰

With this lengthy preamble, we now turn to the question of accounting disclosure.

4. A Model of Accounting Information

We begin with a model of accounting valuation. This allows us to distinguish between accounting and nonaccounting information by specifying the form in which accounting is presented. To keep the accounting valuation process as close as possible to the market's, we assume accounting values are discounted expected values. Given this, the distinction be-

9. See Milgrom (1981) for a discussion of the monotone likelihood ratio property. Notice we use a strict version of this property.

10. CMLRP is not necessary to ensure information content is manifested at the disclosure date. We only need dividend expectations to move at that date with some probability when there is information content. However, because CMLRP implies a higher signal is better news for a given information history, it has a natural tie to "sign" or "magnitude" oriented event studies such as Beaver, Clarke, and Wright (1979). Weaker assumptions might have a natural tie to variance-oriented event studies such as Beaver (1968). Of course, CMLRP does not imply all effects of a disclosure will be felt at the disclosure date. It implies only some unexpected price movement occurs at the disclosure date.

tween accounting and market valuation reduces to the presumption the accounting valuation process uses less information than does the market. We model this by arbitrarily splitting information into accounting and non-accounting information, and assuming the accounting system only uses the accounting information. Appendix B discusses alternative specifications of the accounting process.

4.1 Accounting Valuation

Now label information that arrives in even periods as accounting information and information that arrives in odd periods as nonaccounting information. This simple device allows us to mix the two sources of information; and it captures the institutional feature of information being disseminated between accounting disclosures.¹¹

To ensure the accounting system eventually identifies the cash and therefore the liquidating dividend, we assume T is even. Denote the vector of accounting information realizations by $y^a \equiv \{y_2, y_4, \dots, y_T\} \in Y^a \equiv Y_2 \times Y_4 \times \dots \times Y_T$, where it is understood $y_T = d$. The vector of nonaccounting information realizations is $z \equiv \{y_1, y_3, \dots, y_{T-1}\} \in Z \equiv Y_1 \times Y_3 \times \dots \times Y_{T-1}$. Histories of information realizations are denoted in the manner introduced earlier.

To derive an accounting valuation process, we want to model information processing that uses only the accounting information. For this purpose, we focus on the "accounting probability" of $\pi^a(y^a, d) \equiv \sum_z \pi(y, d)$. Thus, $\pi^a(y^a, d)$ is the projection of $\pi(y, d)$ onto $Y^a \times D$.

We assume the accounting system emulates the market valuation calculation in eq. (A1) with two differences. First, the accounting valuation is confined to base dividend expectations on only the accounting information realizations. Second, the accounting valuation uses the "accounting probability" π^a . Use of the accounting probability is natural given the restriction to accounting information. The specification is intended to capture accounting's use of a valuation language analogous to, but not necessarily coincident with, market valuation. Also, restrictive accounting recognition rules preclude the use of some types of information that are incorporated in market valuation (e.g., order books, price increases in raw materials inventory, new product announcements).

Formally, we assume the book value at even date t is the expected value of the dividend conditional on the available *accounting* information

11. Contemporaneous nonaccounting information releases are easily incorporated into the model. These raise no further difficulties for our analysis than are raised by sequential releases. Contemporaneous nonaccounting releases do, however, complicate efforts to identify reactions empirically.

\bar{y}_t^a , discounted at the same constant interest rate as in the market value calculation. The expected value of the dividend conditional on \bar{y}_t^a is denoted $\hat{d}_t(\bar{y}_t^a) \equiv \sum_d d\pi^a(d|\bar{y}_t^a)$. Thus we assume the book value at date t is

$$A_t(\bar{y}_t^a) \equiv \hat{d}_t(\bar{y}_t^a) (1 + r)^{t-T}. \quad (\text{A2})$$

Equation (A2) defines book value as close as possible to market value, while retaining the distinction between accounting and nonaccounting information. Since Bayesian revision and the same interest rate r are used in eq. (A2), book value is an "unbiased" measure of market value. If the history of the nonaccounting information is not incrementally informative given the history of the accounting information, then book value equals market value at even dates.

The only structural feature of accounting that we invoke is its inability to use all information that is available to the market. No deeper specification is attempted, as this is sufficient to expose the impact of valuation-based scaling on the price-earnings relation. Specifying what the accounting system can use for valuation purposes, however, amounts to placing more structure on π . For example, a particular pattern of dependence is implied if we assume accounting captures everything with a lag.

The same constant interest rate is used in eqs. (A1) and (A2) to emphasize the importance of the restricted information used to calculate book value. We consider in Appendix B the possibility that a conservative accounting system might employ a higher interest rate.¹²

Equation (A2) produces properties analogous to those for eq. (A1). The expected accounting rate of return, $E_{\pi^a}\{[A_{t+2}(\bar{y}_{t+2}^a) - A_t(\bar{y}_t^a)]/A_t(\bar{y}_t^a)|\bar{y}_t^a\}$, is the two-period interest rate $(1 + r)^2 - 1$. Similarly, the dividend expectation process $\hat{d}_t(\bar{y}_t^a)$ is a martingale with respect to the accounting information history \bar{y}_t^a but not necessarily with respect to all information \bar{y}_t .

Next we assume "clean surplus," so that accounting earnings are the change in book value. At even period t we have¹³

$$e_t(\bar{y}_t^a) = A_t(\bar{y}_t^a) - A_{t-2}(\bar{y}_{t-2}^a). \quad (\text{A3})$$

The key feature of eqs. (A2) and (A3) is Bayesian revision given restrictive recognition rules. Figure 3 depicts the sequence of events and calculations. We emphasize the sole difference between the settings of Fig-

12. James Ohlson has pointed out to us in conversation that the use of the same interest rate in the accounting and market valuations is particularly important in a setting with intermediate dividends. In that case, the dividend payout and interest rate histories play important roles in determining the relation between accounting and market values.

13. There is an extensive literature relating to the clean surplus relation. For example, see Paton and Littleton (1940), Dopuch and Drake (1965), Demski and Sappington (1990), and Ohlson (1991).

FIGURE 3

date	t-2	t-1	t (even)	...
nonaccounting information		z_{t-1}		
accounting earnings	$e_{t-2}(\bar{y}_{t-2}^a)$		$e_t(\bar{y}_t^a)$	
market value	$v_{t-2}(\bar{y}_{t-2})$	$v_{t-1}(\bar{y}_{t-1})$	$v_t(\bar{y}_t)$	

ures 1 and 3 is that the accounting information now arrives in the form of accounting values and accounting earnings.¹⁴ The even-period disclosures have been rescaled to arrive in a valuation format. Accounting value is based on the market's valuation algorithm, but with the information base exogenously restricted.

4.2 Unexpected Earnings

Event studies usually work with some measure of unexpected earnings. We consider two such measures, one based on all information, as would be an ideal analyst's forecast, and the other based only on accounting information, as would be a forecast generated by application of time series techniques to accounting variables. We call the former "unexpected earnings" and the latter "TS unexpected earnings" in reference to the time series analogy. Naturally, both are defined only in even periods (where earnings are reported).

Definition: Unexpected earnings in even period t , with expectations as of time $\tau < t$, are $U_{t,\tau}(\bar{y}_t) \equiv e_t(\bar{y}_t^a) - E_{\pi}[e_t(\bar{y}_t^a)|\bar{y}_{\tau}]$.

Definition: TS unexpected earnings in even period t , with expectations as of time $\tau < t$, are $\hat{U}_{t,\tau}(\bar{y}_t^a) \equiv e_t(\bar{y}_t^a) - E_{\pi a}[e_t(\bar{y}_t^a)|\bar{y}_{\tau}^a]$.

(As in the earlier noted definition of unexpected capital gain, we presume the partial information histories are mutually consistent.)

14. To complete the tie to the earlier section, we suppose the accounting earnings in period t can be decoded (perhaps using the full history of prior information) to discover the underlying \bar{y}_t^a . In this way, the primitive information behind the accounting systems remain, while accounting disclosures come packaged in accounting language.

FIGURE 4

		(y_1, y_2^a)			
		(0, 0)	(0, 1)	(1, 0)	(1, 1)
d	0	0.005	0	0	0
	1	0	0.495	0	0
	99	0	0	0.495	0
	100	0	0	0	0.005

5. Relation between Accounting Values and Market Values

We now turn to the price-earnings relation, or more precisely the relation between unexpected capital gain and the two noted unexpected earnings measures. We begin with two examples. In the first, we find a uniformly negative relation. In the second, we find a uniformly positive but nonlinear relation.

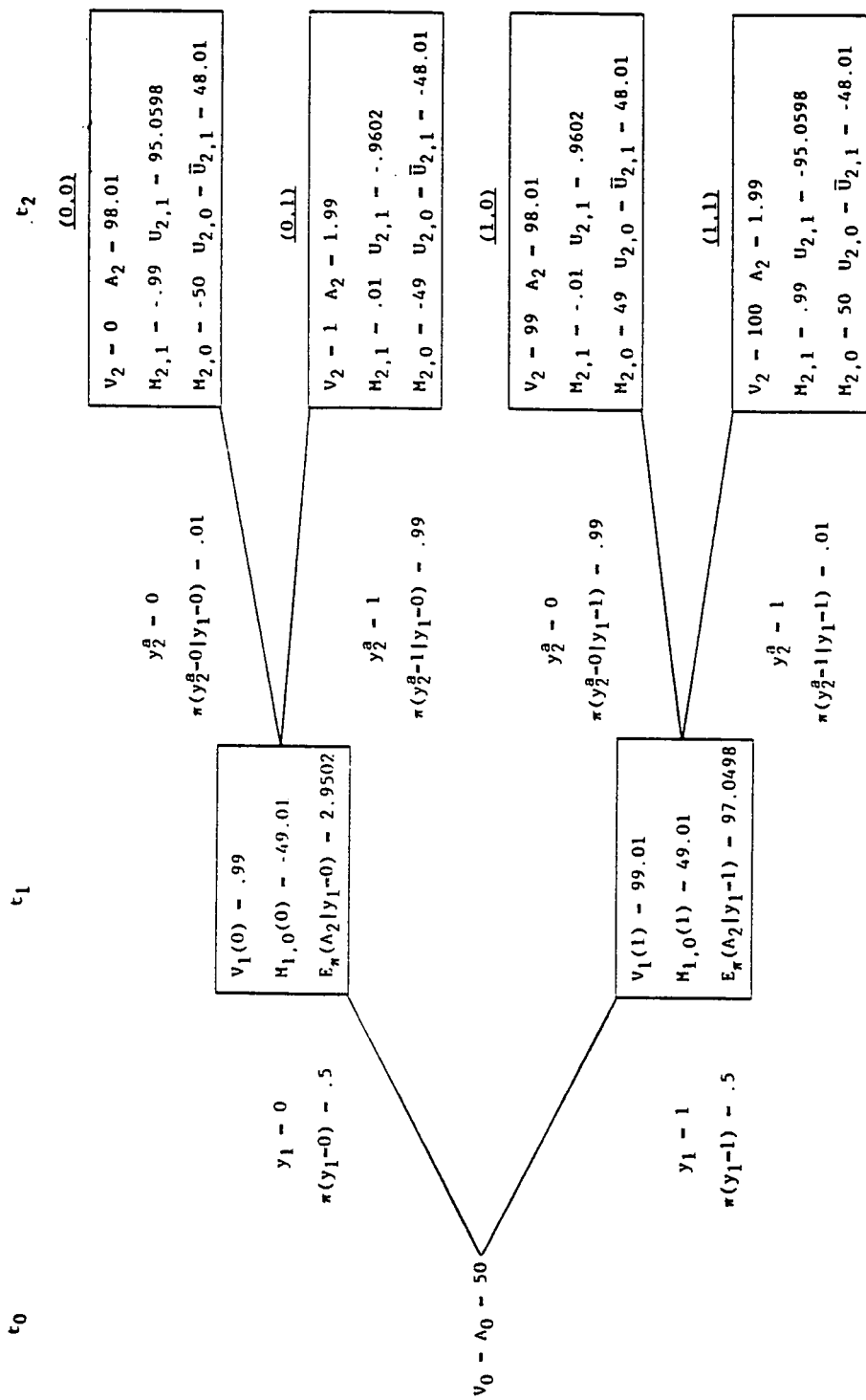
Example 1: Let $r = 0$, $T = 4$, $d \in \{0, 1, 99, 100\}$, $y_1, y_2^a \in \{0, 1\}$, and y_3 be a constant.¹⁵ Figure 4 provides the joint probabilities. The evolution of the example is displayed in Figure 5, which reveals CMLRP is satisfied in each period. Notice $V_1(1) > V_1(0)$ and $V_2(\cdot, 1) > V_2(\cdot, 0)$. Now focus on the four boxes displaying the behavior of the system at $t = 2$. When $y_1 = y_2^a = 0$, unexpected capital gains from $t = 1$ to $t = 2$ are negative, $M_{2,1}(0, 0) = -0.99$, reflecting the conditional bad news in the signal $y_2^a = 0$. However, both measures of unexpected earnings are positive, $U_{2,1}(0, 0) = 95.0598$ and $U_{2,1}(0, 0) = 48.01$, reflecting the unconditional good news in $y_2^a = 0$.

The case of $y_1 = y_2^a = 0$ is not alone in having opposite signs on unexpected capital gain from $t = 1$ to $t = 2$ and unexpected earnings. In fact, this pattern holds for all values of the signals.

The negative relation between unexpected capital gains and unexpected earnings is caused by the restrictive recognition rules under which the accounting system operates. Unconditionally, receipt of an accounting signal of $y_2^a = 1$ is very bad news, since it implies the nonaccounting signal y_1

15. Banks' loan loss allowances corresponds roughly to this example. Both the popular press and academic research suggest the market has at times regarded increases in allowance for loan losses as good news. The market has many sources of information about the quality of a bank's loans. These sources condition the market's reaction to the allowance for loan losses, increases in which are surely unconditional bad news.

FIGURE 5



was most likely a 0, $\pi(y_1 = 0 \mid y_2^a = 1) = 0.99$. The accounting system, by virtue of its restrictive recognition rules, calculates its $t = 2$ values unconditionally. The market valuation process, facing no such constraints on the information it processes, appropriately calculates values conditional on all the available information. Moving to a long return horizon mitigates, but does not eliminate, this effect.¹⁶

Example 2: It is also possible that unexpected capital gain and unexpected earnings track in sensible, but opaque fashion. We use a Bernoulli process to exhibit such a possibility. Here, we have $d \in \{0,1\}$, $\pi(d = 1) = 1/2$, and a signal each period that is, conditional on d , an iid Bernoulli random variable with $y_t \in \{G,B\}$, and $\pi(y_t = G \mid d = 1) = \pi(y_t = B \mid d = 0) = \omega > 1/2$.

This greatly simplifies the conditional expectation dynamics.¹⁷ To wit, let m denote the number of G observations over $t - 2$ periods. That is, \bar{y}_{t-2} contains m G 's and $t - m - 2$ B 's. Then algebraic manipulation produces a revised dividend expectation of $E_\pi(d \mid \bar{y}_{t-2}) = (1 + L^{t-2m-2})^{-1}$, where $L = \omega/(1 - \omega) > 1$. Now suppose the next two signals are each good news, G . Unexpected capital gain over the window will be

$$M_{t,t-2} = [(1 + L^{t-2m-4})^{-1} - (1 + L^{t-2m-2})^{-1}] (1 + r)^{t-T}.$$

Further assume t is an even period, and the history of accounting disclosures contains k realizations of signal G . TS unexpected earnings will then be

$$\hat{U}_{t,t-2} = [(1 + L^{t/2-2k-2})^{-1} - (1 + L^{t/2-2k-1})^{-1}] (1 + r)^{t-T}.$$

Note that $M_{t,t-2}$ reflects the arrival of two G observations, conditional on \bar{y}_{t-2} ; $\hat{U}_{t,t-2}$, on the other hand, reflects the arrival of one G observation, conditional on \bar{y}_{t-2}^a . Both stories are good news overall. The difference is due to recognition rules that restrict earnings to employing the y^a series of observations. This implies unexpected earnings reflect a portion of the information reported by unexpected capital gain, but with a different scaling. That is, $M_{t,t-2} - \hat{U}_{t,t-2}$ reflects additional information and history-dependent scaling differences. On the other hand, $y_t = G$ is good news relative to $y_t = B$ in either domain. (The $U_{t,t-2}$ measure provides a parallel story.)

To reinforce the importance of history in this setting, we introduce some suggestive earnings innovation notation. Let $\varepsilon_{t,t-1} \equiv \bar{d}(\bar{y}_t) -$

16. Focusing on unexpected capital gains from $t = 0$ to $t = 2$ (i.e., $M_{2,0}$) aligns the signs of unexpected capital gains and unexpected earnings in two cases, ($y_1 = 0, y_2^a = 1$) and ($y_1 = 1, y_2^a = 0$), but fails to align the signs in the other two cases. The two cases in which the signs are aligned, however, are far more likely, ex ante, so if the case could be resampled repeatedly, a long return horizon test would find a positive earnings response coefficient.

17. It is also apparent that CMLRP is satisfied.

$\bar{d}(\bar{y}_{t-1}), \epsilon_{t-1,t-2} \equiv \bar{d}(\bar{y}_{t-1}) - \bar{d}(\bar{y}_{t-2})$, and $\epsilon_{t,t-2}^a \equiv \hat{d}(\bar{y}_t^a) - \hat{d}(\bar{y}_{t-2}^a)$. This allows the following restatement of the relation between market and accounting observables:

$$M_{t,t-2} = \hat{U}_{t,t-2} + (\epsilon_{t,t-1} + \epsilon_{t-1,t-2} - \epsilon_{t,t-2}^a) (1 + r)^{t-T}.$$

That is, it is possible to express unexpected capital gain as the TS unexpected earnings plus noise. However, the noise terms make this expression history dependent. Specifically, although $E_{\pi}(\epsilon_{t,t-1} + \epsilon_{t-1,t-2} | \bar{y}_{t-2}) = 0$, it is generally the case that $E_{\pi}(\epsilon_{t,t-2}^a | \bar{y}_{t-2}) \neq 0$. Accordingly, we cannot view dividend revisions as if they are straightforward earnings innovations. Rather, measurement scale issues must be incorporated into the relation between accounting and market observables.

These examples reveal more than a valuation approach to accounting is needed to ensure a clear relation between accounting and market values. Differences in the information on which accounting and market values are based are by themselves sufficient to admit substitute and complement relations among accounting and nonaccounting data. Unexpected earnings, even when all information is used in their calculation, reflect the accounting system's recognition restrictions. The interplay between the market's unrestricted valuation and the accounting system's restricted valuation can be quite subtle. In the following subsections, we present conditions aimed at ensuring market values respond to accounting values in the intuitive direction. We then address the issue of linearity in the price-earnings relation.¹⁸

5.1 Restrictions on the Probability Structure: Additive, Independent Cash Flows

As in the case with abstract information signals, one way to deal with complementarities is to rule them out entirely. Again, the additive, independent cash flow case does exactly that.

18. It is possible most of the response to the accounting release occurs in a later period. Consider the following slight alteration of the example of extreme complementarity: $r = 0, T = 4, D = \{0,1,100,101\}$, y_1 is null, and y_2^a and y_3 are either 0 or 1. The joint probabilities are given below.

		(y_2^a, y_3)			
		(0,0)	(0,1)	(1,0)	(1,1)
d	0	0	0.25	0	0
	1	0	0	0.25	0
	100	0.25	0	0	0
	101	0	0	0	0.25

Proposition 1: Assume eqs. (A1), (A2), (A3), and π is an additive, independent cash flow setting. Then $U_{t, t-1}(\bar{y}_{t-1}, y_t^a) = \hat{U}_{t, t-2}(\bar{y}_t^a) = M_{t, t-1}(\bar{y}_t) = y_t^a - E_{\pi}[y_t^a]$, for all even t , $\bar{y}_t \in \bar{Y}_t$.

With no informational interactions, we have identity between the unexpected capital gain and unexpected earnings. Using a language of valuation, eq. (A2), is particularly powerful here, because the ‘‘unrecognized information’’ does not affect the price-earnings relation in the period of disclosure.

5.2 Restrictions on the Probability Structure: Monopolistic Accounting

In a related vein, suppose π is restricted so none of the nonaccounting signals has any information content. All unexpected changes in value take place at an accounting release, and our valuation assumptions ensure identity between market and accounting variables: $U_{t, t-1}(\bar{y}_{t-1}, y_t^a) = \hat{U}_{t, t-2}(\bar{y}_t^a) = M_{t, t-1}(\bar{y}_t)$. Absent interactions with nonaccounting information, our assumed accounting structure results in a straightforward price-earnings relation.

5.3 Restrictions on the Probability Structure: Conditional Monotone Likelihood Ratios and More

More broadly, to ensure a positive price-earnings relation, we must restrict the probability structure so that positive unexpected earnings tends to be conditional good news for market valuation. One way to do this is to assume that both π and π^a possess CMLRP. As illustrated by Example 1, however, it is possible for π to possess CMLRP, but for π^a not to possess CMLRP. Good news given all the information available to the market may not be good news given the restricted accounting information. In particular, it is possible that the inequality in the definition of CMLRP no longer holds when we remove the nonaccounting information history from the conditioning set. To be able to remove the nonaccounting information from the conditioning set while maintaining the inequality, we assume that the accounting and nonaccounting information are independent conditional on the dividend.

Definition: π possesses conditional independence of the accounting and nonaccounting information if $\pi(y, d) = \pi^a(y^a|d)\pi(z|d)\pi(d)$.

This additional structure ensures that π^a possesses CMLRP.

Lemma: If π possesses CMLRP and the accounting and nonaccounting information are conditionally independent, then π^a possesses CMLRP.

When both π and π^a possess CMLRP, unexpected earnings and unexpected capital gains are positively related.

Proposition 2: Assume eqs. (A1), (A2), and (A3). Also assume π possesses CMLRP and the accounting and nonaccounting information are conditionally independent. Then:

1. $U_{t,t-1}(\bar{y}_{t-1}, y_t^a) > U_{t,t-1}(\bar{y}_{t-1}, y_t^a)$ only if $M_{t,t-1}(\bar{y}_{t-1}, y_t^a) > M_{t,t-1}(\bar{y}_{t-1}, y_t^a)$ for all even t , \bar{y}_{t-1} , y_t^a , and y_t^a ; and
2. $\hat{U}_{t,t-2}(\bar{y}_{t-2}, y_t^a) > \hat{U}_{t,t-2}(\bar{y}_{t-2}, y_t^a)$ only if $M_{t,t-2}(\bar{y}_{t-1}, y_t^a) > M_{t,t-2}(\bar{y}_{t-1}, y_t^a)$ for all even t , \bar{y}_{t-1} , y_t^a , and y_t^a .

The assumptions of Proposition 2 ensure π^a possesses CMLRP. In turn, this implies that "good news" is interpreted as "good news" in the accounting domain. The ranks of unexpected capital gain, unexpected earnings, and TS unexpected earnings are identical. Thus earnings response coefficients are positive, though they need not correspond with the benchmark of one in Proposition 1. Proposition 2 implies nothing about the magnitude of the market's reaction. This is, of course, precisely the picture that emerged in our second example.

The conditions in Proposition 2 are not the minimal sufficient conditions for positive earnings response coefficients. In particular, CMLRP could be replaced by a weaker dominance condition. It is clear, however, that the minimal sufficient conditions to ensure that earnings response coefficients are positive are far from trivial.

Obviously, linearity in the relation between values and earnings requires even more structure than does monotonicity.

5.4 Linear Models of the Relation between Price and Earnings

Linear models of the relation between price and earnings are standard in investigations of the information content or valuation role of earnings.¹⁹ In this section, we provide necessary and sufficient conditions for a linear relation between unexpected capital gain and the two unexpected earnings measures. Given restrictive accounting recognition rules, a linear relation

19. Accounting researchers did not typically use linear models to investigate the relation between price and earnings until the 1980s. Prior research had used a variety of portfolio methods (e.g., see Ball and Brown [1968]; Beaver, Clarke, and Wright [1979]).

between unexpected capital gain and unexpected earnings obtains only if we greatly restrict the probability structure, π . A key is to prohibit interactions between the accounting and nonaccounting information. Examples are cases of additive, independent cash flows and nonrestrictive recognition rules. Proposition 3 records formally the result.

Proposition 3: $M_{t,t-1}(\bar{y}_t) = k_t U_{t,t-1}(\bar{y}_t)$ if and only if $\sum_d d[\pi(d|\bar{y}_t) - \pi(d|\bar{y}_{t-1})] = k_t \{ \sum_d d[\pi^a(d|\bar{y}_t^a) - \sum_{y_t^a} \pi^a(d|\bar{y}_t^a) \pi(y_t^a|\bar{y}_{t-1})] \}$.

Further, there exists a π that satisfies this condition, but the set of π 's that do so have Lesbesgue measure zero in the space of all π 's.

Unexpected capital gain is proportional to unexpected earnings if and only if the market's revision of dividend expectations is proportional to the revision of the accounting system. The two may not, and typically will not, be proportional because the accounting system processes a different, more restricted information set than the market. The proposition also notes proportionality can occur. Proportionality is unlikely, however, in the sense that it only holds for a "small" set of possible distributions. This is consistent with recent empirical evidence in Freeman and Tse (1992) of a nonlinear relation between unexpected returns and unexpected earnings (also see Schroeder [1992] and Subramanyam [1993]).

In short, the price-earnings relation will be linear in our model if the nonaccounting information is null, as we then have full mark-to-market accounting with no information outside that provided by the accounting system. It will also be linear if π is cleverly and tightly restricted, as in the additive cash flow setting of Proposition 2. It might be nonlinear, but ordinal, as in Proposition 3 or the Bernoulli example. It also might be bizarre, as in our example with uniformly negative earnings response coefficients.²⁰

Finally, the price-earnings relation will be linear regardless of π if we drop eq. (A2) and put mark-to-market accounting in its place.

Proposition 4: (A1), $A_t(\bar{y}_t) = V_t(\bar{y}_t)$ and $e_t(\bar{y}_t) = A_t(\bar{y}_t) - A_{t-2}(\bar{y}_{t-2})$ imply $M_{t,t-1}(\bar{y}_t) = U_{t,t-1}(\bar{y}_t)$.

Thus, the price-earnings relation, taking on one obscure form or another in this setting, is driven by the use of the language of valuation coupled with a restrictive recognition rule. Remove the restrictive recognition rule and linearity results, given de facto mark-to-market accounting. But with the restrictive recognition rule, the price-earnings relation will typically be non-

20. Teets (1992) reports a number of simulations based on this example, and finds the negative price-earnings relationship occurs in a sizable percentage of the cases.

linear. This is the inevitable result of additional information being combined with the accounting information in a contextual, history-dependent manner. Recognition rules force a particular scaling of the information release, one that typically stands in the way of linearity.

6. Conclusion

Few subjects in accounting have received as much attention as the relation between stock prices and earnings. Most of this work is empirical; relatively few models exist. Further, the models tend to take accounting earnings literally as economic earnings, to assume they come in linear form, or to regard them as draws from a convenient distribution. This is somewhat in contrast to the empirical literature, which suggests the impact of accounting disclosures depends on context. The reasons are many, but scaling of the information variable called earnings is one. If we use all available information to scale the underlying information, we can rationalize a linear price-earnings relation. If we insist the scaling reflect restrictive recognition rules then, in general, we must use the full history of information to decode the accounting report. This drives the nonlinear relation in our model, as well as the suggestion price can be an important variable in decoding earnings.

There are many ways to expand and extend this work. Alternative descriptions of the accounting process, alternative information processing rules, and alternative valuation settings are possibilities. Perhaps the most attractive direction is to structure the information that is admitted in the accounting valuation. For example, formulating and applying concepts such as matching, conservatism, and selective mark-to-market might naturally restrict the relation between accounting and market values in interesting ways.

APPENDIX A: PROOFS

Proof of Proposition 1

We have shown in the text $M_{t,t-1}(\bar{y}_t) = y_t - E_{\pi}[y_t] = y_t^a - E_{\pi}[y_t^a]$ since t is even. We now show the result is true for unexpected earnings; the independence assumption implies the difference in conditioning variables between unexpected earnings and TS unexpected earnings is of no consequence.

$$\begin{aligned}
 A_t(\bar{y}_t^a) &= \sum_d d\pi^a (d|\bar{y}_t^a) (1 + r)^{t-T} \\
 &= \sum_{\tau=1}^t y_\tau + \sum_{\tau=t+1}^{T-1} E_\pi(y_\tau),
 \end{aligned}$$

by the additive cash flow setting (and its independence assumption). Similarly, $E[A_t(\bar{y}_t^a)|\bar{y}_{t-1}] = \sum_{\tau=1}^{t-1} y_\tau + \sum_{\tau=t}^{T-1} E_\pi(y_\tau)$. Therefore, we have $U_{t,t-1}(\bar{y}_{t-1}, y_t^a) = A_t(\bar{y}_t^a) - E[A_t(\bar{y}_t^a)|\bar{y}_{t-1}] = y_t - E_\pi[y_t^a]$ since t is even.

Proof of Lemma

Introducing conditional independence of the accounting and nonaccounting information into the definition of CMLRP directly simplifies to the conclusion $\pi^a(y_t^a|d, \bar{y}_{t-1}^a)$ satisfies CMLRP.

Proof of Proposition 2

Part 1. π^a possessing CMLRP and $U_{t,t-1}(\bar{y}_{t-1}, y_t^{a'}) > U_{t,t-1}(\bar{y}_{t-1}, y_t^a)$ imply $y_t^{a'} > y_t^a$. Using CMLRP yields $M_{t,t-1}(\bar{y}_{t-1}, y_t^{a'}) > M_{t,t-1}(\bar{y}_{t-1}, y_t^a)$ for all even t .

Part 2. π^a possessing CMLRP and $\hat{U}_{t,t-2}(\bar{y}_{t-2}, y_t^{a'}) > \hat{U}_{t,t-2}(\bar{y}_{t-2}, y_t^a)$ imply $y_t^{a'} > y_t^a$. Using CMLRP yields $M_{t,t-2}(\bar{y}_{t-1}, y_t^{a'}) > M_{t,t-2}(\bar{y}_{t-1}, y_t^a)$ for all even t .

Proof of Proposition 3

The condition follows immediately from the definitions. That the set of π 's satisfying this condition is a set of measure zero in the space of π 's is implied by the fact that the condition defines a linear restriction on that space.

Proof of Proposition 4

With the noted valuation rule, the unexpected earnings measure becomes $U_{t,t-1}(\bar{y}_t) = A_t(\bar{y}_t) - (1 + r)A_{t-1}(\bar{y}_{t-1})$. But this is simply $M_{t-1}(\bar{y}_{t-1})$.

APPENDIX B: ALTERNATIVE ACCOUNTING MEASUREMENT ASSUMPTIONS

The accounting measurement assumptions (A2) and (A3) reflect restrictive recognition rules and Bayesian revision. These two features are crucial to our analysis, because they affect the measurement scale with which in-

formation is conveyed by accounting earnings. Equations (A2) and (A3) also reflect unbiased accounting and clean surplus assumptions. We show in this appendix that unbiased accounting and clean surplus are not crucial, because they have no necessary effect on the information conveyed by accounting sources. In particular, biasing accounting and dirtying surplus only tend to alter the price-earnings relation, which we have already argued tends to be subtle. We also briefly discuss, in the spirit of Proposition 4, the effect of eliminating the assumptions of restrictive recognition rules and Bayesian revision.

We first consider the case of biased accounting. One way in which accounting could be biased is that it could use a higher (more "conservative") interest rate than the market. Denoting the accounting interest rate by r_a and the biased book value at date t by $A_t^B(\bar{y}_t^a)$, eq. (A2) translates into

$$A_t^B(\bar{y}_t^a) \equiv \hat{d}_t(\bar{y}_t^a) (1 + r_a)^{t-T}. \quad (\text{A2:B})$$

Biased earnings are the change in biased book value. Note that the biased book value is proportional to the original book value, with the factor of proportionality, $(1 + r)^{t-T}/(1 + r_a)^{t-T}$. By the same factor of proportionality, biased (TS) unexpected earnings are proportional to original (TS) unexpected earnings. Therefore, the information content of earnings is unaffected. The biased earnings response coefficients would also be proportional to the original earnings response coefficients by the reciprocal of the same factor of proportionality.

Second, we consider the case of "dirty surplus." We choose the case where transitory value changes bypass the income statement, so that changes in earnings are permanent. In particular, we assume that eq. (A2) holds and that accounting return on equity is the interest rate. Denoting the dirty surplus (DS) earnings by $e_t^{DS}(\bar{y}_t^a)$, eq. (A3) translates into

$$e_t^{DS}(\bar{y}_t^a) = rA_t(\bar{y}_t^a). \quad (\text{A3:DS})$$

Equation (A3:DS) evokes Black's (1980, 1991) suggestion to make price-to-earnings ratios less variable. Depending on the context, eq. (A3:DS) could be fairly descriptive of earnings. For example, with the new post-retirement health benefits statement SFAS No. 106, firms have the option of immediately expensing the full amount of the liability (as in eq. [A3]) or expensing the liability over 20 years (more like eq. [A3:DS]).

Unexpected dirty surplus earnings equal the original unexpected earnings times the interest rate r . The same is true for the TS counterparts. Thus, the information content of unexpected earnings is again unaffected.

If we abandon restrictive recognition rules but hold to Bayesian revision, book value and earnings properly reflect all information. Book value then equals market value. Unexpected earnings equal unexpected capital gain in even periods. Earnings response coefficients are equal to one. This is the content of Proposition 4.

Almost anything can happen when we abandon Bayesian revision, because earnings are not constrained to reflect information in a consistent way. A particularly simple example of non-Bayesian revision is a stylized historical cost accounting system where the reported book value is a weighted average of current and prior original book values as given by eq. (A2). Because it would be possible to unravel reported earnings to uncover the original earnings, the information content of earnings is not affected by these measurement procedures. Earnings response coefficients would tend to rise toward $1/r$, however, because earnings reflect only a fraction of the value change implied by the current accounting information realization. This case is similar in many regards to the described case of dirty surplus.

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