# 1 Adverse selection treatment effects

We consider two variations on the analysis of treatment effects associated with adverse selection. For simplicity, we consider two managerial types, 0 and 1.

## 1.1 case one: manager type known

For case one, the manager's type is known but it is unclear what the impact of managerial type is on outcome (welfare). Here we ask what is the treatment effect of type 1.

$$Y_1 - Y_0$$

As is typical, we cannot assess this via data for an individual but with a cross-section of evidence we can pose the question based on population parameters, say the mean. We can ask the counterfactual question for those who chose type 1, where $D$ denotes treatment choice

$$ATT = E\left[Y_1 - Y_0 \mid D = 1\right]$$

or for those who chose type 0 (we'll refer to this choice as untreated)

$$ATUT = E\left[Y_1 - Y_0 \mid D = 0\right]$$

or for a random draw from the population

$$\begin{aligned} ATE &= E\left[Y_1 - Y_0\right] \\ &= \Pr\left(D = 1\right) E\left[Y_1 - Y_0 \mid D = 1\right] \\ &\quad + \Pr\left(D = 0\right) E\left[Y_1 - Y_0 \mid D = 0\right] \end{aligned}$$

## 1.2 case two: manager type unknown (empire building)

For the second case, the manager's type is unknown but the manager's productive decision (for simplicity, consider the decision to be binary, 0 or 1) and the related outcome is observed. Here, we can ask the impact of the decision on outcome (welfare). Of course, this is counterfactual in nature.

$$Y_1 - Y_0$$

Once again we cannot assess this at the individual level but we may be able with cross-sectional evidence to infer population parameters. Similar reasoning to case one yields analogous average treatment effects. For the subpopulation who chose decision 1 we have the average treatment effect on the treated

$$ATT = E\left[Y_1 - Y_0 \mid D = 1\right]$$

for those who chose decision 2 we have the average treatment effect on the untreated

$$ATUT = E\left[Y_1 - Y_0 \mid D = 0\right]$$

and for one chosen at random we have the average treatment effect

$$\begin{aligned} ATE &= E[Y_1 - Y_0] \\ &= \Pr(D=1)\, E[Y_1 - Y_0 \mid D=1] \\ &\quad + \Pr(D=0)\, E[Y_1 - Y_0 \mid D=0] \end{aligned}$$

Also, treatment effects supply indirect evidence on the adverse selection issue. Suppose potential outcome $Y_1$ is a measure of potential (organizational) welfare for the optimal decision and $Y_0$ is welfare for the suboptimal decision, then positive treatment effects for the untreated (those making suboptimal decisions), $(Y_1 - Y_0 \mid D = 0) > 0$, is evidentiary support of empire building managerial type.[1] Further, positive treatment effects for the treated supports a value-enhancing managerial type. However, these inferences are highly dependent on the measure of outcome as discussed next in more detail.

## 1.3 outcome is crucial to performance evaluation

For any (binary) treatment effect analysis, identification depends on potential outcomes with and without treatment. Assessing welfare implications via treatment effects depends critically on collecting appropriate outcome (welfare) data. This is a significant step.

Choice or design of a performance measure is a primary contribution of accounting to performance evaluation. A performance measure is employed based on its informativeness about the manager and the incentives supplied for the manager to enhance organizational welfare. The importance of the outcome or welfare measure seems too often understated. The discussion below focuses on interpretation of the evidence given adequate attention is given to the choice of outcome as an organizational welfare measure. However, it's prudent to bear in mind, potential outcome is a key component of the experiment.

Next, we posit different data generating processes ($DGP$) and ask what implications can be inferred from experimental evidence. For case one, we explore performance (outcome) effects of managerial type directly. While for case two, we explore performance (outcome) effects of decisions by managers of unknown type. In this case, the treatment effect may supply indirect evidence on unknown managerial type.

## 1.4 $DGP_1$: homogeneous outcomes

Homogeneity implies ignorable treatment. In other words, conditional on the regressors (or covariates) treatment is mean independent of potential outcomes with and without treatment, $E[Y_1 \mid X, D] = E[Y_1 \mid X]$ and $E[Y_0 \mid X, D] = E[Y_0 \mid X]$. Hence,

[1] This may also indicate the manager's performance measures are incongruous with organizational welfare. In either case, managerial control is a focal concern. However, if outcome is an incomplete measure of organizational welfare then interpretation of the evidence is more ambiguous.

$$E[Y_1 - Y_0 \mid X] = E[Y_1 - Y_0 \mid X, D]$$

When we speak of homogeneity (or heterogeneity, see discussion below) we refer to the unobservables, say $U_1$ and $U_0$. Homogeneity implies the unobservables are drawn from the same distribution for all organizations and/or subpopulations. For instance, $U_i$ and $U_i \mid D$ for $i = 0, 1$ have the same distribution. Therefore,

$$Y_i = \mu_i(X) + U_i, \quad i = 0, 1$$

implies

$$\begin{aligned} E[Y_1 - Y_0 \mid X] &= E[\mu_1(X) - \mu_0(X) + U_1 - U_0 \mid X] \\ &= \mu_1(X) - \mu_0(X) + E[U_1 - U_0 \mid X] = \alpha(X) \end{aligned}$$

and because of homogeneity this remains true for estimands (like means and quantiles) of subpopulations as well

$$\begin{aligned} E[Y_1 - Y_0 \mid X, D] &= E[\mu_1(X) - \mu_0(X) + U_1 - U_0 \mid X, D] \\ &= \mu_1(X) - \mu_0(X) + E[U_1 - U_0 \mid X, D] = \alpha(X) \end{aligned}$$

Full common support yields the unconditional average treatment effect via iterated expectations.

$$E_X[E[Y_1 - Y_0 \mid X]] = E[Y_1 - Y_0] = \alpha$$

In this case, average treatment effects are nonparametrically point-identified. Suppose the $DGP$ is

| $Y_1$ | $Y_0$ | $TE$ | $Y$ | $D$ | $X$ |
|---|---|---|---|---|---|
| 4 | 2 | 2 | 4 | 1 | 1 |
| 2 | 1 | 1 | 2 | 1 | 1 |
| 0 | 0 | 0 | 0 | 1 | 1 |
| 4 | 2 | 2 | 2 | 0 | 1 |
| 2 | 1 | 1 | 1 | 0 | 1 |
| 0 | 0 | 0 | 0 | 0 | 1 |
| 2 | 1 | 1 | 2 | 1 | 0 |
| 0 | 0 | 0 | 0 | 1 | 0 |
| −2 | −1 | −1 | −2 | 1 | 0 |
| 2 | 1 | 1 | 1 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 |
| −2 | −1 | −1 | −1 | 0 | 0 |

$DGP_1$ : homogeneous outcomes

Homogeneity is strongly evidenced.

| | conditional | | unconditional |
|---|---|---|---|
| estimand | $X=1$ | $X=0$ | $E_X\left[E\left[Y_1-Y_0 \mid X\right]\right]$ |
| $ATT\left(X\right)=$ $E\left[Y_1-Y_0 \mid X, D=1\right]$ | 1 | 0 | $\frac{1}{2}$ |
| $ATUT\left(X\right)=$ $E\left[Y_1-Y_0 \mid X, D=0\right]$ | 1 | 0 | $\frac{1}{2}$ |
| $ATE\left(X\right)=$ $E\left[Y_1-Y_0 \mid X\right]$ | 1 | 0 | $\frac{1}{2}$ |
| $OLS\left(X\right)=E\left[Y_1 \mid X, D=1\right]$ $-E\left[Y_0 \mid X, D=0\right]$ | 1 | 0 | $\frac{1}{2}$ |

various average treatment effects

The final row of the table indicates treatment is ignorable or unconfounded as potential outcomes ($Y_1$ and $Y_0$) are mean conditional independent and identification is not sacrificed if the counterfactual nature of treatment effects is ignored.

The case 1 interpretation of these data is that type 1 managers are organizational welfare-enhancing (relative to type 0). While the case 2 interpretation of these data is that the organization benefits from decision 1 (relative to decision 0) and suggests those managers adopting decision 0 are engaging in empire building rather than pursuing organizational welfare.

## 1.5 $DGP_2$: heterogeneous outcomes $IV$ point-identified

In contrast to the homogeneous case above, here we entertain unobservable outcome heterogeneity. That is, $U_1$ and $U_0$, are heterogeneous or are drawn from potentially different distributions across organizations and/or subpopulations. Since $U_i$ may differ in distribution from $U_i \mid D$ for $i=0,1$, estimands of interest (say, means and quantiles) may differ between subpopulations. For instance, $E\left[Y_1-Y_0\right]$ and $E\left[Y_1-Y_0 \mid D=d\right]$ (or $Q_\theta\left[Y_1\right]-Q_\theta\left[Y_0\right]$ and $Q_\theta\left[Y_1 \mid D=d\right]-Q_\theta\left[Y_0 \mid D=d\right]$) are not necessarily equal and may, in fact, diverge substantially.

A binary instrument $Z$ point identifies $LATE$ and $QTE\left(\theta \mid D_1>D_0\right)$.[2]

$$LATE=E\left[Y_1-Y_0 \mid D_1-D_0=1\right]$$

and

$$QTE\left(\theta \mid D_1>D_0\right)=Q_\theta\left[Y_1 \mid D_1-D_0=1\right]-Q_\theta\left[Y_0 \mid D_1-D_0=1\right]$$

where $Q_\theta\left[\cdot\right]$ is the $\theta$-quantile for the random variable.

[2] Point identification of $QTE\left(\theta\right)$ also requires the $\theta$-quantiles for $Y_1$ and $Y_0$ to be unique.

Suppose the $DGP$ is

| $Y_1$ | $Y_0$ | $TE$ | $Y$ | $D$ | $Z$ |
|---|---|---|---|---|---|
| 3 | 0 | 3 | 3 | 1 | 1 |
| 0 | 0 | 0 | 0 | 1 | 1 |
| −1 | −1 | 0 | −1 | 1 | 1 |
| 2 | 1 | 1 | 2 | 1 | 1 |
| 0 | 0 | 0 | 0 | 1 | 1 |
| −2 | −1 | −1 | −1 | 0 | 1 |
| 3 | 0 | 3 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 | 1 | 0 |
| −1 | −1 | 0 | −1 | 1 | 0 |
| 2 | 1 | 1 | 1 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 |
| −2 | −1 | −1 | −1 | 0 | 0 |

$DGP_2:$ heterogeneous outcomes

These data are quite heterogeneous with a variety of average and quantile treatment effects.

| | |
|---|---|
| $ATT =$ | |
| $E[Y_1 - Y_0 \mid D = 1]$ | 0.5714 |
| $ATUT =$ | |
| $E[Y_1 - Y_0 \mid D = 0]$ | 0.4 |
| $ATE =$ | |
| $E[Y_1 - Y_0]$ | 0.5 |
| $LATE =$ | |
| $E[Y_1 - Y_0 \mid D_1 > D_0]$ | $\frac{4}{3}$ |
| $OLS = E[Y_1 \mid D = 1]$ | |
| $-E[Y_0 \mid D = 0]$ | 0.6286 |

various average treatment effects

| | $QTE(\theta \mid D_1 > D_0) =$ |
|---|---|
| | $Q_\theta[Y_1 \mid D_1 > D_0]$ |
| $\theta$ | $-Q_\theta[Y_0 \mid D_1 > D_0]$ |
| 0.25 | 0 |
| 0.5 | 2 |
| 0.75 | 2 |

various quantile treatment effects

Ignoring the counterfactual nature (the $OLS$ effect) results in overstatement of the average treatment effect. The binary instrument allows identification of quantile treatment effects and the local average treatment effect for the subpopulation of compliers — in this case, $LATE$ is much larger than other average treatment effects.

$LATE$ is identified via *2SLS-IV*

$$E[Y_1 - Y_0 \mid D_1 > D_0] = \frac{E[Y \mid Z = 1] - E[Y \mid Z = 0]}{E[D \mid Z = 1] - E[D \mid Z = 0]}$$

or

$$E\left[Y_1 - Y_0 \mid D_1 > D_0\right] = \alpha$$

identified via

$$(\mu, \alpha) = \arg\min_{m,a} E\left[\kappa \cdot \psi\right]$$

where[3]

$$\kappa = 1 - \frac{D\left(1 - Z\right)}{\Pr\left(Z = 0\right)} - \frac{Z\left(1 - D\right)}{\Pr\left(Z = 1\right)}$$

and

$$\psi = \left(Y - m - aD\right)^2$$

Interpretation is similar to that for the homogeneous case ($DGP_1$) except there is greater outcome variation across organizations and there are some organizations for whom type 1 or decision 1 is not preferred (data points 6 and 12). Even though means and medians suggest treatment leads to improved organizational performance, one must look to lower quantiles to detect counter evidence. Unfortunately, the evidence based on the subpopulation of compliers simply indicates no advantage to treatment (type 1 or decision 1) at lower quantiles.[4]

## 1.6 $DGP_3$: *MTR* partially-identified treatment effects

We continue the theme of potentially heterogeneous outcomes but relax (point) identification conditions. In particular, we address the case where a stochastically independent instrument, $Z$, is not identified. Monotone treatment response ($MTR$) says $Y_{1j} \geq Y_{0j}$ for all individuals $j$ where $Y_{1j}$ refers to potential outcome when optimal decisions for organization $j$ are implemented and $Y_{0j}$ refers to potential outcomes when suboptimal decisions for organization $j$ (such as empire building) are implemented. $MTR$ implies $ATE$ and $QTE\left(\theta\right)$ are partially identified.

Partial identification of the mean based on $MTR$ results in the following identification bounds for the $ATE$

$$\begin{aligned} 0 \quad \leq \quad & E\left[Y_1 - Y_0\right] = E\left[Y_1\right] - E\left[Y_0\right] \leq \\ & \Pr\left(D = 1\right) E\left[Y \mid D = 1\right] + \Pr\left(D = 0\right) y_{\max} \\ & - \left\{\Pr\left(D = 0\right) E\left[Y \mid D = 0\right] + \Pr\left(D = 1\right) y_{\min}\right\} \end{aligned}$$

where $Y = DY_1 + \left(1 - D\right) Y_0$ is observed outcome and $y_{\min}$ and $y_{\max}$ are the population extreme values for potential outcome.

[3] The intercept identified via $E\left[\kappa\psi\right]$ is $\mu = E\left[Y_0 \mid D_1 > D_0\right]$ not the intercept identified via *2SLS-IV*.

[4] Notice, any quantile less than $\frac{1}{3}$ is the same for the subpopulation of compliers. The reversal of the treatment effect resides with the subpopulation of never adopters so this identification strategy fails to detect it.

Partial identification of the $\theta$-quantile based on $MTR$ results in the following two part identification bounds for the $QTE(\theta) = Q_\theta[Y_1] - Q_\theta[Y_0]$

$$\begin{aligned}
&\text{if } 0 < \theta \leq \Pr(D=1), \quad 0 \leq QTE(\theta) \leq Q_\theta[Y \mid D=1] - y_{\min} \\
&\text{if } \Pr(D=1) < \theta < 1, \quad 0 \leq QTE(\theta) \leq y_{\max} - Q_\theta[Y \mid D=0]
\end{aligned}$$

As a baseline for the $MTR$ identification strategy, consider the identification bounds for the mean and quantile treatment effects that can be gleaned from the data alone.

Bounds for the average treatment effect based on the data alone are

$$\begin{aligned}
& \Pr(D=1) E[Y \mid D=1] + \Pr(D=0) y_{\min} \\
& - \{\Pr(D=0) E[Y \mid D=0] + \Pr(D=1) y_{\max}\} \\
\leq \quad & E[Y_1 - Y_0] = E[Y_1] - E[Y_0] \leq \\
& \Pr(D=1) E[Y \mid D=1] + \Pr(D=0) y_{\max} \\
& - \{\Pr(D=0) E[Y \mid D=0] + \Pr(D=1) y_{\min}\}
\end{aligned}$$

Bounds for quantile treatment effects, $QTE(\theta) \equiv Q_\theta[Y_1] - Q_\theta[Y_0]$, based on the data alone are

$$r(\theta, Y_1) - s(\theta, Y_0) \leq QTE(\theta) \leq s(\theta, Y_1) - r(\theta, Y_0)$$

where

$$\begin{aligned}
r(\theta, Y_0) &= \begin{cases} Q_{\frac{\theta - \Pr(D=1)}{\Pr(D=0)}}(Y \mid D=0) & \text{if } \Pr(D=1) < \theta \\ y_0 & \text{otherwise} \end{cases} \\
s(\theta, Y_0) &= \begin{cases} Q_{\frac{\theta}{\Pr(D=0)}}(Y \mid D=0) & \text{if } \Pr(D=1) < 1-\theta \\ y_1 & \text{otherwise} \end{cases} \\
r(\theta, Y_1) &= \begin{cases} Q_{\frac{\theta - \Pr(D=0)}{\Pr(D=1)}}(Y \mid D=1) & \text{if } \Pr(D=0) < \theta \\ y_0 & \text{otherwise} \end{cases} \\
s(\theta, Y_1) &= \begin{cases} Q_{\frac{\theta}{\Pr(D=1)}}(Y \mid D=1) & \text{if } \Pr(D=0) < 1-\theta \\ y_1 & \text{otherwise} \end{cases}
\end{aligned}$$

Suppose the $DGP$ is

| $Y_1$ | $Y_0$ | $TE$ | $Y$ | $D$ |
|---|---|---|---|---|
| 3 | 0 | 3 | 3 | 1 |
| 0 | 0 | 0 | 0 | 1 |
| −1 | −1 | 0 | −1 | 1 |
| 2 | 1 | 1 | 2 | 1 |
| 0 | 0 | 0 | 0 | 1 |
| −1 | −2 | 1 | −2 | 0 |
| 3 | 0 | 3 | 0 | 0 |
| 0 | 0 | 0 | 0 | 1 |
| −1 | −1 | 0 | −1 | 1 |
| 2 | 1 | 1 | 1 | 0 |
| 0 | 0 | 0 | 0 | 0 |
| −1 | −2 | 1 | −2 | 0 |

$DGP_3$: heterogeneous outcomes

These data are quite heterogeneous with a variety of average and quantile treatment effects.

| | |
|---|---|
| $ATT =$ | |
| $E[Y_1 - Y_0 \mid D = 1]$ | 0.5714 |
| $ATUT =$ | |
| $E[Y_1 - Y_0 \mid D = 0]$ | 1.2 |
| $ATE =$ | |
| $E[Y_1 - Y_0]$ | 0.8333 |
| $OLS = E[Y_1 \mid D = 1]$ | |
| $-E[Y_0 \mid D = 0]$ | 1.0286 |

various average treatment effects

| | $QTE(\theta) =$ |
|---|---|
| $\theta$ | $Q_\theta[Y_1] - Q_\theta[Y_0]$ |
| 0.1 | 1 |
| 0.25 | 0 |
| 0.5 | 0 |
| 0.75 | 2 |
| 0.9 | 2 |

various quantile treatment effects

Partial identification based on $MTR$ results in the following bounds for the mean treatment effect

$$0 \leq E[Y_1 - Y_0] \leq 2.9167$$

$ATE$ bounds based on $MTR$

and for quantile treatment effects

| $\theta$ | $Q_\theta\left[Y_1^{low}\right] - Q_\theta\left[Y_0^{upp}\right]$ | $Q_\theta\left[Y_1^{upp}\right] - Q_\theta\left[Y_0^{low}\right]$ |
|---|---|---|
| 0.25 | 0 | 1 |
| 0.5 | 0 | 2 |
| 0.75 | 0 | 3 |

quantile treatment effect bounds based on $MTR$

While partial identification based on the data alone results in the following bounds for the mean treatment effect

$$-2.0833 \leq E\left[Y_1 - Y_0\right] \leq 2.9167$$

$ATE$ bounds based on the data alone

and for quantile treatment effects

| $\theta$ | $Q_\theta\left[Y_1^{low}\right] - Q_\theta\left[Y_0^{upp}\right]$ | $Q_\theta\left[Y_1^{upp}\right] - Q_\theta\left[Y_0^{low}\right]$ |
|---|---|---|
| 0.25 | $-2$ | 2 |
| 0.5 | $-4$ | 5 |
| 0.75 | $-3$ | 4 |

quantile treatment effect bounds based on the data alone

Clearly, the data alone don't offer much help in narrowing the identification bounds for this $DGP$. On the other hand, if $MTR$ is credible it does significantly narrow the bounds. Further, $MTR$ relies less on credibility-challenging conditions than nonrefutable, point identification strategies like ignorable treatment or existence of a stochastically independent instrument. If $MTR$ is credible and potential outcomes effectively capture organizational welfare,[5] then the evidence can help to resolve questions involving adverse selection.

As the $DGP$ is consistent with $MTR$, interpretation is straightforward. For case one when managerial type is known, treatment effects being consistently positive indicates improved outcome (perhaps improved organizational welfare as well, for a sufficiently broad outcome measure) is enhanced by selection of type 1 managers. When managerial type is unknown as in case two, outcome is enhanced by decision 1. Further, a sufficiently broad outcome measure suggests those selecting decision 0 may be engaging in empire building. Of course, with treatment effect identification only bounded there will be added uncertainty regarding any conclusions from the data.

[5] These antecedent conditions are not taken lightly, rather implications the analyst draws from the evidence depend critically on their integrity.