Taxes and the Efficiency-Rent Extraction Trade-off

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Abstract

This paper presents an adverse selection model in which progressive taxation enhances productive efficiency by encouraging a principal (buyer) to be less aggressive in contracting with an agent (seller). Wary of padded cost budgets, the buyer employs a hurdle-rate procurement policy. With a low cost hurdle, the buyer keeps greater profits when transactions are undertaken but trade occurs less often. While the hurdle is unaffected by a flat tax, a progressive tax tilts the buyer's preference: the buyer's benefit from a lower hurdle becomes less pronounced, since the marginal increase in his profits is muted in after-tax terms. The result is increased trade and the possibility of Pareto improvements.

1. Introduction

Tax policies tend to be contentious. Proponents of different approaches heatedly argue the equality and efficiency implications of any proposed tax

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change. An early mention of these issues is found in Sidgwick (1883). He notes that redistribution policies that improve equality can reduce efficiency because they may lead workers to favor leisure to results of labor. Similar sentiments are expressed in Vickrey (1945, p. 330): "the question of the ideal distribution of income, and hence, of the proper progression of the tax system, becomes a matter of compromise between equality and incentives."

A seminal work on optimal taxation is Mirrlees (1971), which formulated the tax problem as one in which a government maximizes a welfare function subject to budget balancing and individuals' self-selection constraints. The optimal solution prescribes a zero tax rate at the highest income level ("no distortion at the top"), and thus the tax function is not progressive everywhere. The inconsistency of the optimal tax scheme with those observed in practice (where progressive taxes are common) was a surprise to many.[1]

In subsequent work, it has been shown that changing the form of the utility function (Diamond 1998) or adding income uncertainty (Varian 1980) can lead to tax rates that increase with income in a Mirrlees economy. Recently, more questions have been raised about the unambiguous efficiency loss associated with tax progressivity.[2] For example, in models of imperfectly competitive wage settings, progressivity in taxes can raise employment and welfare (Sorensen 1997, p. 227). The key is the wage formation process: if taxes on low-paid workers are cut, acceptable pretax wages can fall leading to increased employment. Keuschnigg and Nielsen (2002) study the welfare enhancing role of progressive taxes in a moral hazard setting. By inhibiting risk taking, progressive taxes convert risky income of entrepreneurs into safe income for both entrepreneurs and workers.[3]

In this paper, we present another setting in which progressive taxes prove helpful. In particular, we study a model of procurement in which diffuse information hampers efficient contracting. A seller, who is privately informed of his costs of production, is tempted to overstate costs in order to obtain higher prices from the buyer. In response, the buyer commits to rationing procurement when reported costs exceed a prespecified hurdle. Such rationing reflects a breakdown in negotiations, leading to the oft-studied inefficiencies wrought by adverse selection.

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[1]In fact, Mirrlees (1971, p. 207) writes: "[b]eing aware that many of the arguments used to argue in favor of low marginal tax rates for the rich are, at best, premissed on the odd assumption that any means of raising the national income is good, even if it diverts part of that income from poor to rich, I must confess that I had expected the rigorous analysis of income-taxation in the utilitarian manner to provide an argument for high tax rates. It has not done so."

[2]See Roed and Strom (2002) for a survey of these issues.

[3]Keuschnigg and Nielsen elegantly highlight the delicate relationship between taxes and welfare by varying the specification of the incentive problem. Results change depending on whether shirking by the entrepreneur is aimed at consuming extra outside income or toward leisure. See also Fellingham and Wolfson (1985) for a consideration of the effect of progressive taxes on risk sharing.

In our procurement setting, a taxing authority can enhance productive efficiency by imposing a progressive tax structure on the contracting parties. With progressive taxes, a buyer finds rationing purchases a less attractive means of reducing prices: any incremental income from reducing the cost hurdle is subject to a marginal tax rate above the firm's average tax rate, making the concern of foregone trade more pressing. In effect, while the production-rents tradeoff is pulled equally in both directions by flat taxes, progressive taxes introduce an imbalance favoring increased production.

An alternative but equivalent way of viewing the productivity-increasing aspect of progressive taxes is in terms of induced risk aversion. Under such taxes, an individual who is risk neutral in after-tax lotteries acts as if he is risk averse in pretax lotteries. The induced risk aversion tilts the buyer's preferences toward more frequent trade and a steady cash flow stream.

When the buyer is less aggressive in rationing resources, the increase in the "size of the pie" opens the door for Pareto improvements. We show that the expected tax receipts as well as the buyer's and seller's expected after-tax income can each be higher when a flat tax structure is made progressive.

In contrast to Mirrlees (1971), the tax authority in this paper plays a passive role. It simply announces a tax schedule and sets the stage under which firms subject to taxation negotiate contract (trade) terms. Roughly stated, the standard taxation problem (e.g., Laffont and Martimort 2002, pp. 291–294) deals with the detrimental effect of progressive taxes on the incentives of an individual to undertake personally costly actions vis-a-vis the taxing authority. This paper sidesteps such considerations (neither party undertakes costly actions), instead focusing on interactions among individuals under the auspices of the tax authority. Hence, one way to view the paper's contribution in a broader sense is that it highlights that a progressive tax system may have additional effects that improve productive efficiency when interactions among constituents are prominent.

In an extension section, as a robustness check, we study variants of the basic model. First, we show that similar gains to progressive taxation arise when the tax authority simultaneously confronts a wider constituency. Second, using a binary-type setup, we show the results also apply in standard adverse selection models wherein trade terms are richer (contracting entails more than a procure/not procure decision).

This paper proceeds as follows. Section 2 describes the model. Section 3 identifies the outcome under a flat tax structure. Section 4 demonstrates productive efficiency gains and the resulting Pareto improvements that arise under a judiciously chosen progressive tax plan. Section 5 considers two extensions which highlight the efficacy of the results under different modeling assumptions. Finally, Section 6 concludes.

2. Model

A risk-neutral buyer contracts with a risk-neutral seller for the supply of an intermediate product. The cost to the seller of producing the intermediate product is c, $c \in C = [c_L, c_H]$; without loss of generality, we set $c_L = 0$. The buyer converts the intermediate product into a final good which generates revenue of x for the buyer. For simplicity, assume the buyer's conversion cost is zero. Also, assume the transaction is profitable for the buyer even if the intermediate product is procured at the highest cost, $c_H < x$.

Prior to contracting, the seller privately learns c. While this cost is unobservable to the buyer, it is common knowledge that the probability density function and the cumulative distribution function of c are $f(c)$ and $F(c)$, respectively. Assume $f(c)$ is differentiable and $f(c) > 0$ for all c in C. As is standard in adverse selection models, the distribution is assumed to satisfy the monotone hazard rate condition $H'(c) \geq 0$, where $H(c) = F(c)/f(c)$.[4]

The tax schedule in place prescribes tax rates that are either constant (flat) or that are gradated as a function of pretax income levels (progressive or regressive). Under a progressive (regressive) tax structure, the marginal tax rate is at least weakly increasing (decreasing) in pretax income. At pretax income π, the marginal tax rate is denoted $\tau(\pi)$, $0 \leq \tau(\pi) \leq 1$. Hence, the after-tax income is $\int_0^\pi [1 - \tau(\omega)]\, d\omega = \pi[1 - \bar{\tau}(\pi)]$, where $\bar{\tau}(\pi)$, $0 \leq \bar{\tau}(\pi) < 1$, denotes the average tax rate.

The buyer commits to a menu of contracts $\{s(\hat{c}), d(\hat{c})\}$, $\hat{c} \in C$, where $s(\hat{c})$ is the payment to the seller and $d(\hat{c}) \in \{0, 1\}$, "1" denoting purchase, is the buyer's procurement decision as a function of the seller's cost report $\hat{c}$. Hence, given cost c and cost report $\hat{c}$, the buyer's pretax income (revenue less transfer) is $\pi_B(\hat{c}) = d(\hat{c})x - s(\hat{c})$, and the seller's pretax income (transfer less cost) is $\pi_S(\hat{c}, c) = s(\hat{c}) - d(\hat{c})c$. Each party makes decisions to maximize his expected after-tax income. Note, when collecting taxes, the tax authority is assumed to be able to identify the true cost.

From the revelation principle (Myerson 1979), in the search for the optimal mechanism, it is without loss of generality for the buyer to confine attention to an incentive compatible direct revelation mechanism. The direct revelation mechanism is a simple menu of contracts having as many options as the cardinality of the seller's type space (private information). The incentive compatibility feature refers to the fact that the menu is designed to provide the seller with incentives to report his type truthfully. In effect, the revelation principle offers the simplification that any equilibrium outcome under which the seller would opt to misstate his costs could be achieved equivalently while also eliciting truth: the buyer can commit to a menu that provides a truthful seller what he would have received had he lied.

In particular, given $\bar{\tau}(\pi)$, the buyer chooses procurement and transfer rules to maximize her expected after-tax income subject to individual

[4]In adverse selection models, the monotone hazard rate condition allows for global incentive compatibility constraints to be replaced by their local counterparts. See, for example, Laffont and Tirole (1994, pp. 63–69). The monotone hazard rate condition is satisfied by several distributions such as uniform, normal, logistic, chi-squared, exponential, and Laplace.

rationality (IR) and incentive compatibility constraints (IC). In the individual rationality constraints, the seller's reservation utility is normalized to zero. Also, the output feasibility constraints (OF) in the buyer's contracting program (P) state that the product is either purchased or not.

$$\text{(P)} \quad \underset{d(c),s(c)}{\text{Max}} \int_0^{c_H} \pi_B(c)[1-\bar{\tau}(\pi_B(c))]\,f(c)\,dc$$

subject to:

$$\pi_S(c,c)[1-\bar{\tau}(\pi_S(c,c))] \geq 0 \qquad \text{for all } c \in C \qquad \text{(IR)}$$

$$\pi_S(c,c)[1-\bar{\tau}(\pi_S(c,c))] \geq \pi_S(\hat{c},c)[1-\bar{\tau}(\pi_S(\hat{c},c))] \text{ for all } \hat{c}, c \in C \qquad \text{(IC)}$$

$$d(c) \in \{0,1\} \qquad \text{for all } c \in C \qquad \text{(OF)}$$

Given tax regime *m*, *m* = F (Flat), P (Prog.), or R (Regr.), Π_B^m, Π_S^m, and T^m, respectively, denote the expected after-tax income for the buyer and the seller, and expected taxes under the solution to (P).

3. The Optimal Contract with A Flat Tax

The proposition below characterizes the solution to (P) both with and without the incentive compatibility constraints. In the absence of the (IC) constraints, (P) yields the first-best (full information) solution. Otherwise, the solution is second-best.

PROPOSITION 1:

(i) *In the first-best setting:* $d(c) = 1$ *and* $s(c) = c$ *for all c.*

(ii) *In the second-best setting:* $d(c) = 1$ *and* $s(c) = k^*$, *for* $c \leq k^*$,
$d(c) = 0$ *and* $s(c) = 0$, *otherwise,*
where $k^* \in \underset{k \in C}{\text{argmax}}\, F(k)[x-k][1-\bar{\tau}(x-k)]$.

Proof:

(i) In the event of purchase, the seller is paid precisely c which satisfies his (IR) constraint as an equality. Further, since $x > c_H$ (and $\bar{\tau} < 1$), the buyer always prefers to purchase.

(ii) When c is privately observed by the seller, the seller submits $\hat{c}$ to maximize his expected after-tax income. Under any tax structure, the individual's after-tax income is increasing in his pretax income. The monotonic relationship implies the seller's problem is equivalent to choosing $\hat{c}$ to maximize pretax income.

Suppose $c'' > c'$. From the (IC) constraints:
$s(c'') - d(c'')c'' \geq s(c') - d(c')c''$, and
$s(c') - d(c')c' \geq s(c'') - d(c'')c'$.

The above inequalities together imply:
$c''[d(c'') - d(c')] \leq s(c'') - s(c') \leq c'[d(c'') - d(c')]$.

Since $c'' > c'$, the only way to satisfy the above is to set $d(c'') \leq d(c')$: if the product is purchased when $\hat{c} = c''$ it is also purchased for all $\hat{c} < c''$. This proves the hurdle nature of the procurement decision.

Given the hurdle rule, the (IR) constraints imply $s(c) \geq k$ when the product is purchased, and $s(c) \geq 0$ otherwise. The buyer minimizes the transfers by setting them at their lowest permissible values. This implies the buyer's expected pretax income is $F(k)[x - k]$. The problem, hence, reduces to solving for k^*, where k^* is the k-value that maximizes the buyer's expected after-tax income.

This completes the proof of Proposition 1. ■

Note, the hurdle rate characterization implies the problem is equivalent to one in which the buyer announces a purchase price of k and the seller decides whether or not to produce the good (as in, for example, Antle and Eppen 1985). This paper's focus is on the effect of taxation schemes on the outcome detailed in the proposition. In particular, our interest is in the role of graduated tax rates in the second-best setting. Hence, we start our analysis with the benchmark of flat taxes, i.e., $\bar{\tau}(\cdot) = \tau_F$, $0 < \tau_F < 1$, in Proposition 1.

COROLLARY: *Under flat taxes,* $k^* = k_F$, *where*
If $x - c_H - H(c_H) \geq 0$, $k_F = c_H$;
Else, k_F *is the unique k-value that solves* $x - k - H(k) = 0$.

Proof: Under flat taxes, the buyer chooses k to maximize $\Pi(k) = F(k)[x - k][1 - \tau_F]$. This implies $\Pi'(k) = f(k)g(k)[1 - \tau_F]$, where $g(k) = x - k - H(k)$. The maximum value of $\Pi(k)$ can occur at interior points where $\Pi'(k) = 0$, or at the endpoint c_H. Since $f(k) > 0$ and $\tau_F < 1$, the interior points where the derivative is zero correspond to the roots of $g(k) = 0$. Further, since $g(c_L) = x > 0$ and $g'(k) = -1 - H'(k) < 0$, the sign of $g(c_H)$ determines the number of roots to $g(k) = 0$.

If $g(c_H) < 0$, $g(k)$ has exactly one root, say k_r, and $k_r < c_H$. This root is a local maximum of $\Pi(k)$: $\Pi''(k_r) = -f(k_r)[1 + H'(k_r)][1 - \tau_F] < 0$. Since k_r is the only root of $g(k)$, there are no other local maximum, minimum, or inflection points. This implies $\Pi(k_r)$ is greater than its value at endpoints. Hence, when $g(c_H) = x - c_H - H(c_H) < 0$, $k_F = k_r$.

If $g(c_H) = 0$, then c_H is the unique root of $g(k) = 0$ and, using the above arguments, $k_F = c_H$. If $g(c_H) > 0$, then $g(k) = 0$ has no root. In this case, k_F is the endpoint c_H. ■

Figure 1 graphically illustrates the result in the corollary. In the first-best setting, the buyer benefits from a higher cost hurdle (k) as long as the revenues from procurement exceed the hurdle, or $x - k > 0$. Since

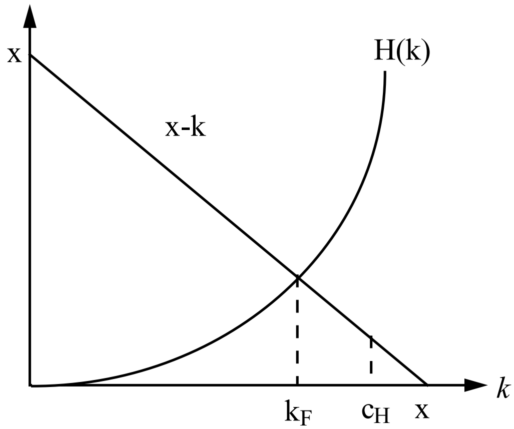

Figure 1: The solution under flat taxes.

$c_H < x$, this implies the first-best hurdle is set at the maximum and trade always occurs. In the second-best setting, private information provides the seller with an opportunity to overstate the product cost. By curtailing procurement for high cost reports, the buyer can curb this tendency. Now the buyer benefits from a higher k only if $x - k > H(k)$, where $H(k)$ reflects the information rents provided to the seller when the hurdle is marginally increased.[5] If the linearly decreasing $x - k$ function intersects the increasing $H(k)$ function to the left of c_H (as in Figure 1), the intersection point is k_F. If the curves intersect at c_H or to the right of c_H, then $x - k \geq H(k)$ for all $k \in [0, c_H]$, implying $k_F = c_H$, i.e., the buyer always procures.

Note, the cost hurdle with flat taxes is the same as when taxes are absent. This is in much the same spirit as the result that flat taxes have no bearing on break-even calculations in traditional accounting cost-volume-profit (CVP) analysis. However, the cost hurdle is affected by taxes when the tax schedule does not impose the same proportionate penalty at all income levels. This has efficiency and distributive implications as highlighted in the next section.

4. Benefits of Progressive Taxes

4.1. Progressive Taxes and Productive Efficiency

As is widely recognized, and as shown in the previous section, information asymmetry problems can lead to productive inefficiencies. In particular, in our procurement setting with flat taxes, such inefficiencies arise if and only if $x - c_H - H(c_H) < 0$, i.e., when $k_F < c_H$ in the corollary. The following

[5]Using integration by parts, the seller's expected information rents $\int_0^k (k - c)\, f(c)\, dc$ equal $\int_0^k H(c)\, f(c)\, dc$.

proposition states that under these conditions, a shift to progressive taxes enhances productive efficiency.

PROPOSITION 2:

(*i*) *Any progressive tax weakly improves productive efficiency relative to a flat tax.*

(*ii*) *If* $x - c_H - H(c_H) < 0$, *then a tax that is strictly progressive strictly improves productive efficiency.*

Proof: The buyer's expected after-tax income $\Pi(k)$ is $F(k)[x-k][1-\bar{\tau}(x-k)$. Hence,

$$\Pi'(k) = f(k)[(x-k-H(k))(1-\bar{\tau}(x-k)) + H(k)(x-k)\bar{\tau}'(x-k)].$$

(i) From the corollary, $x - k - H(k) \geq 0$ for all $k \in [0, k_F]$. Hence, under any progressive tax ($\bar{\tau}'(\pi) \geq 0$), $\Pi'(k) \geq 0$ for all $k \in [0, k_F]$. Thus the cost hurdle with a progressive tax is no less than with a flat tax.

(ii) Under a strictly progressive tax schedule, $\bar{\tau}'(\pi) > 0$ and, hence, $\Pi'(k) > 0$ for all $k \in [0, k_F]$. The result then holds as long as $k_F < c_H$, so there is room to increase k. Under the condition, $x - c_H - H(c_H) < 0$, this is indeed the case. ■

To see the productivity-enhancing role of progressive taxes consider an example wherein c is uniform in $[0, 1]$, so $H(c) = c$, and $x = 1.5$. From the corollary, under a flat tax, trade occurs only for $c \leq 3/4$; the hurdle of $3/4$ is the k-value that solves $x - k - H(k) = 0$. Now consider a shift to a progressive tax where the average tax rate is linearly increasing in pretax income: $\bar{\tau}(\Pi) = 1/2 + (1/8)\pi$. In the progressive tax case, the buyer selects k to maximize $k[3/2 - k][1 - 1/2 - 1/8(3/2 - k)]$, yielding the optimal k value of $5/6$. That is, the shift to progressive taxes results in increased trade. For $3/4 < c \leq 5/6$, trade occurs only under the progressive tax.

Now consider the sharing of productive efficiency gains. With a flat tax of $\tau_F = 3/5$, the buyer's expected after-tax income is $(3/4)(3/2 - 3/4)(1 - 3/5) = 0.225$, the seller's expected after-tax income is $\int_0^{3/4}(3/4 - c)(1 - 3/5)\,dc = 0.1125$, and the tax authority's expected collections are $(3/4)(3/2 - 3/4)(3/5) + \int_0^{3/4}(3/4 - c)(3/5)\,dc = 0.50625$. With the progressive tax and the revised hurdle of $5/6$, similar calculations yield expected after-tax income of 0.2315 for the buyer, 0.1495 for the seller, and tax collections of 0.5218. In the example, progressive taxes not only promote efficiency but also lead to a Pareto improvement.

The example raises several questions. First, while progressive taxes generally lead to productive efficiency gains, can they also be designed to ensure Pareto improvements? Second, can progressive taxes that increase the tax rates in steps rather than continuously be used to achieve both productive efficiency and Pareto improvements? Finally, does a change to a piecewise linear tax structure entail any loss of generality? We turn to these issues in the next subsection.

4.2. Progressive Taxes and Pareto Gains

Consider the case of uniformly distributed cost. The uniform cost assumption permits a simple closed form representation of the optimal hurdle. Also, as it turns out, in the uniform case, an interior cost hurdle condition is not only necessary but is also sufficient for progressive taxes to (Pareto) dominate flat taxes. The construction of such a progressive tax schedule is detailed in the next proposition.

PROPOSITION 3: *With uniformly distributed costs, flat taxes are dominated if and only if $x < 2c_H$. If $x < 2c_H$, for any τ_F, $0 < \tau_F < 1$, a Pareto improvement is obtained by switching to the following progressive tax schedule.*

$\tau_F \leq \tau_F^* = \left[\frac{2c_H}{x} - 1\right]^2$	$\tau_F > \tau_F^*$
$\tau(\pi) = 0$, *if* $\pi \leq \pi^* = 0.5x[1 - \sqrt{\tau_F}]$, $= 1$, *otherwise.*	$\tau(\pi) = \tau^* = \frac{\tau_F - \tau_F^*}{1 - \tau_F^*}$, *if* $\pi \leq x - c_H$, $= 1$, *otherwise.*

Proof: From the proof of Proposition 2, $x - c_H - H(c_H) < 0$ is a necessary condition for flat taxes to be dominated. In the uniform case, $H(c) = c$. Thus, the necessary condition reduces to $x < 2c_H$. To argue sufficiency, the proof proceeds in three steps.

Step 1. Consider the flat tax case. When $x < 2c_H$, from Proposition 1(c), the optimal cost hurdle is the solution to $x - k - H(k) = 0 \Rightarrow k_F = 0.5x$. Under this hurdle:

$$\Pi_B^F = F(k_F)(x - k_F)(1 - \tau_F) = \frac{x^2(1 - \tau_F)}{4c_H},$$

$$\Pi_S^F = \int_0^{k_F} (k_F - c)(1 - \tau_F) f(c)\, dc = \frac{x^2(1 - \tau_F)}{8c_H}, \quad \text{and}$$

$$T^F = F(k_F)(x - k_F)\tau_F + \int_0^{k_F} (k_F - c)\tau_F f(c)\,dc = \frac{3x^2\tau_F}{8c_H}.$$

Step 2. Consider the progressive tax schedule for $\tau_F \le \tau_F^*$. In this case, the optimal hurdle, k^*, is $x - \pi^* = 0.5x[1 + \sqrt{\tau_F}]$.

The argument for the hurdle choice is as follows. For all k, $k \in [0, k^*)$, the buyer purchases the product less often than under k^*, and when he does purchase the product he receives the same after-tax income $x - k^* = \pi^*$; the additional pretax income of $k^* - k$ is paid out entirely in taxes (the marginal tax rate on income exceeding π^* is 1).

For all k, $k \in (k^*, c_H]$, the buyer's marginal tax rate is the same as at k^*. Further, the buyer's expected pretax income $F(k)(x - k)$ reaches a maximum at $k = 0.5x$ and is decreasing in k thereafter. Since $k^* \ge 0.5x$, the buyer's expected pretax (and, hence, after-tax) income is greater at k^* than at any k, $k \in (k^*, c_H]$.

Using $\pi^* = 0.5x[1 + \sqrt{\tau_F}]$, $k^* = 0.5x[1 + \sqrt{\tau_F}]$ and noting $k^* \le c_H$ (equal if $\tau_F = \tau_F^*$), straightforward algebra can be used to verify the following relationships.

$$\Pi_B^P = F(k^*)(x - k^*) = \Pi_B^F,$$

$$\begin{aligned}\Pi_S^P &= \int_0^{k^*-\pi^*} \pi^* f(c)\,dc + \int_{k^*-\pi^*}^{k^*} (k^* - c) f(c)\,dc \\ &= \Pi_S^F + \frac{x^2\sqrt{\tau_F}[1 - \sqrt{\tau_F}]}{4c_H} > \Pi_S^F,\end{aligned}$$

$$T^P = \int_0^{k^*-\pi^*} (k^* - c - \pi^*) f(c)\,dc = T^F + \frac{x^2\tau_F}{8c_H} > T^F.$$

Under the proposed progressive taxes, the buyer's payoff is the same as under flat taxes, while the seller and the tax authority are strictly better off.

Step 3. Consider the tax schedule for $\tau_F > \tau_F^*$. In this case, the buyer sets the hurdle at k^*, $k^* = c_H$. The argument for this is as before: the optimal cost hurdle equals x less the upper bound of the lower income bracket. Earlier, the bound was π^*; here, it is $x - c_H$. So, $k^* = x - (x - c_H) = c_H$. Recognizing $k^* = c_H$, it is easy to verify the following.

$$\Pi_B^P = (x - c_H)(1 - \tau^*) = \Pi_B^F,$$

$$\begin{aligned}\Pi_S^P &= \int_0^{2c_H - x} (x - c_H)(1 - \tau^*) f(c)\, dc \\ &\quad + \int_{2c_H - x}^{c_H} (c_H - c)(1 - \tau^*) f(c)\, dc \\ &= \Pi_S^F \left(3 - \frac{x}{c_H}\right) > \Pi_S^F \text{ since } x < 2c_H,\end{aligned}$$

$$\begin{aligned}T^P &= \int_0^{2c_H - x} (2c_H - x - c) f(c)\, dc + \int_0^{2c_H - x} (x - c_H)\tau^* f(c)\, dc \\ &\quad + \int_{2c_H - x}^{c_H} (c_H - c)\tau^* f(c)\, dc + (x - c_H)\tau^* \\ &= T^F + \frac{(2c_H - x)}{8c_H^2}\left[x^2\left(\tau_F - \tau_F^*\right) + c_H(2c_H - x)\right] > T^F \\ &\quad \text{since } x < 2c_H \text{ and } \tau_F > \tau_F^*.\end{aligned}$$

Again, under the proposed tax schedule, the buyer's payoff is the same as under flat taxes, while the seller and the tax authority are strictly better off. Finally, note for all $\tau_F \in (\tau_F^*, 1)$, $\tau^* \in (0, 1)$; the tax schedule in the right panel is progressive.

This completes the proof of Proposition 3. ■

Under Proposition 3's progressive tax schedule, the tax authority collects more taxes than under the flat tax while the buyer and the seller (i) play in accordance with Program (P) and (ii) receive at least as much expected after-tax income as under the flat tax; neither the buyer nor the seller has reasons to object to using the progressive rather than the flat tax. An upshot of this result is to ask what is the best the tax authority can do while ensuring the desired behavior satisfies (i) and (ii)?

A program that addresses the above issue is formulated in (P'). From Proposition 1(ii), Program (P) is equivalent to the buyer solving for a cost hurdle. This is represented by constraint (1) in (P'). The other two constraints in (P'), labeled (2) and (3), guarantee the buyer and seller are no worse off than with the flat tax. That is, the buyer and the seller's expected after-tax income is at least Π_B^F and Π_S^F, respectively. When $x < 2c_H$, Π_B^F and Π_S^F are identified in Step 1 of the proof of Proposition 3. When $x \geq 2c_H$, $k_F = c_H$ and, hence, $\Pi_B^F = (x - c_H)(1 - \tau_F)$ and $\Pi_S^F = 0.5c_H(1 - \tau_F)$.

Program (P') imposes no restrictions on the progressivity of the tax schedule or on the number of tiers used in constructing the schedule. We refer to the tax schedule $\bar{\tau}(\pi)$ that solves (P') as the optimal tax schedule.

$$(P') \qquad \underset{\substack{k \in C \\ \bar{\tau}(\cdot) \in [0,1)}}{\text{Max}} \quad F(k)[x - k]\bar{\tau}(x - k) + \int_0^k [k - c]\bar{\tau}(k - c) f(c)\, dc$$

subject to:

$$k \in \arg\max_{k' \in C} F(k')[x - k'][1 - \bar{\tau}(x - k')] \tag{1}$$

$$F(k)[x - k][1 - \bar{\tau}(x - k)] \geq \Pi_B^F \tag{2}$$

$$\int_0^k [k - c][1 - \bar{\tau}(k - c)] f(c) dc \geq \Pi_S^F \tag{3}$$

The next proposition identifies the nature of the tax scheme that solves Program (P').

PROPOSITION 4. *With uniformly distributed costs, the optimal tax schedule is*

(i) flat, $\bar{\tau}(\cdot) = \tau_F$, if $x \geq 2c_H$, and

(ii) a two tier progressive tax if $x < 2c_H$.

Proof: Consider the $x \geq 2c_H$ case. Clearly, $\bar{\tau}(\cdot) = \tau_F$ is feasible in (P'). From Proposition 3, it is also optimal.

Next, consider the $x < 2c_H$ case. For Pareto gains to arise, the optimal cutoff in (P') should be at least k_F. And, from the proof of Proposition 3, there exists a schedule that satisfies this and constraints (1)–(3). Hence, it is without loss of generality to confine the search for the optimal tax schedule to progressive taxes and to $k \geq k_F$.

The remainder of the proof argues that a two tier progressive tax schedule is the optimal tax structure to solve program (R), where (R) is a relaxed version of (P') from which (3) is dropped. Finally, we show that this tax schedule (and the associated cost hurdle) satisfies (3), and, thus, is also optimal in the more constrained program (P').

First, we confirm that the progressive tax schedule $\bar{\tau}_R(\cdot)$ and cost hurdle k_R, $k_R \geq 0.5x$, that optimize (R) can be represented by a two-tiered tax schedule in which the marginal tax rate for the higher income bracket is 1.

Construct the following two-tiered tax system:

$$\begin{aligned} \tau(\pi) &= \bar{\tau}_R(x - k_R), \text{ if } \pi < x - k_R, \\ &= 1, \text{ otherwise.} \end{aligned}$$

From the arguments in Step 2 of the proof of Proposition 3, it follows that under the two-tiered tax structure, the buyer chooses the cost hurdle k_R. The buyer's expected after-tax income is $F(k_R)[x - k_R][1 - \bar{\tau}_R(x - k_R)]$, which is the same as under $\bar{\tau}_R(\cdot)$ and k_R.

Also, at all income levels, the average tax rate under the two-tiered tax structure is either the same or higher than under $\bar{\tau}_R(\cdot)$. For $\pi \leq x - k_R$, this is guaranteed by the choice $\tau(\pi) = \bar{\tau}_R(x - k_R)$. Recall, $\bar{\tau}_R(\cdot)$ is progressive, and so $\bar{\tau}_R(x - k_R) \geq \bar{\tau}_R(\pi)$ for $\pi \leq x - k_R$. For $\pi > x - k_R$, this is guaranteed by the choice of $\tau(\pi) = 1$. This, coupled with the fact that k_R is implemented, implies the taxes collected under the two-tiered tax system can be no less than under the original $\bar{\tau}_R(\cdot)$ mechanism. Hence, if $\bar{\tau}_R(\cdot)$ is the optimal progressive tax schedule under (R), so is the two-tiered tax schedule.

Any such two tier structure that solves (R) is also the solution to (P'), as it satisfies (3). The seller's expected after-tax income at the solution is

$$\left[\int_0^{2k_R-x}[x-k_R][1/c_H]dc+\int_{2k_R-x}^{k_R}[k_R-c][1/c_H]dc\right][1-\bar{\tau}_R(x-k_R)]$$
$$=\frac{[3k_R-x][x-k_R][1-\bar{\tau}_R(x-k_R)]}{2c_H}.$$

The above is no less than $\frac{F(k_R)[x-k_R][1-\tau_R(x-k_R]}{2}$ since $k_R \geq 0.5x$. This, in turn, is no less than $\Pi_B^F/2$ since (2) is satisfied. Finally, by the Π_B^F and Π_S^F expressions in step 1 of the proof of Proposition 3, $\Pi_B^F/2=\Pi_S^F$. This completes the proof of Proposition 4. ■

In a discrete cost setup, the same forces are at work with one exception. The inability of the buyer to fine tune the cost hurdle—the hurdle jumps from one discrete cost value to another—limits the τ_F values for which flat taxes are dominated. This is proved in the next proposition for the binary cost case.

PROPOSITION 5: *Assume $c \in \{0, c_H\}$, with each cost equally likely. In this case, flat taxes are dominated by progressive taxes if and only if $x < 2c_H$ and $\tau_F \geq \frac{2c_H}{x}-1$.*

Proof: In the discrete setup, the optimal contract is again a hurdle cost contract. In the two-cost setup, this corresponds to either "Rationing" or "Slack." Under Rationing, the product is purchased if and only if the seller reports costs are low and, in which case, the transfer is 0. The buyer's expected pretax income is $0.5x$. Under Slack, the product is always purchased, and the transfer is c_H. The buyer's expected pretax income is $x-c_H$.

The proof first argues necessity. For Pareto gains to exist, under flat taxes, the buyer must strictly prefer Rationing to Slack (the cost hurdle must be interior):

$$0.5x(1-\tau_F)>(x-c_H)(1-\tau_F)\Rightarrow x<2c_H.$$

Also, for Pareto gains to arise, the contract under nonflat taxes must change to Slack. Hence, another necessary condition is the buyer's expected after-tax income under Slack, even assuming no taxes, is at least as much as under Rationing with flat taxes:

$$x-c_H \geq 0.5x(1-\tau_F)\Rightarrow \tau_F \geq \frac{2c_H}{x}-1.$$

The proof next argues sufficiency. Under flat taxes, it is already shown when $x < 2c_H$, the buyer offers Rationing. Hence, expected after-tax income for the buyer and the seller, and tax collections, equal:

$$\Pi_B^F = 0.5x(1-\tau_F),\ \Pi_S^F = 0, \quad \text{and} \quad T^F = 0.5x\tau_F.$$

Consider the following tax schedule:

$$\begin{aligned}\tau(\pi) = \tau_1 &= 1 - \frac{0.5x(1-\tau_F)}{x - c_H}, \quad \text{if } \pi \le x - c_H,\\ &= 1, \text{ otherwise.}\end{aligned}$$

Given the lower bound on τ_F, $0 \le \tau_1 < 1$. Under this progressive tax schedule, the buyer strictly prefers Slack to Rationing. Under Slack, her expected after-tax income is $(x - c_H)(1 - \tau_1)$. Under Rationing, her expected after-tax income is $0.5[(x - c_H)(1 - \tau_1) + c_H(1 - 1)] = 0.5(x - c_H)(1 - \tau_1)$.

Plugging for τ_1, the expected after-tax income for the buyer and the seller, and tax collections, under the proposed progressive taxes (with Slack contract) equal:

$$\begin{aligned}\Pi_B^P &= (x - c_H)(1-\tau_1) = 0.5x(1-\tau_F) = \Pi_B^F,\\ \Pi_S^P &= 0.5(x - c_H)(1-\tau_1) = 0.25x(1-\tau_F) > \Pi_S^F,\\ T^P &= (x - c_H)\tau_1 + 0.5(x - c_H)\tau_1 + 0.5[c_H - (x - c_H)]\\ &= 0.5x\tau_F + 0.25x\left[\tau_F - \left(\frac{2c_H}{x} - 1\right)\right] \ge T^F,\end{aligned}$$

because the lower bound on τ_F ensures the second term is nonnegative.

This completes the proof of Proposition 5. ■

The binary cost setting highlights the induced risk-aversion effect of progressive taxes. Under Rationing, the buyer obtains a lottery whose pretax payout is x with probability 0.5, and 0 with probability 0.5. In contrast, Slack provides the buyer with a constant payout of an intermediate amount $x - c_H$. Suppose, in the absence of taxes, the buyer prefers the Rationing lottery. The introduction of flat taxes does not change the buyer's ranking over the lotteries because flat taxes impose the same proportionate penalty in each state. In contrast, progressive taxes that impose a sufficiently large penalty on income above $x - c_H$, lead to a switch in the buyer's ranking. Under progressive taxes, an individual who is risk-neutral in after-tax income acts as if he is risk-averse in pretax income. This, then, reconciles with the buyer's preference for the constant payout of Slack rather than the risky gamble of Rationing.

5. Extensions[6]

5.1. Progressive Taxes Imposed on Multiple Firms

The paper has focused on the productive efficiency gains that arise when progressive taxes are imposed on two contracting firms. In reality, however, tax rates are designed with a broader tax base in mind. Thus, taxes cannot be tailored to each particular circumstance or constituent. From Proposition 2, it is clear that any strictly progressive tax structure improves productive efficiency in a range of interactions to which it is applied. However, the ability of the taxing authority to tailor such a tax to achieve Pareto gains (and, thus, a consensus) remains to be seen. In this extension, we expand the setting to include two sets of contracting firms to investigate productive and distributive consequences of progressive taxes.

In particular, consider the discrete cost setup, in which two buyers, indexed 1 and 2, each contract with a different seller. Each seller has the same cost characteristics as before (with costs being independent), and buyer i extracts revenues of x^i, $x^i > c_H$, from procuring its supplier's product. In this case, despite the tax authority being limited to one tax schedule for all four parties, Pareto gains arise with progressive taxes.

PROPOSITION 6: *Assume the conditions in Proposition 5 hold for both buyer-seller relationships, i.e.,* $x^i < 2c_H$ *and* $\tau_F \geq \frac{2c_H}{x^i} - 1$, $i = 1, 2$. *Then, even if the tax authority is limited to using the same tax schedule for all firms, flat taxes are dominated by progressive taxes.*

Proof: Without loss generality, assume $x^1 \leq x^2$. As in the proof of Proposition 5, with flat taxes, each buyer strictly prefers Rationing to Slack since $x^i < 2c_H$. Hence, under flat taxes, expected after-tax income for buyer i, seller i, and tax collections, equal:

$$\Pi_B^F(i) = 0.5x^i(1 - \tau_F), \quad \Pi_S^F(i) = 0, \text{ and } T^F = 0.5x^1\tau_F + 0.5x^2\tau_F.$$

Now, consider the following progressive tax schedule:

$$\begin{aligned}
\tau(\pi) = \tau_1 &= 1 - \frac{0.5x^1(1 - \tau_F)}{x^1 - c_H} && \text{if } \pi \leq x^1 - c_H, \\
= \tau_2 &= \frac{1 + \tau_F}{2} && \text{if } x^1 - c_H < \pi \leq x^2 - c_H \\
= \tau_3 &= 1 && \text{otherwise.}
\end{aligned}$$

[6]We thank the two referees for suggesting the issues addressed in this section.

Under this progressive tax schedule, each buyer strictly prefers Slack to Rationing. Under Slack, buyer 1's expected after-tax income is $(x^1 - c_H)(1 - \tau_1) = 0.5[x^1(1 - \tau_F)]$. Under Rationing, his expected after-tax income is $0.5[(x^1 - c_H)(1 - \tau_1) + (x^2 - x^1)(1 - \tau_2) + (x^1 - x^2 + c_H)(1 - \tau_3)] = 0.25[x^2(1 - \tau_F)]$. Since $2x^1 > 2c_H > x^2$, Slack yields higher after-tax income. In a similar manner, it can be shown that buyer 2's expected after-tax income is also higher under Slack.

Given the progressive tax schedule and the ensuing contract choices, the expected after-tax income for buyer i, seller i, and tax collections, equal:

$$\Pi_B^P(i) = 0.5x^i(1 - \tau_F) = \Pi_S^F(i)$$

$$\Pi_S^P(i) = 0.25[x^2(1 - \tau_F)] > \Pi_S^F(i), \quad \text{and}$$

$$T^P = T^F + 0.5\big[x^1 + \tau_F x^2 - 2c_H\big].$$

The lower bound on τ_F and $x^i < 2c_H$ ensure $T^P \geq T^F$, thereby leading to a Pareto improvement. This completes the proof of Proposition 6. ■

Note two things. First, as before, the progressive tax restores productive efficiency in the buyer-seller relationships. Second, a particular progressive tax choice can ensure such efficiency translates into Pareto gains. Rather than a two-tier structure, however, a three-tier structure is employed. This is consistent with the intuition that if a tax scheme has to function for a variety of relationships, it will be muted relative to the optimal tax scheme designed for one specific relationship. The "cost" of imposing the muted tax scheme arises not in the form of foregone productive efficiency gains, but only in the tax authority's ability to extract the greatest portion of such efficiency gains.

On a related point, we have been silent as to whether the relevant progressive tax is a personal or corporate income tax. Rather, the focus has been on the after-tax income of firm owners. Since the firms' profits flow through both corporate and individual tax filters, the key results imply that a marginal net tax rate that is increasing in income promotes efficiency. However, if a taxing authority seeks to tailor taxes to a specific situation for the sake of promoting Pareto gains, we conjecture that individual taxes may be the more appropriate avenue. This follows because individuals' baseline income levels are on a similar scale, whereas a primary factor in the baseline income of a corporation is its size. Tailoring corporate taxes to the circumstance would require matching the tax rate both to firm size and the firm's attendant incentive problems.

5.2. Progressive Taxes and Other Adverse Selection Problems

Thus far, we have focused on efficiency gains from progressive taxes in the case of procurement of an indivisible product. However, problems of adverse selection can arise in many other arenas, including nonlinear pricing of consumer goods, sales of products with varied quality, capital apportionment in lending relationships, and labor contracting with unobservable productivity (see, e.g., Laffont and Martimort 2002). The key feature of these settings that distinguishes them from the procurement case studied in this paper is that contracting entails not a yes/no decision but a choice amongst a continuum of alternatives such as number of units sold, quantity of capital lent, etc.

In this extension, we consider the procurement setup where the parties agree on a production level $x \in R^{+}$, rather than the de facto choice of $x \in \{0, 1\}$. Besides adding some richness to the paper's main setup, such an analysis also demonstrates that the paper's results can arise under different interpretations of the basic principal-agent model of adverse selection.

For tractability, we continue the two-type case, where now production costs are $c_L(x)$ and $c_H(x)$ for agent (seller) types L and H, respectively. The agent's production cost function satisfies the usual conditions: $c_i'(x) > 0$, $c_i''(x) > 0$, $c_L(x) < c_H(x)$, and $c_L'(x) < c_H'(x)$ for all $x > 0$, and $c_i(0) = c_i'(0) = 0$, $i = L, H$. Procurement of x units yields the principal (buyer) revenues of x. In this continuous production case, x_i and s_i denote the production level and monetary transfer to the agent, respectively, if the agent reports he is of type i. Finally, denote the probability an agent is of type L by p_L. With this setup, the principal's program is presented below.

$$\underset{x_L, x_H, s_L, s_H}{\mathrm{Max}}\ p_L \int_0^{x_L - s_L} [1 - \tau(\pi)]\, d\pi + (1 - p_L) \int_0^{x_H - s_H} [1 - \tau(\pi)]\, d\pi$$

subject to:

$$\int_0^{s_i - c_i(x_i)} [1 - \tau(\pi)]\, d\pi \geq 0 \quad i = L,\ H \tag{IR$_\mathrm{i}$}$$

$$\int_0^{s_i - c_i(x_i)} [1 - \tau(\pi)]\, d\pi \geq \int_0^{s_j - c_i(x_j)} [1 - \tau(\pi)]\, d\pi \quad i = L, H; j \neq i \tag{IC$_\mathrm{i}$}$$

The principal's program represents the standard two-type adverse selection contracting choice, with the inclusion of income taxes. Despite the setting adding the issue of how much to procure when trade is initiated, the paper's main result persists.

PROPOSITION 7: *When procurement quantity can take on a continuum of values, a tax that is strictly progressive strictly improves productive efficiency relative to a flat tax.*

Proof: As with the standard two-type model, it is straightforward to confirm the binding constraints are ($\mathrm{IR_H}$) and ($\mathrm{IC_L}$). Solving these constraints for s_L and s_H and plugging into the principal's objective function yields the following (unconstrained) objective function:

$$p_L \int_0^{x_L - c_L(x_L) - c_H(x_H) + c_L(x_H)} [1 - \tau(\pi)]\, d\pi + (1 - p_L) \times \int_0^{x_H - c_H(x_H)} [1 - \tau(\pi)]\, d\pi$$

Taking a first-order approach to optimization yields the following conditions for the $x_i's$:

$$c_L'(x_L) = 1; \quad \text{and}$$

$$c_H'(x_H) = 1 - \left[\frac{p_L}{1 - p_L}\right]\left[\frac{1 - \tau(x_L - c_L(x_L) - c_H(x_H) + c_L(x_H))}{1 - \tau(x_H - c_H(x_H))}\right] \times [c_H'(x_H) - c_L'(x_H)].$$

Note, the condition for x_L is equivalent to that without private information. However, the analogous first-best condition for x_H, $c_H'(x_{\mathrm{H}}) = 1$, is not satisfied; the second term in the condition for x_H reflects the chosen x_H is below the efficient level. With a flat tax, $\tau(\cdot)$ is constant, so from the first-order condition, a different tax scheme improves productive efficiency as long as $[\frac{1 - \tau(x_L - c_L(x_L) - c_H(x_H) + c_L(x_H))}{1 - \tau(x_H - c_H(x_H))}] < 1$.

Rewriting the condition for productive efficiency yields $\tau(x_H - c_H(x_H)) < \tau(x_L - c_L(x_L) - c_H(x_H) + c_L(x_H))$. A progressive tax meets this condition if $x_L - x_H > c_L(x_L) - c_L(x_H)$, or since $x_L > x_H$, $\frac{c_L(x_L) - c_L(x_H)}{x_L - x_H} < 1$. By the convexity of $c_L(x)$, the left-hand side is less than $C_L'(x_L)$. And, since $c_L'(x_L) = 1$ (from the first-order condition for x_L), the condition is satisfied. ■

Intuitively, the reason progressive taxes promote efficiency in the presence of a continuum of production choices is much like before: progressive taxes cause the principal to discount the use of production cuts as a means of limiting information rents. Though production cuts can limit rents, the resulting gains are realized in those states where income is naturally higher. With higher marginal tax rates in those states, the attractiveness of undertaking such cuts is less pronounced. In the continuous production case, the added efficiency takes the form of an incremental increase in production, rather than an adjusted hurdle.

6. Conclusion

This paper studies the impact of taxes on the efficiency-rents tradeoff in a procurement relationship that operates in an environment of information asymmetry. The results show that the introduction of a progressive tax structure can lead to a productive efficiency gain and sometimes even a Pareto improvement relative to flat taxes.

As in any standard adverse selection model, in order to keep in check the seller's tendency to overstate costs, the buyer commits to limit procurement for high-cost reports. Since a flat tax imposes the same marginal cost at all income levels, it has no impact on this decision. Progressive taxes, on the other hand, add a consideration to the buyer's choice. In choosing how tough a stance to take, the buyer trades off the cost of foregone trade with the increase in income that comes with a tougher posture when trade does occur. With progressive taxes, any increased income that comes with the tougher negotiating posture is taxed at a rate above the firm's average tax rate. This causes the buyer to focus more on the likelihood of trade and less on the savings he can extract when trade does occur. The result is increased productive efficiency that, when taxes are judiciously chosen, can be shared by all interested parties.

This effect of progressive taxes may also have bearing on other control problems. For example, an issue associated with debt financing is that shareholders have incentives to invest raised capital in risky projects since they bear only one-sided risk. Debt holders respond to the shareholders' ex-post aggression by imposing constraining debt covenants and raising interest rates. The use of progressive taxes may prove beneficial in such settings because the high tax rate that is imposed on the successes of risky strategies makes it credible for shareholders to commit to following a more conservative investment strategy. This can make it cheaper for the firm to raise capital in the first place.

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