NOTRE DAME'S PROGRAM
FOR
GIFTED HIGH SCHOOL CHILDREN
SUMMER, 1961

A report by

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MOTIVATION AND HISTORY

Every teacher is fascinated by a gifted youngster and feels particularly responsible for such a youngster. Every professional, sooner or later, faces the problem of transmitting his own dedication and mastery to another generation. Our own involvement began to transcend the above natural concerns as soon as we found ourselves in the role of mentors to the secondary school teachers in our special programs for the teachers of mathematics.

In trying to decide the scope of the curriculum for the teacher we were compelled to re-examine our conception of what mathematical ideas are appropriate for the very young and are accessible to them. For the least that we could do was to make sure that our curriculum for the teacher should provide this teacher with the mastery of the requisite ideas and skills.

In search for a suitable form of instruction we thought of teaching as an art of the developing in the very young of curiosity, of a sense of adventure, and of a sensitivity of perception as well as the mastery of the basic skills. As he practices his art the teacher, naturally, should keep in mind the development of the facility in the use of language appropriate to each stage of the intellectual development of the pupil. We came to realize rather quickly that the teacher's mastery of the art of teaching as we conceived of it does not come easy and that this mastery is certainly not an obvious consequence of some degree of familiarity with factual information. It appeared that the teacher's performance reflected his whole intellectual outlook.

It became quite clear that in order to produce the desired effect upon his charges the teacher himself had to become committed to the habits of observation, conjecture, and critical reappraisal as well as to the habit of precise and concise use of language in his own thinking. In addition to this he had to become aware of the limitations of language as the mechanism for the transmission of ideas, skills, and attitudes to the very young and inexperienced.

What seemed clear to us had to be made convincingly clear to our teacher-students. Some direct participation in the work with the gifted pupils was certainly indicated, for nothing is as convincing as a successful demonstration.

The transition from an academic interest in the able youngsters to a very personal involvement occurred in almost imperceptible stages. The way led from discussions with the teachers and the parents, to counselling the students, to working with about a half dozen gifted boys in the summer of 1957, and to the establishment in the academic year 1957-58 of a Sunday class dubbed "Sunday School" by the youngsters and open to all able children in the environs of Notre Dame. Both the Sunday School and the summer term programs have continually grown in size and in scope.

The principal object of our present report is the summer program for "high ability secondary school students" sponsored at Notre Dame by the National Science Foundation in the summer of 1961.

Our summer group consisted of boys and girls who came from 18 states. These children represented 27 public, 18 parochial and 3 private schools. There were among them 9 seniors, 28 juniors, 18 sophomores and 11 freshmen. Commuters numbered 9 and among these 5 had attended the Sunday School. Among the 57 boarders, 49 came for the first time and 8 attended for the second time.

Their avowed interests ranged from engineering through the physical sciences to mathematics. Biological sciences were also represented, though these were decidedly in the minority (3 out of 66).

Whenever the selection of participants uses some measure of their potential in as much as it can be judged through the quality of their intiative, the intensity of their preoccupation, and the degree of their perseverance, one is bound to net a group representing a wide spectrum of interest.

At the outset, therefore, one is confronted with the dilemma as to what purpose should be served by a mathematics program for a collection of young individuals who have in common only eagerness, curiosity, an unbounded (and hitherto undirected) supply of vitality, and, possibly, an ultimate destiny in science.

One may decide upon a selection of some mathematical skills in the hope that these may prove to be useful at a later date. One may select a (reasonable simple) range of applications and treat jointly the mathematical tools and the ideas of the related science. One may, on the other hand, take advantage of the fact that with a proper selection of material one can remain within mathematics and yet exhibit a whole gamut of vital dilemmas which confront any scientist in any field at some time or another. This last is particularly true today when "theorizing" has become such an important tool in almost every field of human activity.

Even though we did not intend to neglect any of the above alternatives entirely, we did subordinate everything to the aim of providing our charges with a rich experience in scientific thinking while still remaining within mathematics.

CURRICULUM AND ACADEMIC ORGANIZATION

As the principal vehicle to carry us toward our objective we selected number theory. Since most of the youngsters have a great deal of experience with arithmetic, the subject is accessible to them. If taught properly, number theory provides an unusually fine opportunity for the development of the student's powers of observation, his intellectual curiosity, and his capacity for intellectual adventure. As an important by-product the student acquires the facility in the effective, precise, and concise use of language.

Number theory and the algebra of classes (which is studied concurrently with number theory) arise out of very basic, one may even say primitive, physical experience. It is extremely instructive for the student to allow him to participate in the natural development of the subject into a deductive science by letting him share in the continual interplay of observation, experiment, conjecture, and counterexample or proof. It is very important for the student to realize that the asking of good questions is as important tor the growth of an individual or, for that matter, for a science as the answering of questions already posed.

The participants in our summer program varied greatly in the quality of their mathematical experience. The more mature among them were allowed to attend a specially designed course in higher algebra which reinforced and expanded the objectives of the number theory course. It may be mentioned at this point that in its turn number theory, treated as a birthplace of modern algebra, provided the background for a fuller ultimate appreciation of the ideas of abstract algebra.

A very small number of students who took part in our program in 1959 and 1960 were ready for the more technical courses in algebra, in geometry, and in analysis. Such courses are a part of our summer program for the teachers * of mathematics.

Each summer we have had with us a small group of high school youngsters with particularly high qualifications. The have always received very kind and sympathetic treatment from the senior members of our staff in residence at the time.

The number theory course involved five lectures a week given to the whole body of participants. In addition to attending the lectures each student took part in three problem seminars a week. These seminars allowed for discussion in small groups. Further opportunity for informal out-of-class discussions was provided by the presence in the dormitories of very able and very dedicated junior counsellors. This group of counsellors consisted of a small number of our own ablest mathematics majors, one gifted boy from Harvard, and two very able girls, one from the College of Mount St. Joseph and one from Radcliffe College. The use of gifted undergraduates as resident counsellors living near the participants in their dormitory quarters was first tried in the summer of 1959, was continued in 1960 and 1961 and proved to be quite successful.

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* We shall be very glad to send upon request a descriptive pamphlet of 15 pages entitled "To Teach How to Teach How to Do" as long as our supply lasts.

In the Summer of 1960 a project seminar and a project counselling service were organized under the direction of a member of our staff whose interests cover both mathematics and physics. This was done in recognition of the fact that projects are the most prevalent form of student participation activity in the high-school.

Although some of our young people did make use of our project counselling program, this program did not lead to a significant involvement and we came to feel that projects, in their traditional form, do not provide the best outlet for a student's energies, particularly in a short and intensive summer session.

In order to direct into more promising channels the excess energies of the more experienced and the more ambitious of our young charges, we decided to experiment with a course the main purpose of which would be an informal (though informed) exploration of some significant mathematical ideas not usually accessible to the very young. Such a course, we thought, would supplement the more systematic treatment of the subject in number theory and in higher algebra.

By a fortunate combination of circumstances Professor Helmut Röhrl (University of Minnesota), Professor Norman Oler (Columbia University) and Professor Wilhelm Stoll (University of Notre Dame) were able to take part in the development of this idea. It was agreed that in the summer of 1961 every effort should be made to provide the student with a rich geometric experience using intuitive treatment whenever the inexperience of the young participants would call for it.

In the first two weeks, Professor Röhrl discussed basic ideas of combinatorial topology and was able to establish the five color theorem and to discuss the four color problem. Professor Oler took over for the following three weeks and discussed first the theory of braids and then some questions in the theory of convex sets. Finally Professor Stoll completed the course by discussing some questions in the topology of surfaces. Participants were given an opportunity to do a large number of problems and to participate in small discussion groups.

In 1961 the weekend lectures were again used for a stimulating extracurricular activity. We were fortunate to arrange visits by Professor Marston Morse, who lectured on topology and equilibria, by Professor Wilhelm Magnus who discussed combinatorial group theory, by Professor Hans Jonas who spoke of abstract thinking as a major attribute of human intelligence, and by the Very Reverend Ivo Thomas who spoke on the music of Virgil's poetry. Following each lecture, there was a reception for the speaker run by the participants themselves. The informal discussion at each reception made it possible for the members of the young audience and for the distinguished visitor to get to know each other.

The fact that two of the visiting lecturers represented the humanities reflected our concern that our efforts to create a very vivid and compelling picture of science would make the humanities appear dull by comparison unless we arrange for at least a token representation of the humanities on a very high level of excellence.

THE STUDENT REACTION

The students seemed enthralled even if somewhat intimidated by their first serious contact with ideas and by the large measure of responsibility which they had to assume for their own progress. Each student was required to take all of the examinations in each of the basic courses for which he was held responsible. Competition among the participants was very keen, even if very friendly, and contributed much to the maintenance of high standards. The students learned to use the library for supplementary reading and as a means for gathering information related to the topics of special interest to them. The availability of a first rate library appeared to be such a new experience for most of them that in their excitement they withdrew from our library in the first few days of the summer term a host of books on every conceivable subject. It took about ten days for the participants to satisfy their library hunger, to settle down and to apply themselves to the systematic part of the program.

Most of the students have already expressed the hope that they may be able to return to Notre Dame in the summer of 1962. We regret profoundly that at best we shall not have sufficient funds at our disposal to bring even a half of our 1961 participants back next summer.

THE STAFF SUMMER, 1961

Our senior staff consisted of the following:

Professor Stefan Drobot (University of Notre Dame): Analysis.

Professor Milko Jeglic (St. Mary's College, Notre Dame, Indiana): Problem Seminars.

Professor Hans Jonas (New School for Social Research, New York): Visiting Lecturer, Abstract Thinking as one of the Chief Attributes of Human Intelligence, July 23.

Mr. Frank Long (Director of the Chemistry, Mathematics and Physics Library of the University of Notre Dame): Library Counselling.

Professor Wilhelm Magnus (Institute of Mathematical Science, New York University): Visiting Lecturer, Combinatorial Group Theory, July 16.

Professor Marston Morse (Institute for Advanced Study, Princeton, New Jersey): Visiting Lecturer, Topology and Equilibria, July 2.

Professor Norman Oler (Columbia University, New York): Lectures on Braid Theory and Convex Sets, July 3 - July 24.

Professor Helmut Röhrl (University of Minnesota) Lectures on Combinatorial Topology, June 18 - July 3.

Professor Arnold Ross (University of Notre Dame): Number Theory and Algebra lectures

Professor Wilhelm Stoll (University of Notre Dame). Lectures on Topology of Surfaces, July 24-August 3.

The Very Reverend Ivo Thomas (Blackfriars, Oxford, England): Visiting Lecturer, The Music of Virgil, July 9.

Our Counsellors were.

Susan Cary, Sophomore, Radcliffe College, Cambridge, Massachusetts.

Catherine Herr, Senior, College of Mount St. Joseph, Mount St. Joseph, Ohio.

James Burton, Junior, Harvard University, Cambridge, Massachusetts.

Robert Burckel, Graduate Student, University of Notre Dame, Notre Dame, Indiana.

Kevin Cahill, Junior, University of Notre Dame, Notre Dame, Indiana.

Richard Jensen, Senior, University of Notre Dame, Notre Dame, Indiana.

Frank Owens, Junior, University of Notre Dame, Notre Dame, Indiana.

Frank Papp, Sophomore, University of Notre Dame, Notre Dame, Indiana.

John Riedl, Graduate Student, University of Notre Dame, Notre Dame, Indiana.

William Sweeney, Senior, University of Notre Dame, Notre Dame, Indiana

Charles Wong, Senior, University of Notre Dame, Notre Dame, Indiana

SELECTION

Each applicant is sponsored by his mathematics teacher who is either already familiar with the objectives of our program or who can be made aware of these objectives through correspondence. We have found that in most cases the interested teacher can help us to decide whether the youthful prospect is ready for what we have to offer. The final decision to accept or reject a candidate is arrived at by a committee of our teaching staff on the basis of discussion with the teacher-sponsor and of other available evidence of the applicant's achievement and interests such

as an autobiographical application letter, a special questionnaire, and the academic record.

FOOD FOR THOUGHT

We must say that we do not find it easy to single out those qualities of character and of achievement which could be considered as an indication of promise. We try not to be too rigid in our selective process for fear that we may overlook gifted nonconformists.

Remaining within the boundaries imposed by their immaturity, we have tried to bring our young people into contact with the very best scientific thinking of the day. Natural and inevitable limitations in scope which would allow us to set accessible, even if very ambitious, objectives, caused us to stop short of the vital concerns characteristic of experimental science. At present, we do not have the resources to deal with experimental science and we are disturbed by the fact that we cannot direct our young people to anyone who would attempt to do for them in experimental physical science what we have attempted to do for them in mathematics and in abstract thinking generally.

We feel that it would be highly desirable for everyone in our summer group to continue throughout the year some activity which maintains the quality of challenge presented by his summer experience. Unfortunately, for most of our participants this is impossible at present.

For the younger participants of proven ability and seriousness of purpose, it would be extremely worthwhile to maintain some form of challenging summer activity at least until it is time to go to college. Each successive summer can consolidate the gains made in the preceding one and the overall impression which can be produced by continued activity over a period of more than one summer could be incomparably deeper and more lasting.

Working with the very gifted is a very exacting task. One is amply repaid for the effort, however, by the sight of the deep excitement and the keen pleasure show by the youngsters as they meet and grasp new ideas and become aware of new challenges.