

Analysis of the Hockley and Murdock Decision Model

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Hockley and Murdock (1987) proposed a decision model to predict accuracy and response latency in recognition memory across a range of experimental paradigms. A decision is made when the evidence from a memory comparison process plus extraneous noise exceeds a lower or upper criterion. If neither criterion is exceeded, the distance between the criteria is reduced and a new sample of noise is added to the original memory comparison value. This is repeated until a criterion is exceeded. The model is shown to have two shortcomings: First, it produces a reaction time distribution that is multimodal; empirical distributions are generally unimodal. Second, application of the model to various speed-accuracy trade-off phenomena is found to be inadequate: Either the assumptions made to account for speed-accuracy data are post hoc or they are unable to mimic the data. An experiment that manipulates speed-accuracy trade-offs demonstrates that the model cannot produce a trade-off of sufficient range. Alternative conceptions of the model (the addition of a guessing process and a zero-drift random walk) are unsuccessful. The diffusion model of Ratcliff (*Psychological Review* 85, 59-108 (1978); 88, 552-572 (1981)) provides an adequate account of these data and a more parsimonious account of other speed-accuracy phenomena. © 1991 Academic Press, Inc.

Hockley and Murdock (1987) proposed a decision model to predict accuracy and response latency in recognition memory. The model was applied to four item recognition paradigms, and mimicked results for accuracy, mean reaction time, reaction time distributions, and various speed-accuracy trade-off phenomena. However, upon closer examination, we discovered two problems: (a) a multimodal reaction time distribution, and (b) an inadequate account of speed-accuracy trade-off phenomena. We view the latter problem as especially significant because the trade-off of speed for accuracy is a central phenomenon in all of the experimental paradigms examined.

The Hockley-Murdock decision model begins with the output of a memory comparison process that represents the match between a test probe and memory. This value of match is used in an iterative decision process. On each cycle of the decision process, some amount of noise is added to the match value. If the sum is above an upper criterion b , a positive response is made; if the sum is below a lower criterion a , a negative response is made. This constitutes one decision cycle. If the sum is between the two criteria, the distance between the two criteria (a and b) is reduced

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by a constant fraction CCR. Then the sum of a new sample of noise and the *original* match value is compared to these criteria. This process continues on successive cycles until one of the criteria is exceeded and a decision is made. A precursor of the model was proposed by Cartwright and Festinger (1943). That model also had an area of no decision. The larger this area of no decision (analogous to the distance between the criteria in the Hockley–Murdock model), the longer the decision time.

The match (from the memory comparison process) of a test probe to information in memory varies over test probes due to different degrees of match to memory. These are summarized by two distributions that reflect the match values for positive and negative items. In Hockley and Murdock's (1987) applications, the parameters of these distributions are estimated from the data. In a complete description of this match process, the distributions would be derived from a memory model linked to the decision model. In fitting data, the negative distribution is assumed to have a mean of 0.0 and a standard deviation of 1.0. The positive distribution has a mean of μ_o and a standard deviation of μ_o .

To derive estimates of decision latency, the number of cycles until the match plus noise exceeds a criterion must be converted to real-time. Hockley and Murdock (1987) assume that the duration of each successive decision cycle increases according to Eq. (1). This assumption produces decision latency distributions that are positively skewed. By Eq. (1),

$$\text{DecisionLatency} = (k^2 + k + 2) \text{BCT} \quad (1)$$

where k denotes the number of cycles until a criterion is exceeded, and BCT is the base cycle time (set to 17.5 ms by Hockley & Murdock, 1987). Overall response latency is the decision latency plus the time required for other processes such as encoding and response execution. Hockley and Murdock represent this time for other processes by a single normal distribution. They refer to it as the TOS distribution (time for other stages) with mean $\mu_{\text{TOS}+}$ for positive responses, $\mu_{\text{TOS}-}$ for negative responses, and standard deviation σ_{TOS} set equal to 50 ms. Decision latency is convolved with the appropriate TOS distribution to produce response latency. Without the quadratic mapping of Eq. (1), the model produces strictly symmetric reaction time distributions (unless the criteria convergence rate is close to zero, then the model predicts geometric decision time distributions).

The purpose of this article is to demonstrate two fundamental problems with the Hockley–Murdock model. The first problem involves the assumption of increasing cycle durations (Eq. (1)). This assumption results in multimodal reaction time distributions. The second problem concerns the model's account of speed–accuracy phenomena. Note that the model does not accumulate evidence as does a random walk. As a consequence, it has trouble in accounting for the growth of partial information over time (e.g., Meyer, Irwin, Osman, & Kounios, 1988; Ratcliff, 1988b; Reed, 1973, 1976), and in trading speed for accuracy.

PREDICTIONS FOR REACTION TIME DISTRIBUTIONS

In Eq. (1), the assumption of increasing cycle durations predicts increasing step sizes between comparisons that finish on successive cycles. For example, the step size between comparisons that finish on cycle $k=1$ and $k=2$ is only 70 ms (140 – 70 ms, see Eq. 1), but the step size between cycles $k=4$ and $k=5$ is 175 ms (560 – 385 ms). At longer reaction times (greater numbers of cycles), the variance due to the standard deviation in the TOS distribution (50 ms) is insufficient to produce a smooth function over the intervals between these steps. A distribution produced by the model is multimodal; an example is given in the left-hand panel of Fig. 1. This particular distribution results from the parameter values Hockley and Murdock (1987) used to fit data from the Sternberg paradigm, set size 3 (but the multimodality is parameter-independent). A multimodal distribution is contrary to empirical reaction time distributions which are generally unimodal (Ratcliff & Murdock, 1976).

One solution to this problem is to make the base cycle time (BCT) variable rather than constant. The right-hand panel of Fig. 1 shows the distribution for the Sternberg paradigm, set size 3, with BCT drawn from a normal distribution with mean 17.5 (as before) and a standard deviation of 2. Note that the variability is added, then the quadratic mapping of Eq. (1) takes place. The multimodal appearance is eliminated because the variance for the comparisons that finish on a given cycle increases as the number of cycles increases; rather than all $k=1$ comparisons taking 70 ms, they would vary around a mean of 70 ms. Making BCT

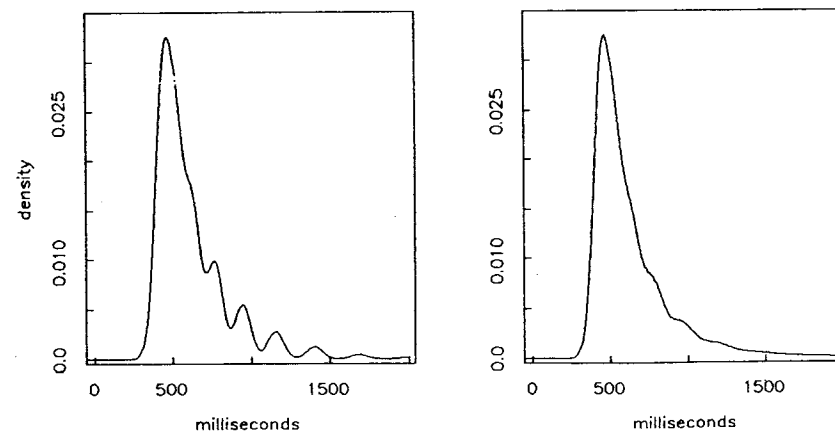


FIG. 1. Hit reaction time distribution for the Sternberg paradigm, set size 3, with constant and variable BCT. In the left-hand panel, parameter values were exactly those used by Hockley and Murdock (1987): CCR = 0.1, $a = -2$, $b = 6.7$, $\mu_o = 4.69$, $\sigma_o = 1$, $\mu_{\text{TOS}+} = 399$, and BCT = 17.5. For the right-hand panel, the same parameter values were used except that BCT was drawn from a normal distribution with mean 17.5 and standard deviation 2.

variable does not significantly change other predictions of the model. For the distribution with $BCT = 17.5$ ms, accuracy was 96.7%, mean latency for hits was 624 ms, mean latency for misses was 1008 ms; with $BCT = \eta(17.5, 2.0)$, accuracy was 96.7%, mean latency for hits was 624 ms, and mean latency for misses was 1012 ms.

The overall shape of the reaction time distribution is also largely unchanged by making $BCT = \eta(17.5, 2.0)$. To examine the shape of the reaction time distribution, a method described by Ratcliff and Murdock (1976) was used (see also Hohle, 1965). Reaction time distributions can be summarized by a distribution that is the convolution of a normal and an exponential distribution. The convolution distribution is described by three parameters: μ and σ of the normal, and τ of the exponential. The μ parameter is an estimate of the leading edge of the distribution; τ provides an estimate of the spread of the tail (see Ratcliff, 1979, for relevant discussion). For $BCT = 17.5$, μ , σ , and τ from the convolution model were 422, 50, and 202 ms, respectively. With $BCT = \eta(17.5, 2.0)$, μ , σ , and τ , were 415, 40, and 212 ms, respectively.

Hazard Functions

Another way of viewing the reaction time distribution is the hazard function (Luce, 1986). A hazard function gives the probability of an event occurring at time t given that the event failed to occur prior to time t . Figure 2 gives the hazard functions for the distributions in Fig. 1. With $BCT = 17.5$, the hazard function has large

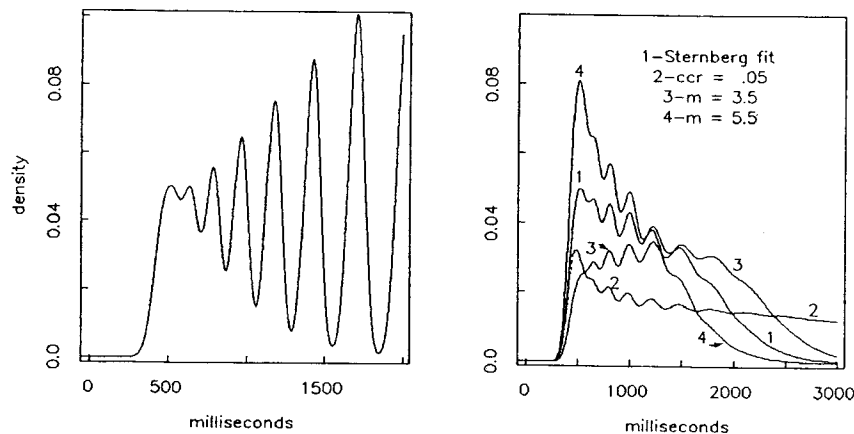


FIG. 2. Hazard functions from the Hockley-Murdock model. In the left-hand panel is the hazard function for the Sternberg paradigm, set size 3, with constant BCT. The corresponding hazard function with $BCT = \eta(17.5, 2.0)$ is Function 1 in the right-hand panel. Also in the right-hand panel are hazard functions for $CCR = .05$ (Function 2), $\mu_o = 3.5$ (Function 3), and $\mu_o = 5.5$ (Function 4). (The other parameters were unchanged from those Hockley and Murdock used to fit the Sternberg data, except $BCT = \eta(17.5, 2.0)$).

fluctuations (left-hand panel), and these fluctuations are not occurring only in the extreme tail of the distribution (1120 ms is the point by which 95% of the responses have terminated). However, making $BCT = \eta(17.5, 2.0)$ smooths the hazard function (right-hand panel, Function 1). The hazard function with variable BCT decreases gradually once an early peak is reached.

Luce (1986) reported that the shapes of hazard functions vary with the intensity of the stimulus (in psychophysical experiments). For stimuli very near threshold, the hazard function rises to an asymptote. For above-threshold stimuli, the hazard function rises to a peak and declines to an above-zero asymptote. At high intensities, Luce (1986) reported that the hazard function rises rapidly to a peak, and falls off sharply.

Hazard functions from memory tasks are similar to the latter two shapes. For the Sternberg paradigm, Ratcliff (1988a) found that the hazard functions rose rapidly to a peak and then declined to an above-zero level in the tail. In addition, some hazard functions were estimated from Ratcliff and Murdock (1976). In the left-hand panel of Fig. 3 are the hazard functions for hits and correct rejections from a study-test paradigm (their Fig. 9, input positions 9-16, output positions 9-16, p. 200). In the right-hand panel of Fig. 3 are the hazard functions from a study-test paradigm with words studied once or twice. The functions are for hits for the once-presented words and correct rejections (Fig. 13 from Ratcliff & Murdock, 1976; output positions 1-8, p. 204). In these two examples, as in Ratcliff (1988a), the hazard function rises to a peak, and remains above zero in the tail of the distribution. (These hazard functions include about 90% of the responses; beyond that, these hazard functions became unstable.)

The Hockley-Murdock model can produce a variety of hazard function shapes.

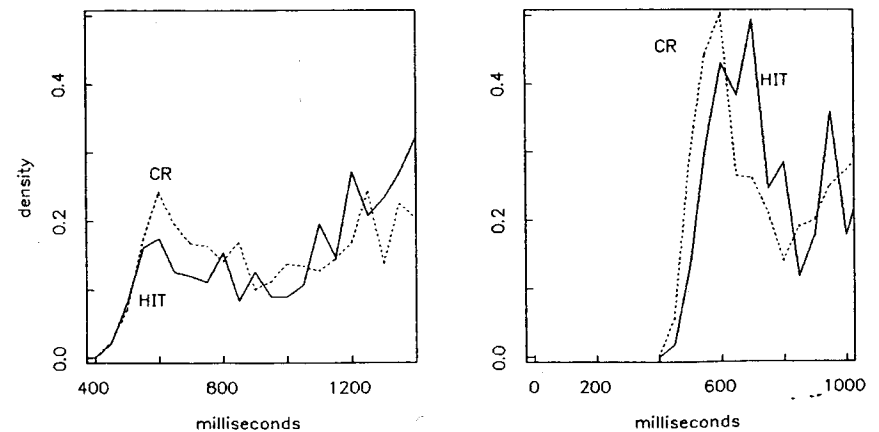


FIG. 3. Hazard functions for hits and correct rejections from Ratcliff and Murdock (1976). The left-hand panel is for a study-test paradigm; the right-hand panel is for the once-presented items in a study-test paradigm with once- and twice-presented items.

We varied the match strength (μ_o and CCR). (Changing the criteria starting positions (b and a) produced hazard functions that were shaped like those produced by changing μ_o and CCR.) The match strength would likely vary as a function of the intensity of the stimulus (perhaps achieved by varying study-time in a memory task). By a decrease in the match strength relative to Fig. 2 Function 1 (representing stimuli near-threshold), the hazard function (Function 3, Fig. 2) rises to an above-zero asymptote (95% of the processes have terminated by 1400 ms). By increasing the match strength (representing increased intensity of the stimulus), the hazard function (Function 4) rises to a peak and falls off sharply (the 95% point is at 870 ms). These are consistent with what Luce (1986) reported from the psychophysical literature. If the criteria convergence rate (CCR) is decreased (relative to Function 1), the hazard function (Function 2) rises to an early peak and falls to an above-zero asymptote (the 95% point is 1980 ms).

It appears that the Hockley-Murdock model can predict a range of hazard functions similar to those observed in the psychophysical literature (and the few examples found in the memory literature). However, it remains to be seen if the shape of the reaction time distribution that corresponds to the hazard function for a given set of parameters, is consistent with observed reaction time distributions. For Functions 2 and 3 in the right-hand panel of Fig. 2, decreasing μ_o and CCR each produce a slowing of reaction time and a skewing of the reaction time distribution. The convolution model fits to the reaction time distributions generated from these parameters give τ equal to 420 and 330 ms for Functions 2 and 3, respectively. In the recognition memory experiments of Ratcliff and Murdock (1976), τ exceeded 300 ms just 8 times in 92 convolution fits (6 of these for error reaction time distributions).

In sum, the result of making BCT variable is to add another parameter and another source of noise to the model, with no gain other than the elimination of the multimodality in the reaction time distribution and the fluctuations in the hazard function. Although the model can account for distribution shape as summarized by the convolution model, it does so only because of the quadratic mapping in Eq. (1). The model can produce a variety of hazard function shapes, but perhaps not when reaction time distribution shape is simultaneously considered.

SPEED-ACCURACY PREDICTIONS

Speed-accuracy trade-off phenomena can be grouped into two classes of dependent measures. Procedures in which subjects are required to respond according to experimenter-controlled deadlines or response signals (time-controlled processing, Ratcliff, 1978) produce a measure of the growth of accuracy as a function of time. In contrast, procedures in which subjects are allowed to respond in their own time allow manipulation of trade-offs in the amount of information required for a response through payoffs or instructions (information-controlled processing, Ratcliff, 1978). These two classes of procedures are examined below and

an experiment is presented that manipulates speed-accuracy trade-off by way of instructions. The Hockley-Murdock model is unable to produce a trade-off between speed and accuracy of sufficient range to fit data from this experiment. This is because the decision model begins with a match value of maximum accuracy (to which noise is added). For this reason, the modifications to the model that Hockley and Murdock (1987) claim affect speed and accuracy (starting positions of the criteria and convergence rate) change latency and accuracy independently of one another.

Time-Controlled Processing

Two of the techniques that have been used to investigate the accuracy of responses as a function of time are the speed-accuracy decomposition technique of Meyer *et al.* (1988) and the response signal method of Reed (1973, 1976). The speed-accuracy decomposition technique mixes regular trials in which subjects respond normally in their own time, with signal trials in which subjects must respond at or before a signal presented at one of several experimenter-determined lags. In analyzing data from this procedure, Meyer *et al.* (1988) assumed that signal trials are a probability mixture of fast-finishing regular responses and guesses. They then used a decomposition technique based on a mixture model (e.g., Ollman & Billington, 1972) that allows the estimation of guessing accuracy (partial information) by factoring out the contribution of the fast-finishing regular responses on the signal trials. In general, results showed that the accuracy of guesses initially rises quickly, then either rises slowly or asymptotes. Kounios, Osman, and Meyer (1987) found this in a sentence verification task, Meyer *et al.* (1988) in a double-word lexical decision task, and Ratcliff (1988b) in a study-test recognition task.

Hockley and Murdock (1987) account for this result in the following way: At any given time, the model is in one of three possible states: the input (match value plus noise) to the decision system is above the upper criterion, below the lower criterion, or between the two criteria (a nonterminated process). A regular response can be made if one of the criteria has been surpassed before the signal is presented. For the case where the value is between the criteria, Hockley and Murdock (1987) examined two possible mechanisms for making decisions for nonterminated processes. One was random guessing. This was not tenable because the data of Kounios *et al.* (1987), Meyer *et al.* (1988), and Ratcliff (1988b) all indicate that the accuracy of guesses increases as a function of retrieval time. A second mechanism was that a response was made based on a single criterion (placed somewhere between the two criteria when the signal is presented). However, Hockley and Murdock (1987) found that this assumption produced estimates of guessing accuracy that were far superior to those obtained by Kounios *et al.* (1987), Meyer *et al.* (1988), and Ratcliff (1988b).

Because neither of these mechanisms was tenable, Hockley and Murdock (1987) proposed that decisions for *nonterminated* processes are based on a probability mixture of random guesses and decisions based on a single criterion, with the probability of a random guess decreasing and the probability of a single criterion

response increasing as the criteria converge (as retrieval time increases). This assumption produces accuracy for partial information that increases as a function of retrieval time. However, the problem with this assumption is that it allows the model to fit *any* pattern of accuracy of partial information (even patterns not seen in the data) by mixing varying amounts of relatively high-accuracy responses, based on a single criterion, with low-accuracy random guesses. Thus, Hockley and Murdock's (1987) proposed account of the speed-accuracy decomposition data cannot be falsified by any pattern of data, so long as guessing accuracy is between zero and the accuracy of regular processes.

Another time-controlled processing paradigm is the response signal procedure (Reed, 1976). In this procedure, every trial is a signal trial and the subject can respond only at the signal (not before the signal as in the speed-accuracy decomposition task). Hockley and Murdock (1987) use the same assumptions to account for performance in this task as in the speed-accuracy decomposition task. That is, decisions are based on fast-finishing regular responses and a probability mixture of random guesses and single criterion responses (for nonterminated processes). Table 1 gives d' accuracy as a function of cycle time for a response signal function generated from parameter values used by Hockley and Murdock (1987) (parameter values given in the table). Accuracy increases as a function of increasing decision time for two reasons: (a) more and more of the decision processes terminate normally by exceeding a criterion; (b) according to the probability mixture assumption, more and more of the decisions involving nonterminated processes are based on a single criterion rather than a random guess. The probability mixture assumption provides a reasonable approximation to data.

For comparison, Table 1 also gives (for the same parameter values) the response signal function that results if decisions for all *nonterminated* processes are based on (a) a single criterion, or on (b) random guesses, rather than a probability mixture

TABLE 1
Three Assumptions about How Response Signal Functions
Can Be Generated by the Hockley-Murdock Model

		Decision cycles (lag in ms)							
		35	70	140	245	385	560	770	1015
Non-terminated responses are a probability mixture of a single criterion and guesses		0.35	1.02	1.57	1.99	2.30	2.52	2.68	2.80
Non-terminated responses are based on a single criterion		1.98	2.09	2.29	2.44	2.56	2.73	2.68	2.73
Non-terminated responses are based on random guesses		0.28	0.69	1.20	1.73	2.14	2.49	2.60	2.73

Note. Entries in the table are d' . The same parameter values were used as in Hockley and Murdock (1987, Fig. 7, p. 347); they were CCR = 0.1, $a = -2.0$, $b = 6.0$, $\mu_c = 3.5$.

of the two. If decisions for nonterminated processes are based only on a guessing mechanism, a reasonable approximation to response signal data also results. Of course, in the speed-accuracy decomposition task, this results in chance accuracy for partial information, contrary to the data.

On the other hand, if decisions for nonterminated processes are based only on a single criterion, accuracy for the first three lags is far superior to data. This illustrates the problem with the Hockley-Murdock model highlighted by time-controlled tasks; the model has nothing analogous to partial information. The initial memory signal represents complete information about the probe and is unchanged during the decision process (noise is added to the original signal). In contrast to a random walk model, the Hockley-Murdock model does not accumulate information over time. The probability mixture assumption produces the appropriate pattern of data, but in an ad hoc manner.

Information-Controlled Processing

In a second method used to study the trade-off of speed and accuracy, termed information-controlled processing, accuracy and reaction time are manipulated by payoffs or instructions. In the Hockley-Murdock model, speed and accuracy are governed by the *initial positions* of the criteria, and their rate of convergence. The model assumes that the initial positions and the convergence rate can both be manipulated by differential payoffs or instructions to subjects. Hockley and Murdock (1987) claim that the greater the distance between the starting positions and the slower their convergence, the slower but more accurate will be the response. The closer together the criteria start and the faster their convergence, the faster but less accurate the response. These two factors affect the ordering of the conditional mean reaction times.

The ordering of conditional mean reaction times has been used to test among various choice reaction time models (see Townsend & Ashby, 1983, for a review). The Hockley-Murdock model, with differential placement of the criteria, together with μ_{TOS+} and μ_{TOS-} being unconstrained, can predict any ordering. However, reaction time ordering is not diagnostic for recognition as it is for choice reaction time. Ratcliff and Murdock (1976) found that in recognition, the order of conditional mean reaction times changes as a function of the output position of the test stimulus, the list length, etc. The pattern of variation is sufficiently complex that none of the candidate models can provide a reasonable account for it. However, this is a limitation not so much of theory as of the current state of empirical knowledge. The number of potentially relevant factors is large, and their effects are not well understood.

The factors of the placement of the criteria and their convergence rate have similar effects; they affect latency more so than accuracy. The reason is that accuracy is primarily a function of the distance between the positive and negative item match distributions (which is unchanged by instructions to subjects or payoffs), while the number of cycles until a criterion is exceeded (the latency) is based on the initial criteria placements and their convergence rate.

The model's account of information-controlled processing has two problems. First, the model predicts that hit rate and correct rejection rate are often inversely related. This is not sensible if the point of increasing the distance between the criterion starting positions is to increase overall accuracy. Second, the model trades off speed for accuracy over too narrow a range and too severely within that range. These problems will be dealt with in turn.

In the Hockley–Murdock model, the distance between the initial positions of the criteria is increased to reflect increasing emphasis on accuracy. This distance between the initial positions of the criteria can be changed in one of two ways. Either each criterion starting position is incremented by the same amount, or one criterion is incremented more than the other. The hit rate and correct rejection rate covary if the criterion starting positions are changed symmetrically (i.e., if the starting positions of a and b change in equal steps or by steps that differ by no more than about $\pm 15\%$); but hit and correct rejection rate are inversely related when the criterion starting positions change asymmetrically (for example, if the initial position of a is changed from -2 to -2.5 , but the initial position of b is changed from 4 to 5.5).

The reason the model produces inversely related hit and correct rejection rates with asymmetric changes has to do with the point toward which the two criteria would converge if neither criterion were exceeded (call this z). If the initial positions

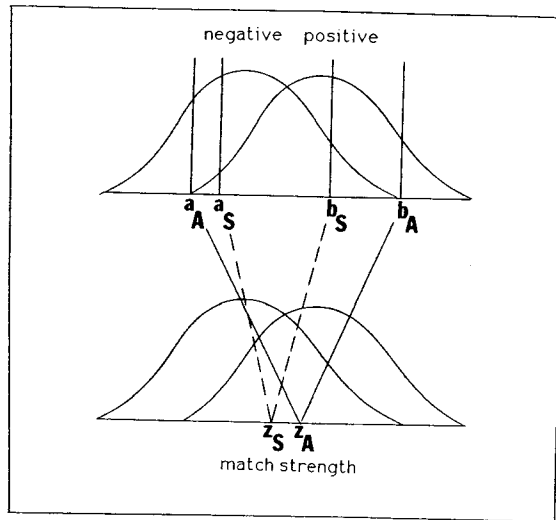


FIG. 4. Illustration of the shift of the convergence point as a function of the initial positions of the criteria. The left-hand distribution represents the match of negative items; the right-hand distribution represents the match of positive items. The point toward which the two criteria would converge if no response is made is z , where z_S is the convergence point for a_S and b_S (initial criterion positions for speed instructions), and z_A is the convergence point for a_A and b_A (initial criterion positions for accuracy instructions).

of the criteria are changed symmetrically (i.e., incremented or decremented equally), the point to which the two criteria converge is unchanged. However, if the initial positions are changed asymmetrically, the point to which the two criteria would converge is shifted in the direction of the criterion whose initial position was changed more. Figure 4 illustrates the shift in the convergence point (z) for criteria starting positions that reflect speed and accuracy instructions.

This shift in z is toward the positive criterion (from z_S to z_A as a result of a larger change in the initial position of b when going from speed to accuracy instructions). The positive criterion is farther away from a given match value than the negative criterion at every cycle (relative to the criterion placements at a_S and b_S). If the test probe is one for which a positive response is correct, b_A is less likely to be exceeded and the probability of a hit is decreased. If the test probe is one for which a negative response is correct, the probability of a correct rejection is increased because a_A is relatively more likely to be exceeded. The opposite is true if the point that the two criteria would converge to shifts in the direction of the negative criterion.

In contrast to accuracy, the latencies for hits and correct rejections are never inversely related as a function of criterion starting position. As the distance between the initial positions of the criteria is increased, latency is slowed because it takes more cycles to exceed a criterion. If the criterion starting positions are changed asymmetrically, so that the positive criterion is increased more than the negative is decreased (as illustrated in Fig. 4), the hit latency is slowed (more than the correct rejection latency) because the positive criterion is further away at every cycle and less likely to be exceeded. However, even though the hit latency is slowed, the hit rate *decreases*. Similarly, if the point the two criteria would converge to shifts toward the negative criterion, the correct rejection latency is slowed (more than the hit latency), but the correct rejection rate *decreases*.

An example follows that illustrates what happens to the hit and correct rejection rates and latencies as a function of asymmetric and symmetric changes in the initial criterion positions. This information will be summarized by a speed–accuracy trade-off function (d' as a function of latency). Its shape demonstrates that the Hockley–Murdock model trades off speed and accuracy independently of each other. When the model produces the largest increase in accuracy, latency changes only slightly; when the model produces the largest slowing of latency, accuracy changes only slightly.

In the Hockley and Murdock model (1987, Fig. 3, p. 345), the initial positions of a and b were varied from $a = -1.5$, $b = 2.5$, to $a = -2.0$, $b = 4.0$, to $a = -2.5$, $b = 5.5$, to model speed–accuracy trade-off (asymmetric changes). Figure 5 gives the proportion correct (top panel) and the latency (bottom panel) for hits and correct rejections, for the parameter values used by Hockley and Murdock (1987). Though correct rejection rate does increase when the criteria start farther apart (as indicated by Hockley & Murdock, 1987), hit rate decreases. This is because the b criterion was changed in greater steps and shifted the point of convergence out with it. Also, because the distance to the positive criterion is greater when b 's initial position is

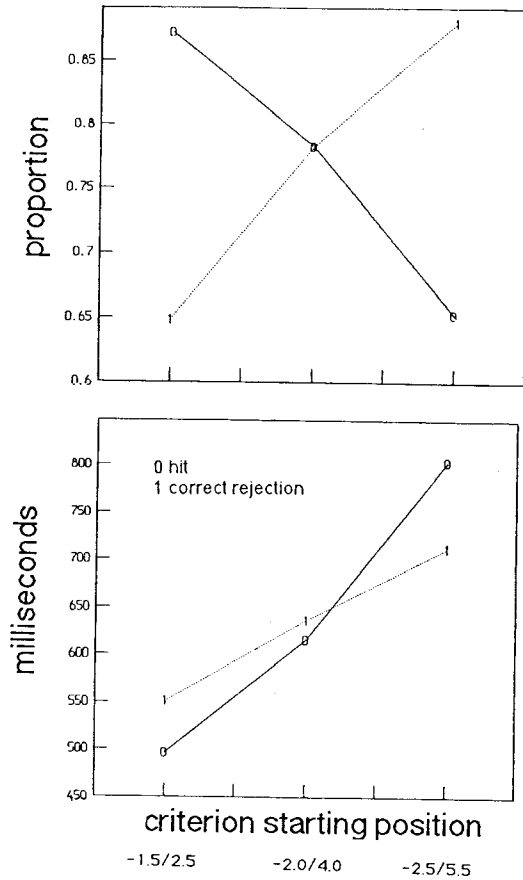


FIG. 5. Hit and correct rejection proportion (top panel) and latencies (bottom panel) for parameter values used by Hockley and Murdock (1987, Fig. 3, p. 345). If initial positions of a and b are changed asymmetrically, hit and correct rejection rate are inversely related.

increased in greater steps, the hit latency is slowed more than the correct rejection latency.

Hit rate and correct rejection rate covary when speed-accuracy instructions are modeled with symmetric criteria change. Figure 6 gives the same information as Fig. 5, with the same parameter values, except that the initial positions of the criteria are changed symmetrically (from $a = b = 0.5$ to $a = -2.5$, $b = 3.5$ in steps of size 1.0). Hit and correct rejection rates both increase as the initial distance between the criteria is increased. However, it is important to note that the largest increase in accuracy is accompanied by little slowing of latency. Conversely, accuracy changes little over the range that latency changes significantly. Thus, speed and accuracy are relatively independent of one another over most of the range.

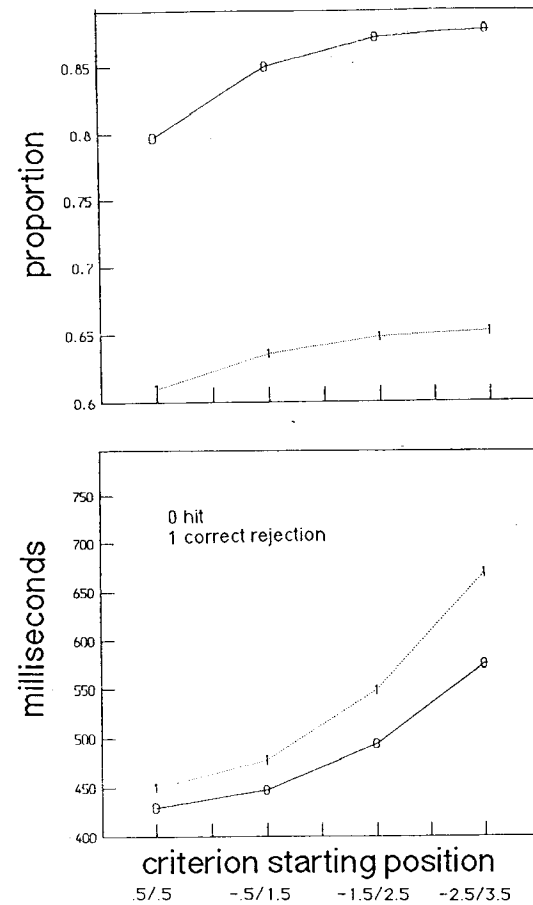


FIG. 6. Hit and correct rejection proportion (top panel) and latencies (bottom panel) for parameter values used by Hockley and Murdock (1987, Fig. 3, p. 345). Initial positions of a and b are changed symmetrically.

Information of the kind shown in Fig. 6 can be summarized to produce a speed-accuracy trade-off function. In Fig. 7, d' accuracy is plotted as a function of hit reaction time for various criterion starting positions. The leftmost point of the speed-accuracy function is obtained from the upper and lower criteria starting already converged to a common point. Successive points on a function are the result of increasing the distance between the initial positions of the criteria (an increased stress on accuracy). The initial rapid rise in d' is accompanied by almost no increase in latency (see Function 4). When even more caution is exercised by further increasing the distance between the criterion starting positions, the resulting slowing of latency is accompanied by little increase in accuracy. In the model, speed

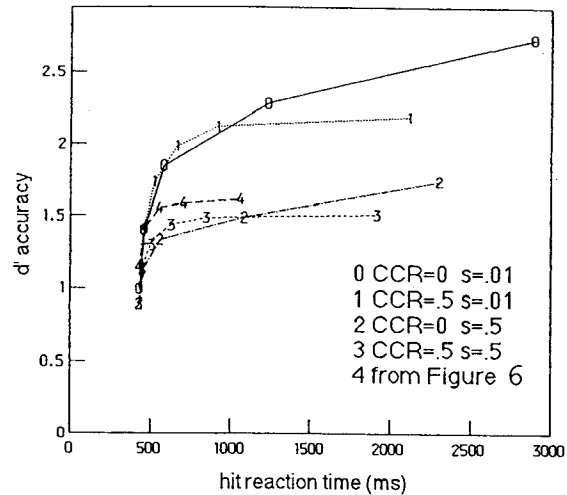


FIG. 7. Speed-accuracy trade-off function modeled by changing the initial positions of the criteria. Accuracy in terms of d' is plotted as a function of hit reaction time (ms). The shape of the speed-accuracy function was explored by manipulating CCR and σ_o : CCR = 0.0 or 0.05, σ_o = 0.01 or 0.5 (denote s). Also included is the speed-accuracy trade-off function for the data from Fig. 6.

and accuracy do not really trade-off so much as they change independently over separate ranges.

Other combinations of parameter values were tried to determine if the model could be made to produce a more gradual trade-off of speed for accuracy. Figure 7 summarizes the effects of CCR and σ_o , on the slope of the speed-accuracy trade-off function. The initial positions of the criteria were changed in equal amounts from a and $b = 0.5$, to $a = -1.5$, $b = 2.5$. The noise variance was set to 1.0, σ_o to 1.0, μ_{TOS+} to 420, σ_{TOS} to 50, and BCT to 17.5.

The most gradual trade-off was found when the criteria were stationary (CCR = 0.0) and the variability of the memory signal was essentially zero ($\sigma_o = 0.01$) (Function 0). Neither of these conditions could be applied to data. Stationary criteria produce very slow latencies and reaction time distributions that are too skewed. No variability in the match of test items to memory is unrealistic because it implies that all items have equivalent strength and are equally memorable. When CCR was 0.5 (Function 1), the speed-accuracy function asymptotes more rapidly, and if σ_o is increased (to 0.5, Function 2), the range of accuracy changes is much reduced. (Increased variability in the memory signal reflects an increased probability that noise variance could push the memory signal beyond the wrong criterion, thereby lowering accuracy.)

These two parameters (CCR and σ_o) are responsible for the shape of the trade-off of speed and accuracy. Other parameters of the model can only shift the whole speed-accuracy function horizontally (μ_{TOS}) or vertically (μ_o). Fits to data require non-zero values of both CCR and σ_o (for example, Functions 3 and 4). Therefore,

the model must produce these sharply bending speed-accuracy trade-off functions for sets of parameter values that produce fits to data for a single speed-accuracy condition.

A more gradually sloped function could be produced if responses were instead a probability mixture of random guesses and regularly-terminated responses. However, this alternative model has problems accounting for time-controlled data (as was described above). It also predicts that the minimum reaction time remains constant as accuracy is emphasized (because the guessing component is responsible for the minimum). We shall see that data from the speed-accuracy experiment show that the minimum reaction time increases as accuracy is emphasized, contrary to this alternative.

To evaluate the ability of the model to fit data from speed-accuracy manipulations, an experiment was conducted. In the experiment, three levels of speed-accuracy trade-off were manipulated using differential instructions in a regular reaction time procedure. Subjects studied 16 words and were tested on those 16 words plus 16 unstudied words. They made a positive response to a word from the study phase and a negative response to an unstudied word. There were three instruction conditions: conditions that stressed speed, accuracy, or a balance between the two.

METHOD

Subjects

Twenty-six Northwestern University undergraduates participated in exchange for course credit. Subjects were run in groups of size 1 to 4.

Design and Procedure

Presentation of stimuli, timing, and response collection were under micro-computer control. Words were presented on a terminal screen, and keypress responses were made on the keyboard.

The experiment used a study-test recognition memory procedure. On each study trial, 16 words were presented for study at a 1-s rate. The recognition test phase followed immediately and consisted of 16 studied words plus 16 unstudied words tested in random order. Subjects were required to press "/" to indicate that the test word was from the study phase, and "Z" to indicate that the test word was not previously studied.

The trade-off of speed for accuracy was manipulated through instructions. There were 27 trials in the experiment, nine of each instruction type. These were blocked and tested in one of six possible counterbalanced orders. For speed instructions, subjects were told to keep their response time under 500 ms and were given reaction time feedback on their terminal for 1 s. For normal instructions, they were told to keep their response times between 500 to 700 ms. They received reaction time feedback for 1 s on every trial and "ERROR" for 1 s after each error. For accuracy instructions, subjects were told to be as accurate as possible. For each error, "ERROR" appeared on the screen for 2 s; no reaction time feedback was given.

The experiment began with a practice phase. Subjects were told to keep their response times between 500 to 700 ms on the first practice trial. The next two blocks of practice trials were accuracy trials, then two blocks of speed trials, and finally two blocks of normal trials. Then the experiment proper commenced.

For each subject, the words were drawn at random without replacement from a pool of 1650 words selected from the Kucera and Francis (1967) word norms. The words were all two syllables, four to eight letters in length, and varied in frequency from a count of 6 per million to 472 per million.

RESULTS AND DISCUSSION

Six subjects were eliminated for responding at or below chance in the speed condition. Figure 8 gives hit and correct rejection rates as a function of latency. Hit rate and correct rejection rate both increased as accuracy was stressed. The convolution model was fitted to the grouped reaction time distributions (Ratcliff, 1979) for each of the instruction conditions. (Ratcliff (1979) showed that this ensured that the grouped distribution retained the shape of the individual subjects' reaction time distributions for such recognition memory procedures.) Table 2 gives the convolution parameter summary for hits and correct rejections. The tail (τ) and the leading edge (μ) of the hit (and the correct rejection) reaction time distributions increased as accuracy was emphasized. In other words, as accuracy is emphasized, the fastest (minimum) responses get slower (μ increases) and the slower processes get slower (τ increases).

Application of the Hockley-Murdock Model

The Hockley-Murdock model was fitted to the hit and correct rejection rates and their latencies. (Distribution shape was not fit explicitly because it was of secondary interest.) Parameters μ_o , μ_{TOS+} , and μ_{TOS-} were estimated to fit the normal instruction data and were held constant across the other instruction conditions (σ_o was set equal to 1.0 as it was in many of the fits of Hockley & Murdock, 1987). Only b , a , and CCR were allowed to vary as a function of the instructions. The following parameter values were estimated using the SIMPLEX parameter search algorithm (Nelder & Mead, 1965): $\mu_o = 2.02$, $\mu_{TOS+} = 433$ ms, and $\mu_{TOS-} = 463$ ms. (The SIMPLEX algorithm minimizes the sum of squared differences, for all the parameter sets that define the simplex, are less than some criterion, which we set to .00000001.) For the normal instruction condition, $b = 3.29$, $a = -1.46$, and $CCR = 0.094$. The top panel of Fig. 8 gives the fits to the data.

The values of b , a , and CCR for the speed and accuracy conditions should be evaluated relative to those for the normal instruction condition. According to the model, when accuracy is stressed the value of CCR should decrease and/or the initial distance between b and a should increase. An adequate fit was found for either of these. When speed is stressed, the model should decrease the distance

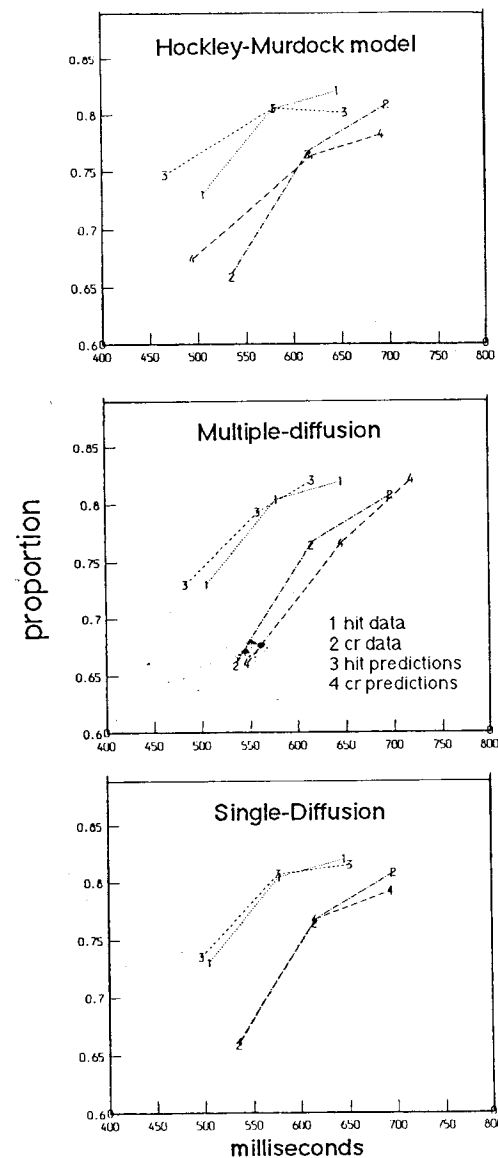


FIG. 8. Hit and correct rejection rate as a function of hit and correct rejection latency (ms) for the speed-accuracy experiment. Fits are given for the Hockley and Murdock (1987) model (top panel), the multiple-diffusion model (middle panel), and the single-diffusion model (bottom panel).

TABLE 2
Predictions from the Hockley and Murdock Model
and the Diffusion Model (Ratcliff, 1978, 1981)

Instructions	Hit			CR		
	μ	σ	τ	μ	σ	τ
	Data					
Speed	408	47	109	430	55	117
Normal	465	34	129	500	48	134
Accuracy	500	25	166	541	33	182
	Hockley-Murdock					
Speed	464	50	0	497	51	0
Normal	429	39	157	460	40	160
Accuracy ^a	407	32	244	437	30	254
Accuracy ^b	439	43	216	469	43	228
	Multiple-Diffusion					
Speed	435	10	70	476	26	70
Normal	451	20	108	525	37	120
Accuracy	467	26	143	556	44	160
	Single-Diffusion					
Speed	440	15	94	411	6	63
Normal	456	21	149	432	12	124
Accuracy	474	26	196	450	18	171

^a $b = 3.12$, $a = -1.19$, $CCR = 0.028$.

^b $b = 3.96$, $a = -2.02$, $CCR = 0.094$.

between a and b and/or increase CCR. However, the model was unable to change accuracy sufficiently in this way and was unable to provide an adequate fit to the data.

For the accuracy instructions, the parameter values that best approximated the data were $b = 3.12$, $a = -1.19$, and $CCR = 0.028$. When only b and a were allowed to vary (with $CCR = 0.094$ as for the normal instruction data), an almost equivalent fit was found for $b = 3.96$ and $a = -2.02$. For the former fit, the smaller CCR slowed reaction time; for the latter fit this was accomplished by starting the criteria further apart (relative to the normal condition). Increasing the distance between the initial positions of the criteria is preferable to reducing the value of CCR because the resulting reaction time distribution is not as highly skewed (see Table 2, $\tau = 244$ vs. 216).

For the speed instructions, the data were best approximated if $b = a$ (both = 0.81); CCR is irrelevant since the criteria start converged. To try to get accuracy to change over the range required by this experiment, the initial positions of the criteria must be brought together. This property of the model is illustrated

in Fig. 7 where it was shown that the majority of the increase in accuracy occurs when the initial positions of the criteria go from starting already converged (all processes finish on cycle 1) to where the criteria start slightly separated. The 0.06 increase in hit rate and 0.1 increase in correct rejection rate are the maximum that the model can produce given the value of μ_0 estimated to fit these data (the amount of increase would be little different no matter what the value of μ_0). The model does not cover the range of the data, and the range that is achieved requires reducing the Hockley-Murdock model to a single-criterion model.

The model with a single criterion (the initial positions of the two criteria are equal) is arguably not an allowable member of the family of possible Hockley and Murdock (1987) models; there is only one criterion and no convergence. When the initial positions of the criteria are close together or equal, almost all processes finish on the first cycle. The resulting reaction time distribution is normal (see Table 2, $\tau = 0.0$, $\sigma = \sigma_{TOS} = 50.0$) because Eq. (1) does not come into play. This is contrary to the data. Furthermore, if equal criterion starting positions are allowed, a response with high accuracy results when primarily non-decision aspects (i.e., only μ_{TOS}) contribute to the latency (this is evident in Table 1, for the 35-ms accuracy level for responses based on a single criterion).

The Hockley and Murdock (1987) model does an unsatisfactory job accounting for the data of the speed-accuracy experiment. It cannot change accuracy over the range required by the experiment. To even come close to changing the accuracy this much, the dual-criteria model must revert to a single-criterion model. Of secondary interest, the model does predict τ to increase as accuracy is emphasized, but the increase is more extreme than the data (from Table 2, hit $\tau = 0, 157, 216$). Contrary to the data, the model does not predict μ to increase as accuracy is emphasized (hit $\mu = 464, 429, 439$). However, distribution shape was not fit explicitly, only accuracy and latency.

Because of the difficulties in accounting for the speed-accuracy experiment, the problems of the multimodal reaction time distributions and hazard functions, and the ad hoc probability mixture assumption used in time-controlled processing tasks, alternative formulations of the Hockley-Murdock model will be considered. These include the addition of a guessing process to the regular Hockley-Murdock model, and modification of the model to a zero-drift random walk decision process. Because neither of these attempts is successful, an alternative framework will also be considered in an attempt to fit the data of the speed-accuracy experiment, the diffusion model of Ratcliff (1978, 1981).

ALTERNATIVE MODELS

Hockley-Murdock Plus Guessing. A version of the Hockley-Murdock model was tried in which guessing is incorporated under speed instructions (akin to the fast-guess models of Ollman, 1966, and Yellott, 1967, 1971). This was done because the decision model was unable to account for the range of the speed-accuracy

trade-off through criterion positioning and changes in the convergence rate alone. The same parameter values were used as in the fit to the normal instruction data; however, if the match of the test probe plus noise did not exceed either criterion on cycle 1, there was a 20% chance that a guess would be made. On cycle 2, there was a 40% chance of a guess if the match plus new noise sample did not exceed either converging criterion. If a criterion had not been exceeded by cycle 5, a guess would be made 100% of the time. This modification of the model provides a good approximation to the mean latency and accuracy data. It does so without requiring that the criteria start already converged, thereby avoiding the problems inherent in that fit to the data.

However, there is a problem with this guessing model. Responding on the basis of exceeding a criterion would always be preferable to making a guess because it would result in much higher accuracy. Because subjects are free to change the initial positions of the criteria, subjects should start the criteria close together to optimize performance. This would make the frequency of a guess rare and would not produce a sufficient decline in accuracy for the speed condition. This restates a fundamental problem with the decision model; the model begins processing at essentially full accuracy and uses criterion positioning and convergence rate primarily to change latency, not accuracy. Therefore, a modification of the model was sought which would (a) accumulate accuracy over time and (b) involve a more direct linking of latency to accuracy. As this is a property of sequential sampling models, a zero-drift random walk version of the Hockley–Murdock model was examined.

Zero-Drift Random Walk. In this model, the criteria are stationary, and the noise (added on each cycle to the match strength) is accumulated. The starting point of the walk is determined by the match of the test probe. A strong match of a test item to memory will bias the process to begin close to the upper criterion and to terminate at the upper criterion. A weak match will bias the process to begin close to the lower criterion and terminate at that criterion. The problem with this model is that as the distance between the criteria is increased (reflecting increased stress on accuracy), latency slows but accuracy drops toward chance. If the criteria are far apart, the bias in the starting point of the walk is outweighed by the great distance to either criteria. The decision outcome is entirely determined by the noise in the walk and not the biased starting point. Consequently, the probability of termination at each criterion approaches 50%. This model predicts the opposite of the speed–accuracy trade-off effect obtained in data.

Because of the problems with the original version of the Hockley and Murdock (1987) decision model, and the lack of success with two variations, we now contrast this model with sequential sampling models. Potential candidates among sequential sampling models include the runs model of Audley (1960) and Laming (1968), the relative judgement theory of Link and Heath (1975), the counter model of Pike (1973), the simple random walk model of Stone (1960), and the accumulator model

of Vickers (1970). We will consider only one member of this class, the diffusion model of Ratcliff (1978, 1981) (it is representative, and it is the only one that has been applied to recognition memory procedures).

THE DIFFUSION MODEL

In the diffusion model, items are stored separately. At retrieval, a test probe is compared with each item in memory and evidence is accumulated in parallel for each comparison. The comparison process is modeled by the continuous version of a random walk called the diffusion process. Goodness-of-match determines the drift rate in the diffusion process, i.e., the rate of accumulation of evidence. This drift rate has variance s^2 . The goodness-of-match of a test probe to a memory item is assumed to vary over items; it can be summarized by a normal distribution with variance σ^2 (set equal to 0.18^2), with mean u for old items, and mean v for new items. Evidence accumulates continuously beginning from a starting point z toward a decision at one of two decision boundaries (at a and 0). The more extreme the goodness-of-match, the more rapid the drift to a boundary. A positive decision is made if the diffusion process for one comparison reaches the positive boundary, while a negative decision is made when all the diffusion processes have reached the negative boundary. Response latency is based on the resulting decision latency plus a factor T_{ER} that represents the time for non-decision aspects of the task (similar to TOS in Hockley & Murdock, 1987).

Because the diffusion model accumulates evidence over time, it can more naturally deal with partial information in time-controlled processing than can the Hockley–Murdock model. To model the Meyer *et al.* (1988) paradigm, Ratcliff (1988b) assumed that responses on signal trials were based on fast-finishing regular responses that had exceeded one of the decision boundaries, or guesses based on a single criterion if a boundary had not been exceeded. For the response signal paradigm, Ratcliff (1978) assumed that the decision boundaries were moved far from the starting point and all responses were based on a single criterion. However, Ratcliff (1988b) showed that response signal data could also be predicted from a mixture of regularly terminated responses using decision boundaries plus guesses initiated by the signal based on a single criterion.

To account for speed–accuracy trade-off data, the diffusion model assumes that there are three criteria that can be adjusted by the subject. The first two are the distance from the upper boundary to the starting point ($a - z$) and the distance from the starting point to the lower boundary ($z - 0$). If accuracy is stressed, these distances are increased. The third criterion is the zero point between the match (u) and nonmatch (v) distributions (Ratcliff, 1985). If subjects are penalized for false positives over false negatives (for example), they could shift the zero point so that there exists a bias to interpret evidence as being negative. However, as long as the difference between u and v is held constant, d' remains constant. Speed–accuracy manipulations are assumed to vary primarily the distances from the starting point to the decision boundaries.

The diffusion model was fitted to the hit and correct rejection rates and latencies from the speed-accuracy experiment. The parameters u , v , s , and T_{ER} were estimated, but were fixed across conditions; a (the positive boundary) and z (the starting point) were allowed to vary with instructions. It was not necessary to vary the zero point between u and v for the fits achieved. (This contrasts with Ratcliff, 1985, where the relative probability of a *yes* or *no* response varied and was modelled by changes in the zero point.) The SIMPLEX (Nelder & Mead, 1965) algorithm was used to find a set of parameters that closely approximated the hit and correct rejection rates and mean latencies. The resulting parameter values were $u = 0.276$, $v = -0.493$, $s = 0.112$, and $T_{ER} = 400$ ms. The value of z gradually increased as accuracy was emphasized, reflecting increasing caution through movement away from the negative boundary (speed $z = 0.020$, normal $z = 0.033$, accuracy $z = 0.042$). In addition, the distance from z to a gradually increased as accuracy was emphasized (speed $a - z = 0.052$, normal $a - z = 0.067$, accuracy $a - z = 0.079$). The middle panel of Fig. 8 gives the fit to the data.

The model is able to predict the increase in hit rate and the increase in correct rejection rate as accuracy is emphasized, with only a and z changing. It should be noted that the diffusion model is capable of changing accuracy over an even wider range than indicated here. This might be necessary if speed versus accuracy was manipulated through explicit payoffs. Of secondary interest, the shapes of the reaction time distributions produced by the diffusion model are more in line with the data than those of the Hockley-Murdock model (see Table 2). In contrast to the Hockley-Murdock model, the diffusion model does predict μ to increase as accuracy is stressed, as well as predicting an increase in τ (see Table 2).

One aim of the Hockley-Murdock model was to provide a decision process that would be consistent with a number of memory models whose output for recognition is a single value of match between the test item and memory. The memory models include those with distributed representations such as the linear association model of Anderson (Anderson, Silverstein, Ritz, & Jones, 1977), CHARM (Eich, 1982), Murdock's (1982, 1983) TODAM model, and Pike's (1984) matrix model, and models with a localized representation such as the SAM model of Gillund and Shiffrin (1984) and the MINERVA 2 model of Hintzman (1984, 1986).

The diffusion model of Ratcliff (1978) cannot be linked to a distributed memory model (such as TODAM, Murdock, 1982). The diffusion model requires the separate representation of each item in memory; the test probe contacts each memory item in parallel and a diffusion comparison process is executed for each. Distributed memory models do not possess separate representations of each item in memory. However, Ratcliff (1981) presented a single-diffusion model that could be linked to a distributed memory model. Other sequential sampling models could also serve this purpose.

Single-Diffusion Model

In the multiple-diffusion model, a test probe is compared with each item in memory and evidence is accumulated in parallel for each comparison. The decision

process is exhaustive for a negative response and terminating for a positive one. In the single-diffusion model, the comparison process is driven by the matching strength of a test probe to memory (e.g., the initial value used by the Hockley-Murdock model). The more extreme the goodness-of-match (positive or negative), the more rapid the drift to the appropriate boundary. The single diffusion process begins at z and terminates when the process exceeds one of the boundaries. A positive response is made if the upper boundary is exceeded, a negative response is made if the lower boundary is exceeded.

The single comparison diffusion model was fit to the data of the speed-accuracy experiment. The parameters u , v , and T_{ER} were estimated, but did not vary across conditions; a and z did vary with instructions. In this version of the model, u and v correspond to the means of the positive and negative strength distributions, with variance $\eta^2 = 0.18^2$ (from Ratcliff, 1981). The variance of the drift was given by $s^2 = 0.08^2$ (from Ratcliff, 1981). The parameter values (estimated using SIMPLEX, Nelder & Mead, 1965) were $u = 0.184$, $v = -0.164$, and $T_{ER} = 400$ ms. The value of z gradually increased as accuracy was emphasized (speed $z = 0.022$, normal $z = 0.038$, accuracy $z = 0.050$). In addition, the distance from z to a gradually increased as accuracy was emphasized (speed $a - z = 0.042$, normal $a - z = 0.053$, accuracy $a - z = 0.064$). The bottom panel of Fig. 8 gives the fit to the data. This version of the diffusion model also predicts that both μ and τ increase as accuracy is emphasized (see Table 2).

Either version of the diffusion model provides a superior fit to the data of the speed-accuracy experiment, with parameter values that change in systematic ways (i.e., the distance from the starting point to the boundaries increases). Application of the Hockley-Murdock model to time-controlled processing tasks revealed a model that was too flexible. The probability mixture assumption for time-controlled processing is able to fit almost any pattern of data, even some that do not occur. In contrast, the diffusion model is tightly constrained. For example, Ratcliff (1988b) fit the diffusion model to regular trial data in the speed-accuracy decomposition task of Meyer *et al.* (1988), and was able to produce a close approximation to the observed guessing accuracy with only one free parameter, the time of onset of guessing in response to the signal. The level of accuracy and shape of the guessing accuracy function were fixed by the regular trial fits.

GENERAL DISCUSSION

We have reviewed several problems that raise questions about the utility of the Hockley and Murdock (1987) decision model. First of all, reaction time distributions are multimodal and hazard functions show extreme fluctuations. This is the result of the duration of each decision cycle becoming increasingly longer according to Eq. (1) (and the variance of TOS not increasing as the number of cycles increase), an assumption necessary to produce skewed decision latency

distributions. These problems can be remedied by making the base cycle time (BCT) variable rather than constant, though this is an ad hoc fix.

The more important problems stem from the model's application to time-controlled and information-controlled processing tasks. The model begins with full information and uses noise to drive the decision process; consequently, partial information in time-controlled processing must be derived from a probability mixture of random guesses and single criterion responses. The model can produce any pattern of data from chance to regular accuracy by varying the amount of each of these processes in the mixture. Not only is this assumption too powerful, it is not sensible. Subjects should not make random guesses when they could be more accurate by (a) basing all nonterminated process decisions on a single criterion, or (b) making a higher proportion of relatively high-accuracy regular responses by starting with the criteria closer together.

Serious problems with the decision model also arise for applications to information-controlled processing tasks. These problems are the most serious because they result in mispredictions. First, the model is limited to modeling trade-offs through symmetric changes in the initial positions of the criteria; otherwise hit and correct rejection rate are inversely-related (contrary to data). Second, the model trades speed and accuracy largely independently of one another. The range of the primary change in accuracy is accompanied by little change in latency; the largest change in latency is accompanied by little change in accuracy. Finally, the model was unable to vary accuracy over a range sufficient to account for data, even if the initial positions of the criteria were allowed to be equal (as was demonstrated by the fit to the speed condition in the experiment). It was argued that a single-criterion version of the model was not an allowable member of the class of possible models of the Hockley and Murdock (1987) framework.

Two alternative versions of the model were examined: (a) adding guessing to the original model; (b) modifying the decision process to be a zero-drift random walk. Neither of these alternatives was successful. On the other hand, two versions of the diffusion model (Ratcliff, 1978, 1981) provided reasonable accounts of the speed-accuracy experiment and parsimonious explanations of time-controlled processing tasks. Together, the single- and multiple-diffusion models offer a common framework for the linking of a decision model to a memory model with a localized or a distributed memory representation. Based on the problems with the original version of the Hockley and Murdock (1987) decision model, and the lack of success of its two variations, it seems that a major revision of the Hockley and Murdock (1987) model is required if the model is to compete with other candidate models.

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