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A Mathematical Model for Paced Serial Addition[1]

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The paced serial addition task is reminiscent of a series of tasks studied in the later 1950s which were concerned with recall under conditions of pacing. The information processing operations in this task are completed in about a second and the paradigm allows a large amount of data to be collected in a short time. Group performance shows a primacy effect lasting a few seconds followed by a steady state. In this paper, a model is developed for steady-state performance consisting of continuous time and discrete components that generalize to a semi-Markov model. The model accounts for results from the two experiments presented and deviations of the model from the data point out distinct strategies being employed by the subject. The components of the model have direct psychological interpretation and allow many of the processes underlying performance to be specified.

1. INTRODUCTION

The paced serial addition task (PSAT) is typical of a group of tasks studied around the late 1950s, which were all concerned with some aspect of paced performance (for example, Mackworth & Mackworth, 1959; Sampson, 1956; Kay, 1953; Kirchner, 1958; Pollack & Johnson, 1963; with a review by Posner, 1963). There were two main interests in these studies: first, the theoretical implications for memory span and information processing under continuous paced conditions and, second, a pragmatic concern with the performance of subjects on tasks similar to those found in industry and other practical situations. In this paper a mathematical model is developed for the PSAT which enables the processes underlying performance to be examined in terms of recent views and theories. This, therefore, provides a basis for the examination of these earlier studies in terms of the current approaches to memory and information processing (Ratcliff, 1974).

The PSAT is set apart from most experimental paradigms currently employed in the field of short-term processing (with the notable exception of Shepherd & Teghtsoonian, 1961; Donaldson & Murdock, 1968), because each trial lasts 5 to 10 min.,

[1] This work is based in part on my Ph.D. dissertation at Auckland University. I would like to thank my supervisor, Professor H. Sampson, for his advice and encouragement.

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but information processing operations, such as encoding, storage, and retrieval, take 1 to 2 sec to complete. The following are the main characteristics of the PSAT.

Each trial consists of 241 digits between 1 and 9, randomly ordered. These are presented to the subject one after the other. The subject's instructions are to add each digit to the one that immediately preceded it and call out the answer. The experimenter controls pacing rate (rate of presentation) and stimulus duration in the case of visual presentation. Both remain constant throughout the trial of 241 items. The subject is given no immediate feedback on the accuracy of his responses. The result at any serial position falls into one of three categories, correct response, error response, or omission.

Effects on group performance of variables, such as pacing rate, halves of task and stimulus duration, have been examined in some detail by Sampson and his co-workers (Sampson, 1956, 1958a, b; Sampson & MacNeilage, 1960; Corballis & Sampson, 1963). In these studies it was found that group performance along a trial showed an initial decay in proportion correct to a constant level of perfomance (Sampson & MacNeilage, 1960) and this can be represented by:

$$p_i = ae^{-i/\tau} + c, \tag{1}$$

where i is the position in the series; p_i is the proportion of correct responses at i; and c, τ, and a are fitted parameters that represent the final level of performance, range of initial decay in performance and initial level above c, respectively. This model can be used in comparing group trends and thus makes contact with much of the previous work on the PSAT.

The development of the paper is as follows: First, a description of an experiment is presented together with a brief description of important group results; second, a model for individual performance is developed (the main focus of the paper); third, this model is applied to the data from the experiment, fourth, a second experiment is presented and the model applied, and fifth; psychological processes involved in the task are discussed.

2. Experiment 1 and Group Performance

Method

Forty-eight male Auckland University students (unpaid volunteers), whose mean age was 21.3 years (standard deviation 1.5 years), were given the following written instructions.

The experiment you are about to take part in is designed to investigate aspects of dynamic human information processing. A series of numbers will be presented aurally from a tape recorder and will be evenly spaced. Your task is to add the present (*n*th) number to the previous

(($n - 1$)th) number in the series and call out the answer. For example, if the stimulus string was 9 4 3 8 2 7 4 ... the response string should be 13 7 11 10 9 11 ... The rate of presentation is too rapid for you to get 100% correct, so when you lose track, try and start again as soon as possible.

The first series of number is of 61 items, then the next three trials are of 241 items.

Following this, subjects were given an unpaced auditory example to see if the instructions had been understood. The criterion for proceeding with the experiment was the ability of subjects to perform this unpaced test without error.

Subjects were divided randomly into three groups, one per pacing rate. All subjects received a preliminary trial at 1.6 sec/digit using 61 items, then three trials at either 1.2, 1.6, or 2.0 sec/digit (one group at each pacing rate) using the same 241 digits for each trial, followed 1 week later by two trials of 241 digits under the same pacing conditions as before.

The long series of 241 items was made by splicing together four copies of a 61 item series, omitting the first digit in the last three series. Thus, each subject operated on the same series 20 times (4×5 trials) at the same pacing rate. This enabled estimates to be obtained of the number of error responses attributable to individual subject difficulties with the number sequence. The stimuli were presented from a Phillips cassette tape recorder and responses were recorded manually on a specially designed result sheet. Subjects were given a progress report on their performance at the end of each trial.

Groups Trends

The group model, Eq. (1), was fitted to the data using a nonlinear least-squares method. Using a χ^2 goodness-of-fit statistic, the group model was found to adequately represent the data (see Appendix A) and values of c and τ as a function of practice and presentation rate are shown in Fig. 1. Appendix A also gives two typical fits of Eq. (1) to the data, which demonstrate the primacy effect and asymptote of performance. It is important to know the range of the primacy effect because the individual model presented later is designed to deal only with asymptotic performance.

Group Error Performance

Two easily classifiable types of error response were observed and labeled as follows: "3" when the nth stimulus is added to the n − 2nd stimulus instead of the n − 1st; "4" when the previous response is added to the present stimulus. Other errors were not easily classifiable and were labeled "5."

A random block ANOVA design was used to test if the ratio of the different error types was the same under all conditions of practice. Results showed that the difference between the error types was significant ($\alpha << 0.01$ at each pacing rate) but the difference in error types with practice was not significant ($\alpha > 0.05$ at each pacing rate). This

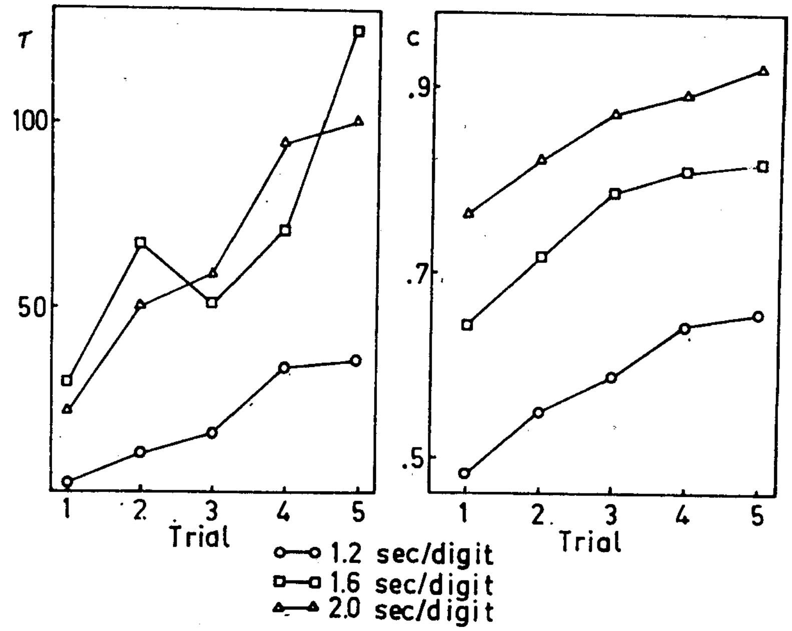

FIG. 1. Group model parameters c and τ as a function of practice and rate of presentation.

suggests that if errors are caused by a failure in the memory retrieval system, then the system operates in the same way as the subject becomes practiced. On this basis it is reasonable to group these three kinds of error into one category for further analyses.

3. The Model

Before proceeding with development of the model, it is necessary to investigate the nature of individual asymptotic performance. Let us define a response run as a string of correct responses and errors, not containing an omission, lying between two omissions. Similarly an omission run is a string of omissions, not containing a response, lying between two responses. It was found that the following hypotheses could be accepted:

(a) Mean length of a response run is independent of the length of the preceding omission run.

(b) Mean length of an omission run is independent of the length of the preceding response run.

(c) Mean length of an omission run is the same whether a correct response or an error response ends the preceding response run.

These hypotheses were tested using t-statistics calculated for each comparison for each subject under each practice condition. The following results were obtained for trial 1, 1.2 sec/digit (other results are similar and will not be presented). The

t-statistics for (a), (b), and (c) for subject 9 and (a) and (b) for subject 4 were significant at a 5% level. This can be attributed to a pair-adding strategy adopted by these subjects. This strategy arises when the subject finds his performance level is low and that more correct responses can be made by adding each successive pair of digits and so giving only every second possible response. This strategy is most easily seen using an autocorrelation analysis. The autocorrelation function at even shifts is significantly higher than at odd shifts, showing that every second response is of the same type. Apart from these two subjects, there was only one other significant statistic out of the remaining 42. These results show that assertions (a), (b), and (c) can be accepted.

In the PSAT there are very few errors, typically 5%, but 20% of omission runs are initiated by an error. Hypothesis (c) above shows that the two types of omission run can be grouped together. Also, as a first step, error responses are classified along with correct responses in the first model for response runs.

From a psychological point of view it seems that processes operating while a subject is engaged in a response run are different from those operating while engaged in an omission run. In a response run, the subject is capable of producing a certain number of responses before an interruption occurs, whereas an omission run gives an estimate of the recovery time from an interruption. Therefore, these two performance components are treated separately.

Response Run Model

Perhaps one of the simplest models that can be postulated for processes occurring in a response run is a geometric model. In this, the probability that an omission occurs at any position in a response run is $(1 - p)$ and this means that the probability of interruption is independent of the prior length of the run. Thus, the probability of occurrence of a response run of length k is

$$\Pr(\text{run length } k) = (1 - p)p^{k-1}. \tag{2}$$

Given a random sample of N response runs, the likelihood function L (joint probability density function for a random sample) is defined by

$$L = \prod_{i=1}^{\infty} [(1 - p)p^{i-1}]^{n_i}, \tag{3}$$

where n_i = number of runs of length i. From this the maximum likelihood estimate $\hat{p}$ is given by

$$\hat{p} = (\bar{x} - 1)/\bar{x}, \tag{4}$$

where $\bar{x} = \sum_{i=1}^{\infty} n_i i / \sum_{i=1}^{\infty} n_i$ is the mean response run length. Also the asymptotic variance estimate v_p^2 is given by

$$v_p^2 = p(1 - p)^2/N. \tag{5}$$

Minimum variance unbiased estimators of p and v^2 also exist (Chapman & Robson, 1960) and may be written

$$\hat{p}_u = \frac{\bar{x} - 1}{\bar{x} - (1/N)}$$

$$v_u^2 = \frac{p_u(1 - p_u)^2}{N} \left(\frac{1}{1 + (p_u/N) - (2/N)} \right).$$

The sample size is large for data in Experiment 1 and so the unbiased estimates are essentially the same as maximum likelihood and asymptotic variance estimates.

This model suggests two main questions about the psychological processes involved:

(a) How is the system organized while the subject is operating in a response run?

(b) What processes result in or precede interruptions?

Model for Correct Responses and Errors

The response run model, Eq. (2), can be rewritten as a Markov process with the following state transition matrix and initial probability vector:

$$\begin{array}{c} \\ R \\ 0 \end{array} \begin{array}{c} \begin{array}{cc} R & 0 \end{array} \\ \begin{pmatrix} p & 1-p \\ 0 & 1 \end{pmatrix} \end{array} \quad \begin{array}{c} \text{Pr(initial)} \\ \begin{pmatrix} 1 \\ 0 \end{pmatrix} \end{array} \tag{6}$$

where R is the response and 0 is the omission state. This formulation is equivalent to Eq. (2) and the maximum likelihood estimate of p is just the usual estimate of the transition probability p:

$$p = \frac{\text{number of transitions } R \text{ to } R}{\text{total number of transitions from } R} = \frac{n\bar{x} - n}{n\bar{x}} = \frac{\bar{x} - 1}{\bar{x}}.$$

Let us now extend Eq. (6) to account for the correct response error response distinction. Assuming independence, the state transition matrix and initial probability vector become:

$$\begin{array}{c} \\ 1 \\ X \\ 0 \end{array} \begin{array}{c} \begin{array}{ccc} 1 & X & 0 \end{array} \\ \begin{pmatrix} r & q & 1-r-q \\ s & t & 1-s-t \\ 0 & 0 & 1 \end{pmatrix} \end{array} \quad \begin{array}{c} \text{Pr(initial)} \\ \begin{pmatrix} 1-u \\ u \\ 0 \end{pmatrix} \end{array} \tag{7}$$

where states 1, X, and 0 represent correct response, error response, and omission, respectively; u represents the probability of starting a response run with an error response; and r, q, s, and t are transition probabilities. The proportion of errors is small with respect to correct responses so that the parameter r in Eq. (7) is approximately the same as p in Eq. (6); therefore, the two models converge. Tests of this model are described as the model is applied to the data of Experiment 1.

Because response runs and omission runs are independent ((a), (b), and (c) above), Eq. (7) is written as a finite absorbing Markov chain (treated fully in Kemeny & Snell, 1960, Chap. 3). This is done because each response run is treated as an independent Bernoulli trial and parameters are estimated on this series of Bernoulli trials over the experimental trial. After describing the omission run model, it will be shown how this model and the omission run model can be combined to form a semi-Markov model.

Omission Run Model

It was argued earlier that the number of responses omitted gives a crude measure of recovery time. Let us suppose that recovery takes place through a-stages, the times $T_1, \ldots, T_a$ spent in these stages being independently exponentially distributed with probability density function $\rho e^{-\rho t}$. Then recovery time T is $T_1 + \cdots + T_a$ and has a gamma distribution

$$f(t) = \frac{\rho(\rho t)^{a-1} e^{-\rho t}}{(a-1)!}. \tag{8}$$

The assumption that all processes in recovery proceed at the same rate is a desirable approximation because of the small omission run lengths which give few degrees-of-freedom in goodness-of-fit tests. Also, if a is fixed prior to fitting the data, then a one-parameter distribution is obtained (just as an exponential is "really" a gamma with parameter 1). It was found that $a = 3$ gave the best fit to a small sample of data. These two assumptions may seem a little restrictive, but we shall see that the resulting estimates of ρ show stable trends for each subject and provide an estimate of the average rate of recovery. The fact that Eq. (8) was derived using assumptions of independence and serial processing does not mean that the processes involved must have these properties. Townsend (1972) and Murdock (1971) have shown that models derived using such assumptions can be mimicked by parallel processing and hybrid systems.

Although the probability density function for recovery (Eq. (8)) is a continuous function, the data are in discrete form. Therefore, to perform parameter estimation, Eq. (8) must be manipulated to give discrete expected values of recovery.

The cumulative distribution function of the gamma is given by

$$F(t) = 1 - e^{-\rho t}\left(1 + \frac{\rho t}{1!} + \cdots + \frac{(\rho t)^r}{r!}\right).$$

The probability of recovery in the interval t_{i-1} to t_i for $a = 3$ is given by

$$\begin{aligned}\phi(t_i, \rho) &= F(t_i) - F(t_{i-1}) \\ &= \{e^{-\rho(t_i - t_1)} - e^{-\rho t_i}\} + \{\rho(t_i - t_1)e^{-\rho(t_i - t_1)} - \rho t_i e^{-\rho t_i}\} \\ &\quad + \{\tfrac{1}{2}\rho^2(t_i - t_1)^2 e^{-\rho(t_i - t_1)} - \tfrac{1}{2}\rho^2 t_i^2 e^{-\rho t_i}\},\end{aligned} \tag{9}$$

where t_i is the time to the ith stimulus following the first stimulus after interruption. This, therefore, is the probability density function for an omission run of length i.

Given a random sample of N omission runs, with n_i of length i, the likelihood function is

$$L = \prod_{i=1}^{\infty} [\phi(t_i, \rho)]^{n_i}.$$

Now

$$\ln(L) = \sum_{i=1}^{\infty} n_i[-\rho t_i + \ln\{e^{\rho t_1} - 1 + \rho(t_i - t_1)e^{\rho t_1} - \rho t_i + \tfrac{1}{2}\rho^2(t_i - t_1)^2 e^{\rho t_1} - \tfrac{1}{2}\rho^2 t_i^2\}].$$

Thus, the maximum likelihood estimate $\hat{\rho}$ satisfies

$$\left.\frac{\partial \ln(L)}{\partial \hat{\rho}}\right|_{\rho=\hat{\rho}} = 0,$$

i.e.,

$$\sum_{i=1}^{\infty} n_i \left[-t_i + \left\{\frac{t_i e^{\hat{\rho} t_1} - t_i - \hat{\rho} t_i^2 + \hat{\rho} t_i(t_i - t_1)e^{\hat{\rho} t_1} + \tfrac{1}{2}\hat{\rho}^2(t_i - t_1)^2\, t_1 e^{\hat{\rho} t_1}}{e^{\hat{\rho} t_1} - 1 + \hat{\rho}(t_i - t_1)e^{\hat{\rho} t_1} - \hat{\rho} t_i + \tfrac{1}{2}\hat{\rho}^2(t_i - t_1)^2 e^{\hat{\rho} t_1} - \tfrac{1}{2}\hat{\rho}^2 t_i^2}\right\}\right] = 0. \quad (10)$$

The solution of (10) can be obtained numerically for each set of data (the value of n_i).

The asymptotic variance of ρ can be found as before as follows:

$$\frac{\partial^2 \ln(L)}{\partial \rho^2} = \sum_{i=1}^{\infty} n_i \left[\frac{g(t_i, \rho)}{\phi(t_i, \rho)e^{\rho t_i}}\right],$$

where

$$g(t_i, \rho) = \frac{-(t_i e^{\rho t_1} - t_i - \rho t_i^2 + \rho t_i(t_i - t_1)e^{\rho t_1} + \tfrac{1}{2}\rho^2(t_i - t_1)^2\, t_1 e^{\rho t_1})^2}{[\phi(t_i, \rho)e^{\rho t_i}]}$$
$$+ [t_i^2(e^{\rho t_1} - 1) + \rho t_1 e^{\rho t_1}(2t_i^2 - 3t_i t_1 + t_1^2) + \tfrac{1}{2}\rho^2(t_i - t_1)^2\, t_1^2 e^{\rho t_1}].$$

Now

$$E\left[\frac{\partial^2 \ln(L)}{\partial \rho^2}\right] = \sum_{i=1}^{\infty} E[n_i] \left[\frac{g(t_i, \rho)}{\phi(t_i, \rho)e^{\rho t_i}}\right]$$
$$= \sum_{i=1}^{\infty} N\phi(t_i, \rho) \left[\frac{g(t_i, \rho)}{\phi(t_i, \rho)e^{\rho t_i}}\right]$$
$$= N \sum_{i=1}^{\infty} e^{-\rho t_i} g(t_i, \rho).$$

Thus, the asymptotic variance is given by:

$$v_\rho^2 = -\frac{1}{N\sum_{i=1}^{\infty} e^{-\rho t_i} g(t_i, \rho)}. \quad (11)$$

Semi-Markov Model

The model for correct and error responses and the omission run model can be combined to produce a semi-Markov model (Barlow & Proschan, 1965; Hunter, 1969). A semi-Markov process can be defined as follows.

Consider a stochastic process which moves from one to another of a finite number of states $A_1, \ldots, A_m$ with successive states forming a Markov chain, whose transition matrix is given by $X = (x_{ij})$. Furthermore, the process stays in a given state a random length of time whose distribution function $Y_{ij}(t)$ depends on the state being visited, A_i, as well as the one to be visited next, A_j.

Let us write Z_t for the state being occupied at time t, then $\{Z_t\,;\, t \geqslant 0\}$ is called a semi-Markov process. For the PSAT, the transition matrix X is given by:

$$\begin{array}{c} \\ 1 \\ X \\ 0 \end{array} \begin{array}{c} \begin{array}{ccc} 1 & X & 0 \end{array} \\ \begin{pmatrix} r & q & 1-r-q \\ s & t & 1-s-t \\ 1-u & u & 0 \end{pmatrix} \end{array} \tag{12}$$

and the time distribution $Y(t)$ is given by:

$$\begin{array}{c} \\ 1 \\ X \\ 0 \end{array} \begin{array}{c} \begin{array}{ccc} 1 & X & 0 \end{array} \\ \begin{pmatrix} g(t) & g(t) & g(t) \\ g(t) & g(t) & g(t) \\ \phi(t_i) & \phi(t_i) & \phi(t_i) \end{pmatrix} \end{array} \tag{13}$$

where $g(t)$ is a distribution function which is zero for t greater than the interstimulus interval. It should be noted that at fast rates of presentation one omission run may immediately follow another because the subject may recover, get ready to produce the next response but fail because the next stimulus has arrived. This would lead to a modification of Eq. (12) with a nonzero entry in cell 0, 0.

Although this model unifies the response and omission run models, it goes no further than these models and the experiments will be analyzed in terms of the component models.

4. Results from Application of the Model to Experiment 1

The response and omission run models are evaluated in much the same way and so are treated together in this section. A χ^2-statistic is used to test these two models using observed and expected values of the number of runs of each run length.[3]

[3] In fitting this model the first response run was ignored on the assumption that this accounted for the primacy effect.

TABLE 1

Individual Model Results for Experiment 1

Subject	Pacing rate (sec/ digit)	p	v_p [a]	χ_p^2 [b]	dfp [c]	ρ	v_ρ	χ_ρ^2	$df\rho$	Number of runs
1	1.2	0.280	0.047	0.38	1	1.62	0.12	8.26	1	67
2	1.2	0.614	0.042	0.71	2	1.75	0.15	0.15	1	51
3	1.2	0.766	0.034	5.41	2	1.72	0.17	0.49	1	37
4	1.2	0.165	0.038	—	—	2.24	0.15	37.39	1	81
5	1.2	0.317	0.042	0.37	2	3.26	0.24	2.14	1	84
6	1.2	0.557	0.048	3.59	2	1.26	0.11	3.55	3	47
7	1.2	0.717	0.037	0.81	2	1.99	0.19	2.34	1	41
8	1.2	0.415	0.051	4.52	2	1.32	0.10	1.64	3	55
9	1.2	0.235	0.043	0.15	1	1.99	0.14	10.62	1	75
10	1.2	0.491	0.047	6.71	2	1.71	0.13	1.49	2	58
11	1.2	0.581	0.051	2.51	2	0.86	0.08	8.08	3	39
12	1.2	0.569	0.047	3.64	2	1.27	0.11	5.38	3	47
13	1.2	0.509	0.049	0.42	1	1.38	0.11	0.97	2	52
14	1.2	0.767	0.032	9.81	2	3.92	0.44	—	—	41
15	1.2	0.578	0.042	0.69	2	2.68	0.22	1.55	1	60
16	1.2	0.562	0.045	7.67	2	1.65	0.14	0.86	2	53
17	1.6	0.910	0.021	—	—	3.27	0.61	—	—	18
18	1.6	0.694	0.036	2.94	3	2.03	0.19	0.05	1	50
19	1.6	0.728	0.036	1.14	2	1.95	0.20	0.03	1	41
20	1.6	0.284	0.046	0.66	1	1.16	0.08	13.57	2	68
21	1.6	0.683	0.039	5.81	2	2.08	0.20	—	—	45
22	1.6	0.673	0.036	5.53	3	2.94	0.29	—	—	46
23	1.6	0.765	0.032	0.69	2	2.55	0.27	0.45	1	45
24	1.6	0.744	0.035	1.24	2	1.73	0.17	—	—	41
25	1.6	0.921	0.020	—	—	—	—	—	—	14
26	1.6	0.742	0.038	0.01	1	0.99	0.10	—	—	34
27	1.6	0.669	0.037	3.26	3	2.69	0.26	—	—	53
28	1.6	0.518	0.043	3.69	2	1.81	0.14	2.02	1	65
29	1.6	0.871	0.024	—	—	3.24	0.50	—	—	26
30	1.6	0.797	0.030	0.74	1	2.64	0.30	—	—	37
31	1.6	0.856	0.025	—	—	2.12	0.36	—	—	28
32	1.6	0.583	0.044	8.42	2	1.19	0.10	3.91	2	53
33	2.0	0.888	0.024	—	—	2.23	0.35	—	—	21
34	2.0	0.818	0.028	—	—	3.66	0.67	—	—	36
35	2.0	0.765	0.032	0.11	2	2.95	0.38	—	—	43
36	2.0	0.881	0.023	—	—	1.90	0.27	—	—	23

Table continued

TABLE 1 (*continued*)

Subject	Pacing rate (sec/digit)	p	v_p [a]	χ_p^2 [b]	dfp [c]	ρ	v_ρ	χ_ρ^2	$df\rho$	Number of runs
37	2.0	0.855	0.025	—	—	—	—	—	—	30
38	2.0	0.568	0.043	0.85	2	1.20	0.10	4.88	2	58
39	2.0	0.620	0.039	9.07	3	2.16	0.19	—	—	60
40	2.0	0.750	0.125	—	—	—	—	—	—	4
41	2.0	0.888	0.023	—	—	2.35	0.36	—	—	23
42	2.0	0.796	0.031	0.77	1	1.44	0.15	0.29	1	35
43	2.0	0.791	0.031	1.38	1	1.98	0.22	—	—	37
44	2.0	0.793	0.030	5.26	2	2.18	0.25	—	—	38
45	2.0	0.884	0.024	—	—	2.53	0.41	—	—	22
46	2.0	0.914	0.020	—	—	3.18	0.69	—	—	18
47	2.0	0.812	0.029	0.03	1	3.64	0.67	—	—	35
48	2.0	0.896	0.022	—	—	3.28	0.69	—	—	21

Note. These results are for trial 1 at the three pacing rates. See Table 1A for further data.
[a] v is the square root of the asymptotic variance.
[b] A small table containing critical values of χ^2 is presented below for direct comparison.
[c] Degrees-of-freedom is given by number of frequency classes minus two. Frequency classes are grouped when the number of entries is less than five thus reducing degrees-of-freedom.

TABLE 1A

Critical Values of χ^2 for Various Probability Levels α and Degrees-of-Freedom

	Degrees-of-freedom			
α	1	2	3	4
0.01	6.64	9.21	11.34	13.28
0.05	3.84	5.99	7.82	9.49
0.20	1.64	3.22	4.64	5.99
0.50	0.46	1.39	2.37	3.36
0.80	0.06	0.45	1.01	1.65

Response and Omission Run Models

An illustrative sample of results for the response and omission run models applied to Experiment 1 are shown in Table 1 and Fig. 2. In Table 1 it can be seen that some of the χ^2 values are statistically significant even at a probability level of $\alpha = 0.01$, which means that the data are not in full accord with the postulated probability density function. Several of the worst fits of the models to the data for trial 1 at 1.2 sec/digit are now examined.

Subjects 1, 4, and 9 developed a strategy of adding every successive pair of digits;

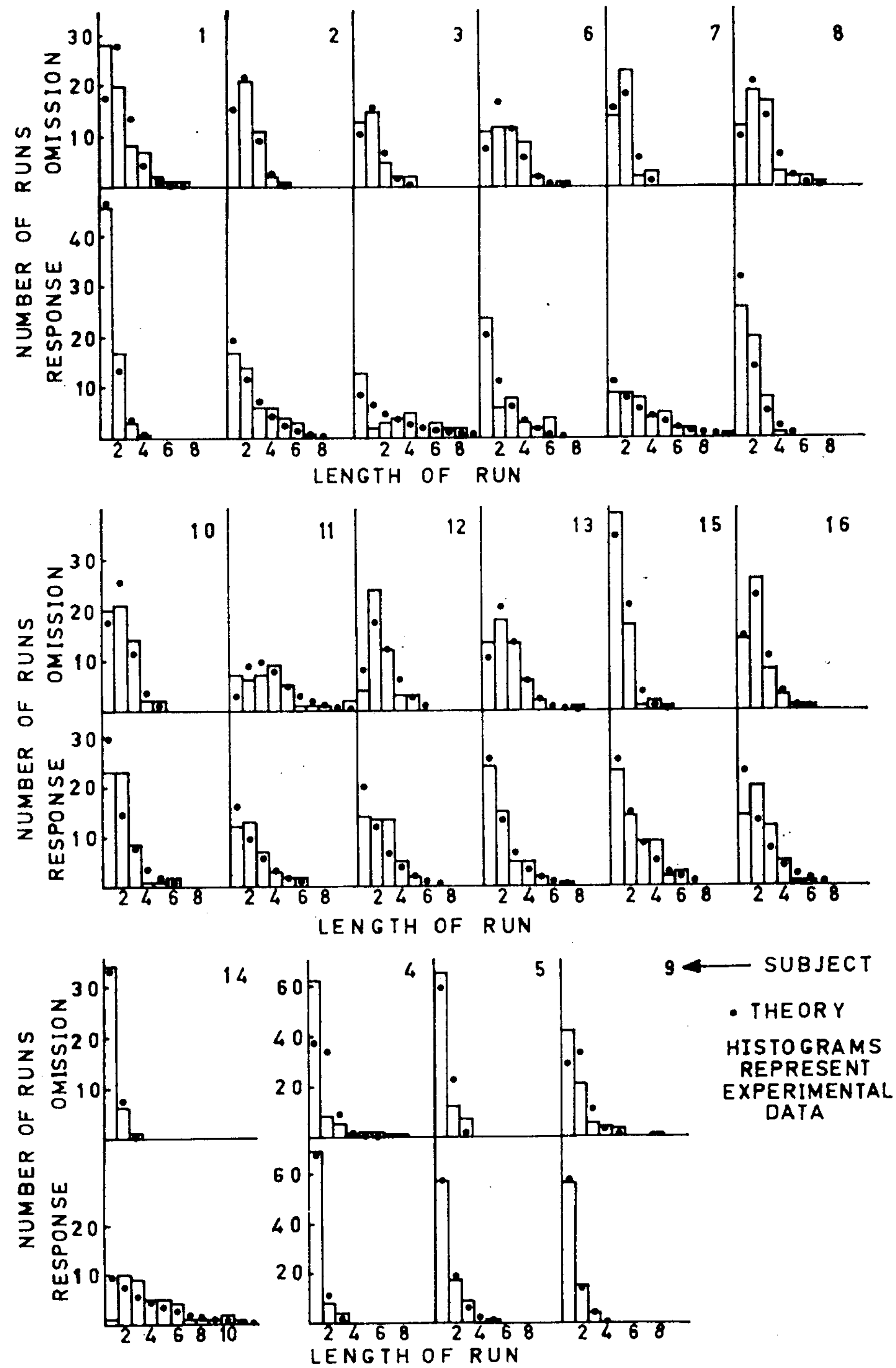

FIG. 2. Experimental results and theoretical fits to response and omission run models for trial 1 at 1.2 sec/digit (16 subjects) in Experiment 1.

this strategy can be seen from an autocorrelation analysis. The omission run model fits poorly because of the large number of omission runs of length 1, but the response run model fits well because there are few frequency classes. Thus, an obvious strategy accounts for the poor fits of the model.

Subjects 10, 14, and 16 had fewer response runs of length one than would be expected from the model, leading to large χ^2 values. There are two possible explanations for this.[4] First some subjects may have been adopting the strategy of only beginning a response run when reasonably sure of not stopping after one response (see Experiment 2 for further justification of this). Second the processing system is acting differently for the first compared with later items in the response run. This is taken into account in the initial probability vector of the model for correct responses and errors, Eq. (7), and the omission to response transitions in the semi-Markov model, Eq. (12). In Fig. 2 it can be seen that good fits could be obtained by ignoring runs of length 1 for subjects 10, 14, and 16.

The two models appear to fit individual data quite well for trial 1 at 1.2 sec/digit except when an easily identifiable strategy is employed. A suitable test for overall goodness-of-fit of the response and omission run models is made by computing the overall χ^2 for the 16 subjects, excluding the above cases. If we assume that the runs for different subjects are independent, then $\bar{X}^2 = \sum_{i=1}^{13} \chi_i^2$ is distributed chi-square with m degrees-of-freedom, where m is the sum of the degrees of freedom for the χ_i^2 statistic of each subject. The value of $\bar{X}^2$ for response runs is 23.6 with 21 degrees-of-freedom ($\alpha \approx 0.30$) and $\bar{X}^2$ for omission runs is 27.6 with 23 degrees of freedom ($\alpha \approx 0.25$). Both these values are acceptable so with the above exceptions we can conclude that the response and omission run models fit well over the whole group of subjects.

At slower rates of presentation, the χ^2 test cannot be applied in many cases because of the small number of runs leading to no degrees-of-freedom.

These goodness-of-fit results are encouraging but any model should provide further evidence of its usefulness such as stable parameter trends. Fig. 3 shows changes in p and ρ with practice over the 5 trials for 16 subjects at 1.2 sec/digit. Also included are error bars that represent the square root of the variance estimate for each parameter value. It can be seen that these curves are regular to within the standard deviation estimates and a simple binomial test shows that p and ρ increase with practice. These show that stable trends for each individual are extracted from the data by the response and omission run models.

Although no individual subject was tested at different rates of presentation in Experiment 1, a parametric one-way analysis of variance can be performed on the data in Table 1 (separate analyses for p and ρ values) to test if p and ρ change with

[4] It was felt necessary to provide a discussion of these individual deviations because most occurred consistently with practice. For example, value of χ^2 (2 df) for response run fits for subject 16 over the five trials at 1.2 sec/digit are 7.67, 6.00, 9.33, 7.82, 4.28, and in each case the large χ^2 value arises from a small number of runs of length 1.

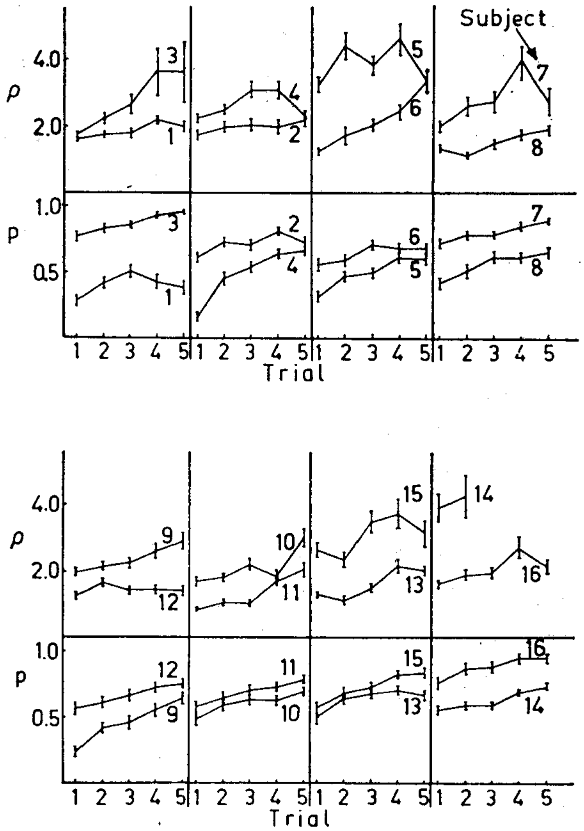

FIG. 3. Variation of parameters p and ρ with practice for the 16 subjects tested at 1.2 sec/digit in Experiment 1.

rate of presentation. The variances of the individual parameters are not constant so strictly speaking the ANOVA should not be used; however, the ANOVA is a robust test and should at least indicate trends. The computed F values are $F(p) = 17.18$ and $F(\rho) = 2.30$ and $F_{0.05}(2,47) = 3.20$. Because the F values lie well away from 3.20, it is reasonable to conclude that p increases as rate of presentation decreases and ρ is constant with rate of presentation.

Model for Correct and Error Responses

Before developing tests of this model and applying them to Experiment 1, two further analyses of error performance are discussed.

Because the task was repeated 20 times over the same series of 60 sums (see the Method section), it is possible to examine the performance of each subject on every digit combination in the series. Several kinds of idiosyncracies occur, for example a particular addition may be incorrect ($7 + 6 = 15$), a subject may fail at a certain addition ($8 + 5 = ?$). In Experiment 1 only 15% of total errors at 1.2 sec/digit could be identified this way (5% and 7% at 1.6 and 2.0 sec/digit, repectively); therefore, to a good approximation this kind of error can be ignored in the following analyses.

Earlier it was argued that the processing system acts differently for the first compared with later items in the response run. The pattern of errors initiating response runs supports this argument. For example, very few response runs begin with a response adding error (type 4). Therefore, errors beginning runs are not used in calculating parameters s and t but are used in calculating u.

To test the model for correct and error responses we must determine if the process can be represented by a first order Markov chain. As was mentioned earlier, if the response run model is adequate and if there are few errors, then the transition correct to correct response (1 to 1) has constant probability and only first-order effects. This property of the 1 to 1 transition provides a basis for tests of the other transitions. The small number of errors in any subject's response protocol is a major problem in providing reasonable tests for first order dependence in the transitions 1 to X (error), X to 1, and X to X for individual subjects. However, this lack of data can be overcome by combining data for all subjects for each condition of pacing and practice. This grouping is probably reasonable because two of the following tests are based on the geometric distribution and the sum of geometric distributions with similar parameters is very nearly geometric. Therefore, the following results should not be treated as strict hypothesis tests.

Consider runs of responses starting with a correct response, preceded by an omission and containing an error. If there were constant probability of an error occurring at any position in a response run, then the distribution of response run length up to an error would be geometric

$$\Pr\,(k \text{ correct followed by an error}) = r^{k-1}q. \tag{14}$$

If the parameter q increased with position in the response run (e.g., a "fatigue" effect) then the value of r calculated from Eq. (14) would be larger than the average value of r calculated for the group and if q decreased (e.g., "confusion" effect) then the computed value of r would be smaller. Thus, the comparison of values of r and χ^2 goodness-of-fit test to Eq. (14) are suitable tests for the transition 1 to X.

Similar tests are made to see if an error affects only the next response and not performance later in the response string. The distribution of correct response run length following an error (not beginning a run) and up to an omission or error is again geometric with parameter r:

$$\begin{aligned}&\Pr(k \text{ correct preceded by an error and followed by an}\\ &\quad \text{omission or error}) = sr^{k-1}\,(1 - r).\end{aligned} \tag{15}$$

If an error had disruptive effects after the next response then the value of r obtained from Eq. (15) would be smaller than the group value of r and a high value of χ^2 would be expected.

Values of the transition probablilities are calculated using

$$r = \frac{N(1 \text{ to } 1)}{N(1 \text{ to } 1) + N(1 \text{ to } X) + N(1 \text{ to } 0)}, \quad q = \frac{N(1 \text{ to } X)}{N(1 \text{ to } 1) + N(1 \text{ to } X) + N(1 \text{ to } 0)}$$

$$s = \frac{N(X \text{ to } 1)}{N(X \text{ to } 1) + N(X \text{ to } X) + N(X \text{ to } 0)}, \quad t = \frac{N(X \text{ to } X)}{N(X \text{ to } 1) + N(X \text{ to } X) + N(X \text{ to } 0)} \tag{16}$$

where $N(A \text{ to } B)$ is the number of transitions from state A to state B. A z-statistic may be used to compare r with r_q and r_s (the values of r calculated from Eq. (14) and (15), respectively) and this is calculated from

$$Z = \frac{x_1 - x_2}{x(1-x)(1/N_1 + 1/N_2)} \quad \text{and} \quad x = \frac{x_1 N_1 + x_2 N_2}{N_1 + N_2}, \tag{17}$$

where x_i is the value of the parameter i and N_i is the total number of transitions, i.e., the bottom line of the expressions in Eq. (17).

Table 2 contains these comparisons together with group parameters for p, r, q, s,

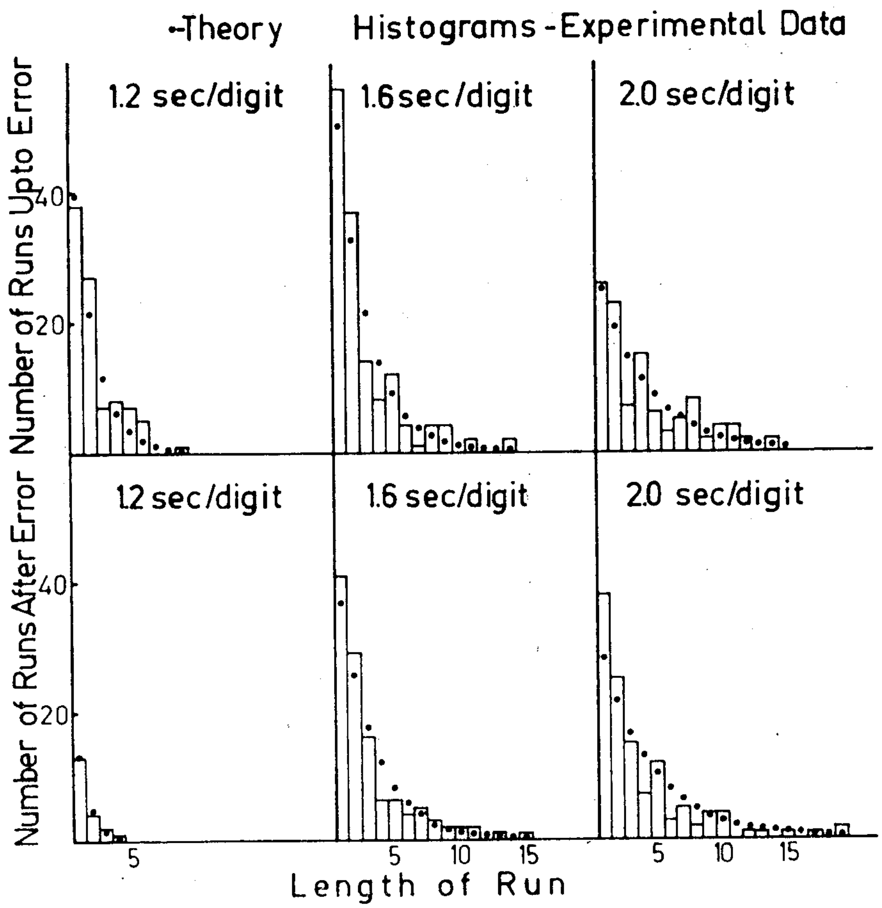

FIG. 4. Group histograms and fits of the geometric distribution tests for constant probability of q and s.

TABLE 2

Group Average Results of Tests and Parameter Estimations for the Model of Correct and Error Responses in Experiment 1

	Trial 1 1.2 sec/digit				Trial 1 1.6 sec/digit				Trial 1 2.0 sec/digit			
Parameter name	Parameter value	No. of transitions or runs	χ^2 Statistic	z Statistic	Parameter value	No. of transitions or runs	χ^2 Statistic	z Statistic	Parameter value	No. of transitions or runs	χ^2 Statistic	z Statistic
u	0.040	898			0.030	662			0.041	504		
r	0.468	1847			0.695	2238			0.809	2718		
q	0.046	1847			0.067	2238			0.062	2718		
s	0.174	87			0.507	201			0.661	177		
t	0.044	87		0.09, (t, q)	0.055	201		0.66, (t, q)	0.057	177		0.27, (t, q)
r_q [a]	0.528	85	3.96, $df=3$	1.08, (r_q, r)	0.673	144	8.23, $df=5$	0.56, (r_q, r)	0.764	107	10.71, $df=6$	1.16, (r_q, r)
r_s [b]	0.355	20	—	1.01, (r_s, r)	0.691	119	2.75, $df=4$	0.09, (r_s, r)	0.774	124	4.35, $df=5$	0.97, (r_s, r)
p [c]	0.535	888		3.28, (p, r)	0.741	662		2.28, (p, r)	0.829	504		1.06, (p, r)

[a] r_q is determined from Eq. (14).
[b] r_s is determined from Eq. (15).
[c] p is the mean p calculated from the individual values, Table 5.

TABLE 3

Results of the Correct and Error Response Model for Subject 10, 1.2 sec/digit

Parameter name	Trial 1		Trial 2		Trial 3		Trial 4		Trial 5	
	Parameter value	No. of transitions	Parameter value	No. of transitions	Parameter value	No. of transitions	Parameter value	No. of transitions	Parameter value	No. of transitions
u	0.068	59	0.096	52	0.151	53	0.040	53	0.080	50
r	0.367	109	0.455	101	0.468	111	0.410	100	0.438	121
q	0.101	109	0.188	101	0.189	111	0.250	100	0.248	121
s	0.273	11	0.250	24	0.400	25	0.353	33	0.525	40
t	0	11	0.208	24	0.160	25	0.243	33	0.250	40
z-statistic ($t = q$)	1.106		0.224		0.338		0.081		0.025	

t, u (simply the proportion of response runs beginning with an error) and values of χ^2 for the fits of Eqs. (14) and (15). Figure 4 shows histograms and theoretical fits for Eqs. (14) and (15). The values of χ^2 together with the nonsignificant z-statistics for r, r_q and r, r_s comparisons indicate that there are only first-order effects.

An obvious psychological hypothesis for error responses is that they occur randomly in a response run with constant probability q. An alternative hypothesis is that if mistakes were made in perceiving stimuli, then more pairs of error responses than chance would be expected and t would be greater than q. Comparisons in Table 2 show t is approximately the same as q.

Subject 10 accounted for one-third of the error responses of the 1.2 sec/digit group at trial 5, and a significant proportion of earlier trials. It is instructive to tabulate the values of the parameters r, q, s, t and u for this subject over the 5 trials and these are shown in Table 3.

The value of the z-statistic was not significant for the comparison between t and q as for the group. The values of r and q increase from the first trial and their behaviour is relatively smooth, r as expected from the response run model results and q by choice of strategy. The subject stated after the first or second trial that he was going to make as many responses as possible, correct or error and so make more correct responses than otherwise. This indicates that subjects may have some control over error performance. The parameter s increases with practice but more erratically than r or q because of the small number of transitions from the error state. This suggests that with practice the subject can learn not to permit an error to interrupt performance.

Therefore from this and the group data, the Markov model (Eq. (7)) adequately represents response run performance. A full description of asymptotic performance of any subject requires 5 parameters, r, q, s, u, and ρ. For purposes of comparison between subjects or investigation of overall performance, the number of parameters can be reduced to two namely p and ρ.

5. Experiment 2

A second experiment was performed to study changes in model parameters with change in pacing rate for individual subjects. It seemed reasonable to ask subjects to give an account of what operations were used (Reitman, 1970) so that these reports could be used along with experimental results to deduce psychological processes involved in the task.

Method

Six subjects were chosen from those who participated in Experiment 1 in order that practice effects be minimized. The subjects were tested at four pacing rates beginning with 0.8 sec/digit and increasing by 0.4 sec/digit at each subsequent trial.

To avoid the pair-adding strategy the following verbal instructions were presented:

Most subjects at the fastest presentation rate just add successive pairs of digits, e.g.,

2 7 4 6 3 9 ···
9 10 12···

This strategy enables them to get by without any memory component and memory recall is what I am trying to study in this experiment. So that you don't use this strategy, I want you to concentrate on getting as long runs of responses as possible and when interruption occurs, to try and get back on the track as soon as possible.

Following this a practice trial of 241 digits at 0.8 sec/digit was presented followed by the four experimental trials of 241 digits. The series was the same as used in Experiment 1 and was presented through 8Ω Sound headphones from a Ferrograph tape recorder. After each trial subjects were asked for introspective reports.

Results

There is a design problem in this experiment in that any changes in p and ρ could be attributed to practice. The subjects in this experiment took part in Experiment 1 and these results (Fig. 3) can be used to give estimates of the size of practice effects. A linear regression line can be fitted to the practice data for each subject's performance in Experiment 1. This line can be expected to be an upper limit on the increase in the parameter with practice because the subjects are highly practiced and their performance should be approaching asymptote.

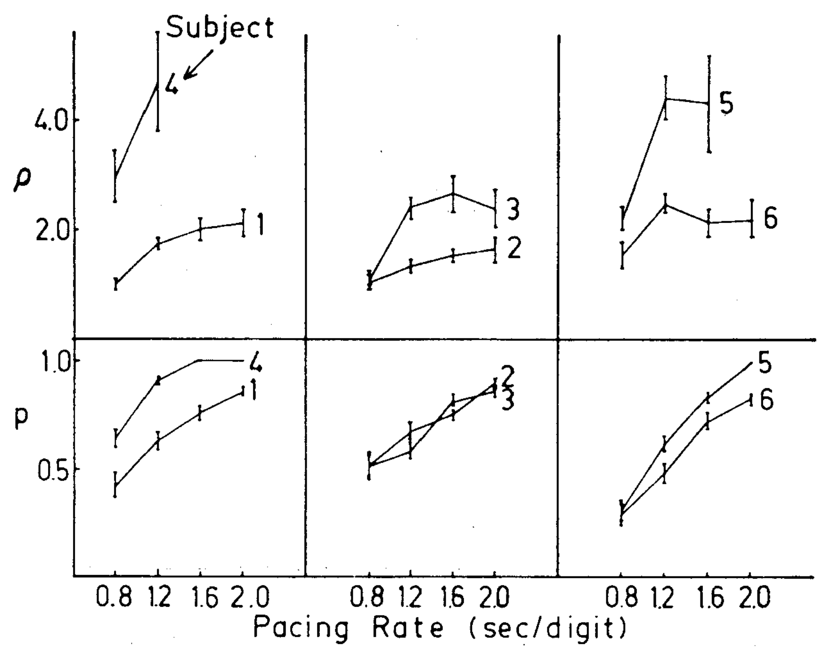

FIG. 5. Variation of parameters p and ρ with rate of presentation for six subjects in Experiment 2.

For the parameter p, the difference in parameter values between adjacent pacing rates (e.g., $p_{1.2}$–$p_{0.8}$) is significantly greater than the slope of the regression line at a 5% level except for 3 comparisons out of 15. These results suggest that even for individual subjects the hypothesis that p increases with decrease in pacing rate can be accepted.

For the parameter ρ slopes and estimates of standard deviations in slopes of linear regressions for practice from Experiment 1 and pacing rate from Experiment 2 were calculated and there was no significant difference in these slopes. Results at 0.8 sec/digit

TABLE 4

Results for the Individual Model for Experiment 2

Subject	Pacing rate (sec/digit)	p	v_p [a]	χ_p^2	df	ρ	v_ρ	χ_ρ^2	df	No. of runs
1(8)[b]	0.8	0.423	0.059	0.72	1	1.08	0.10	3.42	4	41
1	1.2	0.630	0.042	0.40	2	1.75	0.15	0.19	2	48
1	1.6	0.754	0.033	0.17	2	2.03	0.20	—	0	42
1	2.0	0.857	0.024	1.13	1	2.13	0.28	—	0	28
2(12)	0.8	0.514	0.057	17.69	1	1.03	0.10	2.17	4	37
2	1.2	0.678	0.044	5.49	2	1.35	0.13	0.10	2	37
2	1.6	0.760	0.035	0.41	1	1.55	0.16	0.13	1	38
2	2.0	0.899	0.022	—	0	1.66	0.23	—	0	22
3(9)	0.8	0.520	0.058	4.39	1	1.10	0.11	5.95	4	35
3	1.2	0.590	0.043	5.53	2	2.43	0.21	6.30	2	52
3	1.6	0.813	0.029	1.23	1	2.68	0.32	—	0	35
3	2.0	0.864	0.024	0.70	1	2.40	0.35	—	0	27
4(7)	0.8	0.643	0.041	14.56	2	2.95	0.26	0.53	1	49
4	1.2	0.910	0.022	—	0	4.66	0.88	—	0	17
5(5)	0.8	0.314	0.053	1.21	1	2.94	0.22	8.96	1	66
5[c]	0.8	0.314	0.053	1.21	1	2.24	0.20	1.14	2	45
5	1.2	0.625	0.038	3.58	3	4.43	0.44	—	0	59
5	1.6	0.838	0.026	—	0	4.33	0.82	—	0	27
6[b]	0.8	0.292	0.054	1.62	1	1.56	0.14	0.50	3	51
6	1.2	0.485	0.044	2.93	1	2.48	0.19	4.19	1	67
6	1.6	0.725	0.035	0.10	1	2.16	0.24	—	0	34
6	2.0	0.879	0.024	—	0	2.21	0.36	—	0	20

[a] v_p and v_ρ are the square root of the asymptotic variance for p and ρ.

[b] The number in parentheses refers to the subject's identification in Experiment 1. Subject 6 was a pilot subject at 0.8 sec/digit and the results were not presented.

[c] Adjusted omission run data, see Fig. 7.

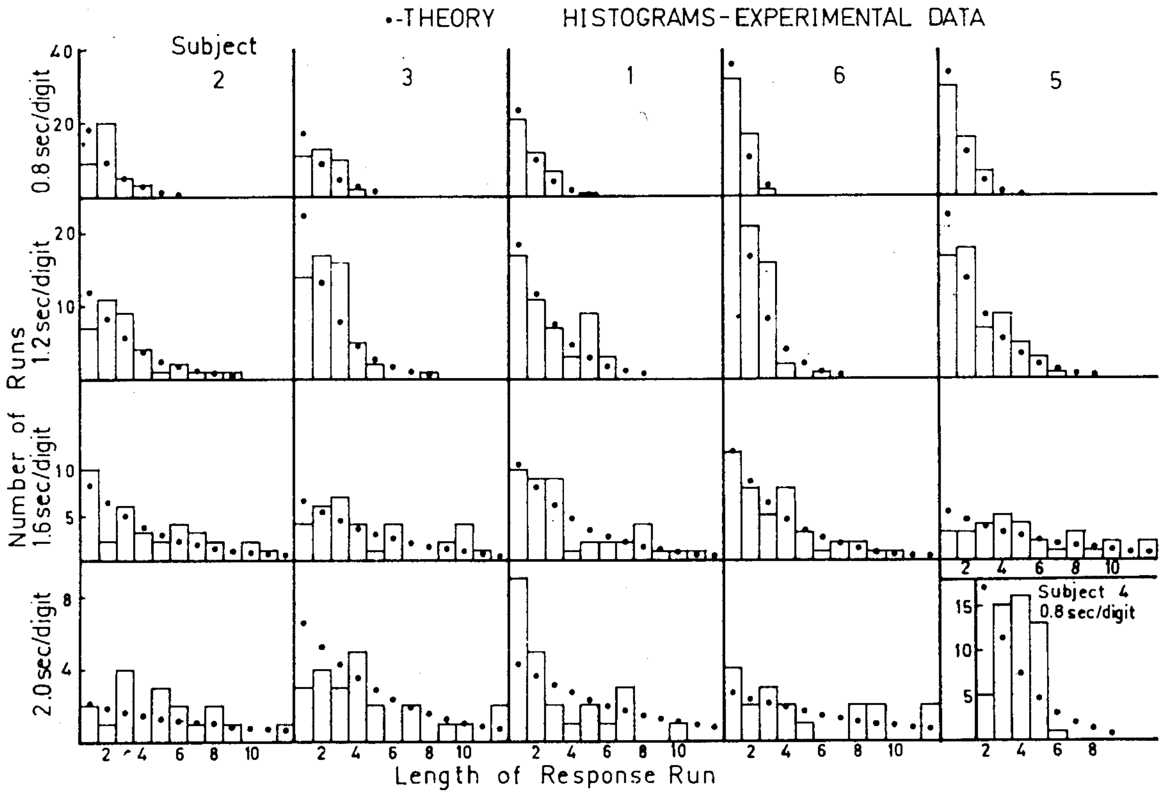

FIG. 6. Response run histograms for six subjects in Experiment 2.

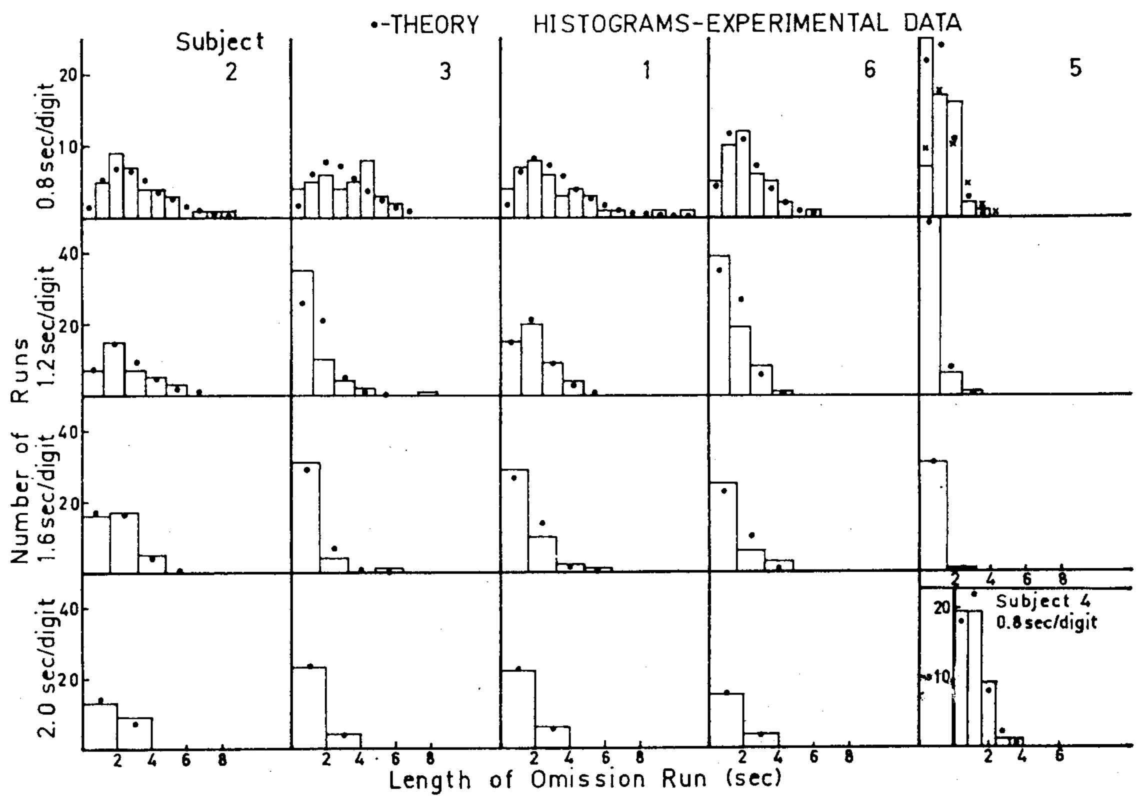

FIG. 7. Omission run histograms for six subjects in Experiment 2.

TABLE 5

Results of Correct and Error Response Model for Experiment 2

		0.8 sec/digit		1.2 sec/digit		1.6 sec/digit		2.0 sec/digit	
Subject	Parameter name	Parameter value	No. of transitions	Parameter value	No. of transitions	Parameter value	No. of transitions	Parameter value	No. of transitions
3	r	0.345	55	0.509	108	0.621	140	0.695	164
3	q	0.254	55	0.176	108	0.221	140	0.177	164
3	s	0.176	17	0.280	25	0.528	36	0.794	34
3	t	0.176	17	0.160	25	0.139	36	0.118	34
3	u	0.086	35	0.173	52	0.057	35	0.111	27
3	$z_{t,q}$	0.662		0.191		1.088		0.804	
1	r	0.301	55	0.522	115	0.700	160	0.738	172
1	q	0.164	55	0.130	115	0.106	160	0.128	172
2	r	0.500	72	0.655	110	0.776	165	0.890	210
2	q	0.028	72	0.027	110	0.018	165	0.024	210
4	r	0.608	125	0.913	195	0.996	239	1.000	239
4	q	0.088	125	0.015	195	0	239	0	239
5	r	0.301	103	0.619	155	0.845	207	0.987	236
5	q	0.019	103	0.026	155	0	207	0.008	236
6	r	0.219	64	0.460	124	0.695	151	0.809	183
6	q	0.078	64	0.040	124	0.073	151	0.060	183
Number of response-adding errors initiating a response run for the group.		1		2		1		1	
Group values of u		0.020	279	0.056	266	0.038	209	0.064	125

were not included in this analysis because ρ is significantly smaller which is probably due to a significant proportion of omission runs consisting of two recoveries. This analysis is somewhat weaker than that for p but indicates that ρ is approximately constant for changes in pacing rate for individual subjects. Table 4 shows results for the response and omission run models together with goodness-of-fit estimates, Fig. 5 shows changes in p and ρ with pacing rate for individual subjects and Figs. 6 and 7 show histograms for response and omission runs, respectively, together with fitted values.

As in Experiment 1, one subject (subject 3) made enough error responses to allow the Markov model for correct and error responses to be applied. These results are presented in Table 5 together with values of r and q for the other 5 subjects, values of u for the group and z-scores for the comparison between t and q for subject 3.

For subject 3, t is approximately the same as q at all pacing rates and for the group u is about the same as in Experiment 1. For individual subjects q is sometimes constant at all pacing rates and sometimes decreases as pacing rate increases. As in Experiment 1, few response-adding errors start response runs.

Introspective Reports

Introspective reports were obtained by asking subjects to say how they went about performing the task, then probing with questions such as "What caused interruption?" "Did you do any rehearsal?" These reports are summarized and listed below.

(a) At fast pacing rates (0.8 and 1.2 sec/digit for poorer performers) all subjects stated that many interruptions were due to loss of the previous stimulus or due to falling behind the stimulus sequence in responding.

(b) For the faster rates of presentation, all subjects said that if they noticed they had made an error then the response run was interrupted.

(c) All subjects at fast pacing rates also reported they could not or would not start a response run if the response was going to be slow, otherwise they would lapse into the pair-adding strategy.

(d) At intermediate rates of presentation (1.6 sec/digit for poorer performers and 1.2 and 1.6 sec/digit for better performers) loss of the previous stimulus in recall was again said to be a major cause of interruption. However there was often time to correct an error or recover from the disturbance and so maintain the response run.

(e) At 2.0 sec/digit, all subjects indicated that "wandering attention" was a major cause of interruption.

(f) Subject 5 found that often if the next stimulus had arrived before he was able to respond then he was able to hold enough information to miss that response and get the next.

(g) All subjects reported, at slower rates of presentation, that immediately after the response the previous stimulus was recalled and then rehearsed.

Now reports (c) and (f) will be examined together with results from the model and these will enable us to investigate strategies or where the model breaks down (other reports are considered in the next section).

Report (c) suggests that the number of response runs of length 1 should be smaller than that expected from the model at fast pacing rates and that there should be some larger response latencies than at slower rates. From Fig. 6, it can be seen that all subjects exhibit a smaller number of response runs of length 1 than predicted from the model at faster pacing rates, with subjects 2 and 4 extreme examples. Also the omission run histograms show several large latencies at faster pacing rates which probably accounts in part for the smaller values of ρ at these rates of presentation.

As in Experiment 1 very few response-adding errors initiated response runs. Subject 1 made a response-adding error that was surrounded by omissions. At the end of the trial he confirmed that this was a response-adding error rather than a random error coinciding with the response-adding result. This is consistent with report (c).

From report (f) we would expect a larger number of omission runs of length 1 than predicted by the model for subject 5 and this is observed. By reducing the number of runs of length 1 and reapplying the model, a reasonable fit occurs (subject 5, 0.8 sec/digit, Fig. 7, crosses are adjusted fits).

If the interstimulus interval is smaller than the time required for a processing cycle and if the probability of an interfering event is small then the response run model would be expected to break down. The distribution of response run length versus number of runs should show a small number of runs of short length increasing up to a peak at length 3 or 4 (for example), then dropping to zero sharply. This is precisely what the response run histogram for subject 4 at 0.8 sec/digit shows.

At slow rates of presentation, report (e) suggests that vigilance effects may be important (Broadbent, 1971, p. 23). The resulting omission runs may correspond to gaps in performance, "blocks" reported by Bills (1931).

It seems that at fast rates of presentation the model may not represent the data adequately. This is because subjects fall behind in processing or find it easier to respond in pairs.

6. Psychological Processes

In this section an attempt is made to relate the model to some current theories in memory and information processing.

Processes Involved in Response Runs

Two experiments were performed by Corballis and Sampson (1962) in an attempt to identify processes involved in the PSAT. The paced pure addition task (PPAT) involved addition with no immediate memory recall, whereas the paced memory task (PMT) involved memory recall with no addition. Both were presented visually. The PPAT had pairs of digits mounted on the same slide, the subject being required to call out the sum. PMT presentation was the same as for the PSAT except that the subject was required to call out the nth stimulus after the $n + 1$st had passed. Results showed that the PMT exhibited similar trends to the PSAT but the PPAT showed no decrement in performance even at 1 sec/digit (group obtaining 95% correct). Thus, we can eliminate encoding, response output and addition stages as major sources of performance decrement in the PSAT because these stages are present in the PPAT, which shows no decrement. Also, response time measurements (Ratcliff, 1974) have shown that time to perform these operations is almost independent of interstimulus interval, which provides further evidence that these stages are not important in producing performance decrement. The PMT results show that recall from immediate memory plays an important role in accounting for performance decrement. This paced memory task is similar in many respects to the tasks mentioned in the introduction (e.g., Mackworth & Mackworth, 1959; Kay, 1953; Pollack & Johnson, 1963) and so features in common between the PMT and PSAT are also shared by these tasks.

Individual subjects were shown to have adding difficulties with certain digit combinations but there were no major group or individual trends. Groen and Parkman (1972) analyzed response times for one-digit additions and found that adults' response latencies can be explained by a model that assumes a memory look-up process with homogeneous retrieval times, apart from occasionally reverting to a counting process. Thus, the addition stage does not constitute a performance limiting factor dependent on the particular digit combination required to be added.

Interruption in processing is an important factor in prerformance on the PSAT and failure to recall the previous stimulus item, prior to the addition stage, is likely a major source of this interruption. At the rates of presentation used in this experiment and with the short recall period, interference would be expected to be the major source of interruption (Atkinson & Shiffrin, 1971). Two obvious sources of interference in the PSAT are input and output interference. Welford (1968) discusses the experiments of Kay (1953), which are similar in many ways to the PMT (Corballis & Sampson 1963). The conclusion reached is that if acquisition of a new item coincides with recall of an item then there are severe interference effects even to the extend that retrieval of one item can clear the "memory store." Introspective reports (a) and (d) in the last section support this hypothesis that interruption is due to recall failure. It is also likely that the addition stage produces interference in stored information. This can be seen by identifying the addition stage with distracting activity in the Brown–Peterson paradigm (Peterson & Peterson, 1959).

Thus interference effects from addition, response and stimulus presentation appear to be major sources of interruption in the PSAT except when the presentation rate is fast and the subject falls behind in processing. Response time measurements (Ratcliff, 1974) show that the processing cycle is about 1 sec long and if the pacing rate is faster than this, the subject falls behind in responding until he ends the run. The size of the interference effects can be derived from p, the response run model parameter, except at these fast rates of presentation.

Figure 8 illustrates the sequence of operations and items in immediate memory when the subject is engaged in a response run. Earlier, it was noted that two types of error were easily recognized and classified, namely, type 3 corresponding to incorrect

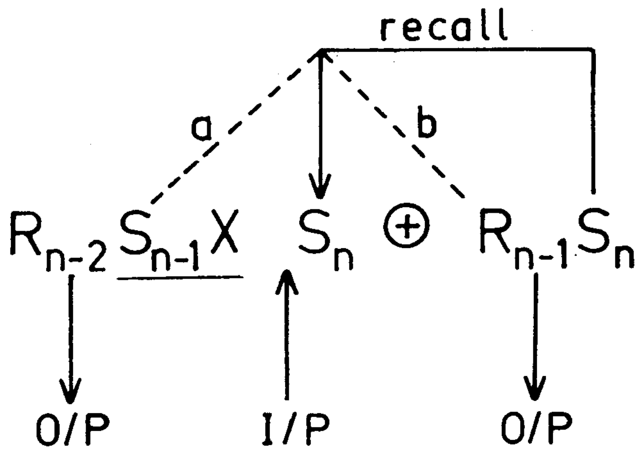

FIG. 8. Sequence of events and items in short-term memory when the subject is engaged in a response run. R, response; S, stimulus; $\oplus$, addition stage; X, rehearsal of the underlined preceding item.

retrieval *a* and type 4 corresponding to incorrect retrieval *b* in Fig. 8. The remaining errors were not easily classified. There are two forms of deviation from correct responses, omissions and errors, and there seems to be some correspondence between these two, and item and order information, respectively. (For further discussion of item and order information see Murdock, 1974.) For example, omissions may be due to loss of item information and errors (especially type 3 and 4) appear to be due to loss of order information, with items on either side of the target being used in the addition. However, a simple strength theory (Norman & Wickelgren, 1969) is also capable of accounting for errors and omissions. Interference would be expected to reduce the strength of items in immediate memory and if the second strongest was to be retrieved (R_{n-1}, the last response would be the strongest, Fig. 8), then errors would occur when the strength of one of the other items was ranked second in strength. The most likely candidates are S_{n-1} and R_{n-1} (Fig. 8) leading to the observed error pattern. Omissions would occur when the strength of the second strongest item was below a retrieval criterion.

Earlier, it was noted that an error often immediately precedes an omission run. Introspective reports (b) and (d) support this and the parameter s in the Markov model (Eq. (7)) describes the probability of recovery from an error. It seems that some error-checking process (Montague, 1972, p. 231) occurs with normal serial addition processing. At slow rates of presentation, it is possible to recover from an interruption resulting from an error without omission, but at fast rates an omission run usually results. The recovery either takes the form of a correction to the present response or a correct response to the next stimulus occasionally after an "um", a flinch, or some other sign.

This section shows that processes involved in response run performance are capable of being explained by current theory in memory research.

Processes Involved in Omission Runs

The model developed to account for recovery adequately represents omission run data and gives a measure of recovery rate. Empirical results show that recovery proceeds at the same speed at all except the faster pacing rates and the mean length of an omission run is independent of the preceding response run. It is reasonable to conclude therefore that processes involved in recovery from interruption are independent of rate of presentation and previous performance in the trial (at asymptote). The model, however, cannot deal with fast recovery which occurs at intermediate and faster pacing rates when the subjects falls behind in processing and deliberately misses a response in order to continue processing (see introspective report (f)). In these cases, the model helps isolate this strategy. Also, at fast pacing rates there are often relatively long omission runs. In these cases, the subject may be slow in producing a result and so does not respond but starts the recovery process again. As before the model helps identify this process.

The processes involved in recovery from interruption may be expected to be rather complicated. The omission run model is based on the rather strong assumptions of serial, exponentially distributed processes but is capable of being mimicked by other classes of model (Townsend, 1972). Therefore, this model may not be too powerful in helping specify mechanisms, but it does model recovery processes which are rarely considered in modern cognitive psychology.

Primacy Effect

In the PSAT the delay period from presentation of one item to the next is brief and the interfering activity (the input, output, and addition processes) has short duration. Thus, we might expect proactive inhibition to build up over the experimental session and because the retention period is brief it is reasonable that recovery takes place quickly in the intertrial interval. A strength theory would deal with this by supposing that there is a change in d' and β as the trial progresses leading to poorer performance. Thus, the primacy effect can be accounted for in terms of proactive inhibition.

7. Summary

In this paper, a model has been developed for the PSAT that has both a clear psychological interpretation and allows many of the psychological processes underlying performance to be specified.

Initially, a steady state of performance and a primacy effect were distinguished. The primacy effect showed characteristics that enabled it to be attributed to proactive inhibition building up as the trial proceeded. A model was developed to account for individual performance at asymptote. This was composed of three submodels, a model concerning performance in a response run which groups all responses together, a model that differentiates between correct and error responses (with the response run model as a limiting case) and a model for recovery from interruption dealing with omission runs. The latter two models generalize to a semi-Markov model.

The main performance limiting factors, as indicated by the models, are interruption, while the subject is performing in a response run and recovery from such an interruption. It is argued that interruptions are mainly due to interference and recovery involves complex processes which take place at a rate independent of rate of presentation. Errors are shown to be mainly due to incorrect memory retrieval.

Appendix A

Because average group performance has often been used as a dependent variable (Sampson, 1960; Corballis & Sampson, 1963; Gronwall, 1972), it is worthwhile developing the group model further. In this appendix both asymptotic variance estimates and goodness-of-fit measures are derived for the group model results from Experiment 1.

Asymptotic Variance Estimates

Asymptotic variance estimates are given by diagonal elements of the covariance matrix,

$$C = I^{-1},$$

where the information matrix

$$I = -E\left(\frac{\partial^2 l(\boldsymbol{\theta})}{\partial\theta_i\,\partial\theta_j}\right)$$

$\boldsymbol{\theta} = (\theta_1, \ldots, \theta_k)$ are the unknown parameters (in this case a, τ, and c) and $l(\boldsymbol{\theta})$ is the natural logarithm of the joint probability density function.

If we make the simplifying assumptions that the number of correct responses is distributed binomially with parameter p_i, and the proportion correct at any position is independent of what happened before (this approximation will only break down seriously for omission runs at faster rates of presentation), then the joint probability density function

$$f_{\boldsymbol{\theta}}(y_1, \ldots, y_N) = \prod_{i=1}^{N} \left[\binom{n}{y_i} p_i^{y_i} (1 - p_i)^{n-y_i}\right]$$

where n is the number of subjects in the group, y_i is the number of correct responses at position i, and N is the length of the response string.

Thus,

$$l(\boldsymbol{\theta}) = \sum_{i=1}^{N} \ln \binom{n}{y_i} + \sum_{i=1}^{N} [y_i \ln p_i + (n - y_i) \ln(1 - p_i)].$$

After some manipulation the information matrix can be written as

$$I = \begin{bmatrix} \left[\sum_{i=1}^{N} \frac{n e^{-\frac{2t_i}{\tau}}}{p_i(1-p_i)}\right] & \left[\sum_{i=1}^{N} \frac{an\left(\frac{t_i}{\tau^2}\right) e^{-\frac{2t_i}{\tau}}}{p_i(1-p_i)}\right] & \left[\sum_{i=1}^{N} \frac{n e^{-\frac{t_i}{\tau}}}{p_i(1-p_i)}\right] \\ \left[\sum_{i=1}^{N} \frac{an\left(\frac{t_i}{\tau^2}\right) e^{-\frac{2t_i}{\tau}}}{p_i(1-p_i)}\right] & \left[\sum_{i=1}^{N} \frac{a^2 n\left(\frac{t_i^2}{\tau^4}\right) e^{-\frac{2t_i}{\tau}}}{p_i(1-p_i)}\right] & \left[\sum_{i=1}^{N} \frac{an\left(\frac{t_i}{\tau^2}\right) e^{-\frac{t_i}{\tau}}}{p_i(1-p_i)}\right] \\ \left[\sum_{i=1}^{N} \frac{n e^{-\frac{2t_i}{\tau}}}{p_i(1-p_i)}\right] & \left[\sum_{i=1}^{N} \frac{an\left(\frac{t_i}{\tau^2}\right) e^{-\frac{t_i}{\tau}}}{p_i(1-p_i)}\right] & \left[\sum_{i=1}^{N} \frac{n}{p_i(1-p_i)}\right] \end{bmatrix}. \quad (18)$$

The variance estimates were calculated by numerically inverting the information matrix Eq. (18), using the estimated values of a, τ, and c.

TABLE A

Group Model Results, Variance Estimates, and Goodness-of-Fit Measures for Experiment 1

Pacing rate[a] (sec/digit)	Trial	a	σ_a	τ	σ_τ	c	σ_c	W
1.2	1	0.820	0.341	2.5	1.0	0.481	0.008	311.0
1.2	2	0.411	0.056	10.4	2.6	0.550	0.008	341.6
1.2	3	0.365	0.042	16.2	3.6	0.588	0.009	288.3
1.2	4	0.272	0.031	34.2	8.2	0.643	0.011	284.7
1.2	5	0.261	0.029	36.1	8.9	0.656	0.011	266.2
1.6	1	0.213	0.037	29.8	9.8	0.644	0.010	273.1
1.6	2	0.166	0.024	67.6	28.9	0.717	0.020	275.7
1.6	3	0.127	0.023	51.0	23.3	0.787	0.013	259.1
1.6	4	0.152	0.018	70.9	26.5	0.810	0.017	214.4
1.6	5	0.125	0.044	140.1	117.5	0.818	0.051	266.8
2.0	1	0.201	0.029	22.5	6.7	0.763	0.008	258.3
2.0	2	0.166	0.015	50.5	13.6	0.822	0.011	212.9
2.0	3	0.099	0.016	59.2	27.3	0.871	0.012	232.3
2.0	4	0.086	0.018	94.3	57.9	0.892	0.020	248.9
2.0	5	0.046	0.016	100.0	103.3	0.931	0.019	211.8

Note. The preliminary trials at 1.6 sec/digit gave the following proportion correct 0.633, 0.636, 0.596 for the three groups, respectively.

[a] A different group was used for each pacing rate.

Goodness of Fit

Equation (1) leads to $\hat{p}_i = \hat{a}e^{-i/\hat{\tau}} + \hat{c}$. Then

$$Z^2 = \frac{(y_i/n - \hat{p}_i)^2}{\hat{p}_i(1 - \hat{p}_i)/n}$$

is distributed χ_1^2. Therefore, $W = \sum_{i=1}^{240} Z_i^2$ is distributed χ^2_{236} with (240-1)-3 degrees-of-freedom. The 1% confidence limits on χ^2_{236} are 182.9 and 294.7.

Results

Table A contains values of the group parameters, variance estimates and goodness-of-fit statistics W for Experiment 1. The goodness-of-fit estimates show that the fits are reasonable so that group hypotheses may be tested using the group parameters and asymptotic variance estimates. Increases in c are found to be smooth to within two standard deviations indicating that the steady state shows stable practice and pacing effects. Although standard deviations for τ are large, results do show that a high level of performance can be maintained for longer periods with practice.

Figure A shows fits of Eq. (1) to two sets of data and indicates graphically the primacy effect and asymptotic performance.

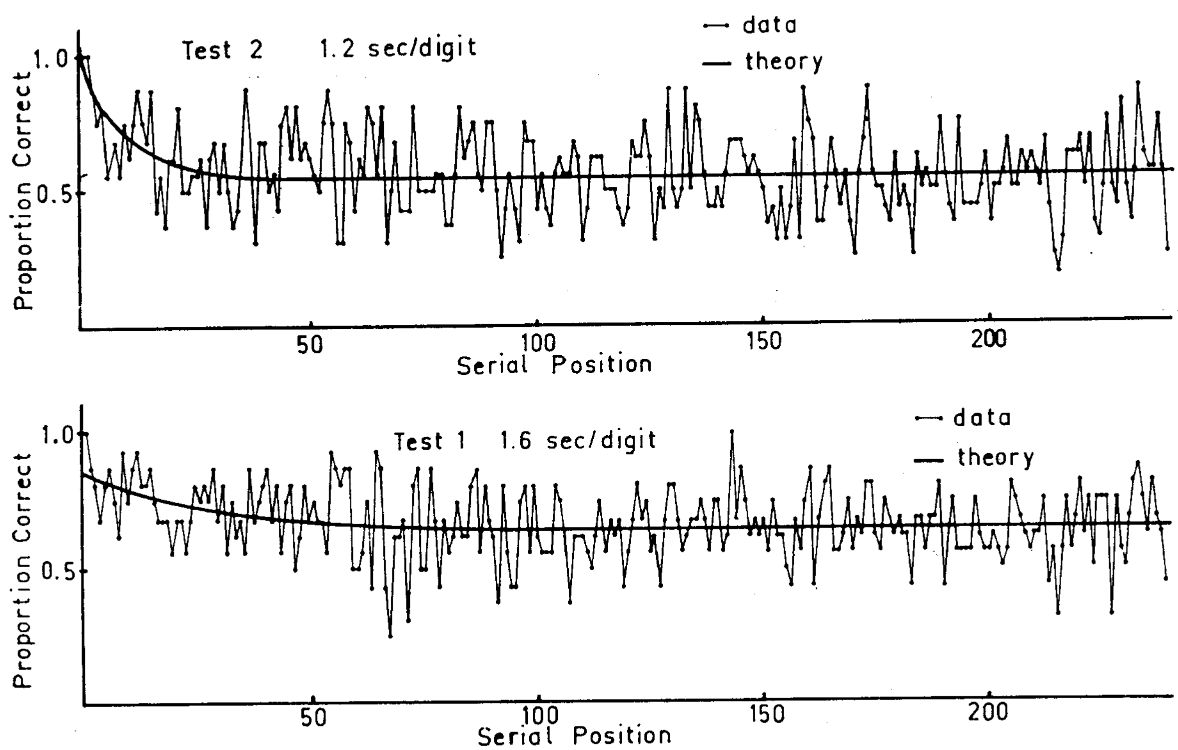

FIG. A. Typical fits of exponential to group proportion correct.

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