

Example (not all Hermitian operators are observables)

- Let's use a SSR i.e. $[\hat{O}, \hat{J}] = 0$ for all obs. \hat{O} on \mathcal{H}

↳ If have \hat{Q} self-adjoint s.t. $[\hat{Q}, \hat{J}] \neq 0$ then \hat{Q} is not an observable

↳ Let $\mathcal{H} = \bigoplus_i \mathcal{H}_i$ be eigenspace decomp. under \hat{J} (i.e. \mathcal{H}_i are supersectors)

↳ Take $|i\rangle \in \mathcal{H}_i$ & $|j\rangle \in \mathcal{H}_j$ ($i \neq j$). Note that $|i\rangle\langle j| + |j\rangle\langle i|$ is Hermitian

↳ $\langle i| (|i\rangle\langle j| + |j\rangle\langle i|) |j\rangle \neq 0 \quad \therefore [\underbrace{|i\rangle\langle j| + |j\rangle\langle i|}_{\text{Hermitian}}, \hat{J}] \neq 0$

So it can't be an observable!