10 October 2013 **Math 2173: Quiz 3** Name: KEY Rec. Time_____

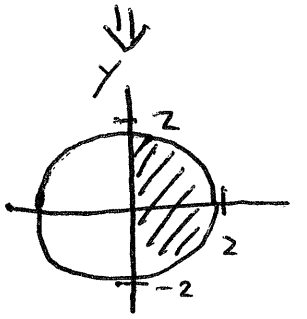
Problem 1. [7 points] Rewrite the following integral in **cylindrical** coordinates.

$$\int_0^2 \int_{-\sqrt{4-x^2}}^{\sqrt{4-x^2}} \int_{x^2+y^2}^{5} e^{-x^2-y^2}\, dz\, dy\, dx$$

DO NOT EVALUATE.

$0 \le x \le 2$

$-\sqrt{4-x^2} \le y \le \sqrt{4-x^2}$



$0 \le r \le 2$

$-\frac{\pi}{2} \le \theta \le \frac{\pi}{2}$

$$\Rightarrow \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \int_0^2 \int_{r^2}^5 e^{-r^2}\, r\, dz\, dr\, d\theta$$

Problem 2. [7 points] Rewrite the following integral in **spherical** coordinates.

$$\iiint_R y\, dV$$

R is the portion of the *first octant* between the surfaces given by $x^2+y^2+z^2=4$ and $x^2+y^2+z^2=9$.

DO NOT EVALUATE.

$0 \le \theta \le \frac{\pi}{2}$

$0 \le \phi \le \frac{\pi}{2}$

$y = \rho \sin\phi \sin\theta$

$\rho^2 = 4 \Rightarrow \rho = 2$

$\rho^2 = 9 \Rightarrow \rho = 3$

$$\Rightarrow \int_0^{\frac{\pi}{2}} \int_0^{\frac{\pi}{2}} \int_2^3 \rho \sin\phi \sin\theta\ \rho^2 \sin\phi\, d\rho\, d\phi\, d\theta$$

Problem 3. [6 points] Compute the **Jacobian determinant** $J(u,v)$ for the transformation given by

$$x = \frac{u}{v} \qquad \text{and} \qquad y = u + 2v.$$

$$J(u,v) = \left| \begin{pmatrix} \frac{\partial x}{\partial u} & \frac{\partial x}{\partial v} \\ \frac{\partial y}{\partial u} & \frac{\partial y}{\partial v} \end{pmatrix} \right|$$

$$= \left| \begin{pmatrix} \frac{1}{v} & \frac{-u}{v^2} \\ 1 & 2 \end{pmatrix} \right|$$

$$= \frac{1}{v} \cdot 2 - \left(-\frac{u}{v^2}\right) \cdot 1$$

$$= \frac{2}{v} + \frac{u}{v^2}$$

$J(u,v) =$ $\boxed{\frac{2}{v} + \frac{u}{v^2}}$