Computing Equilibrium: From the Energy Economy to Airline Networks

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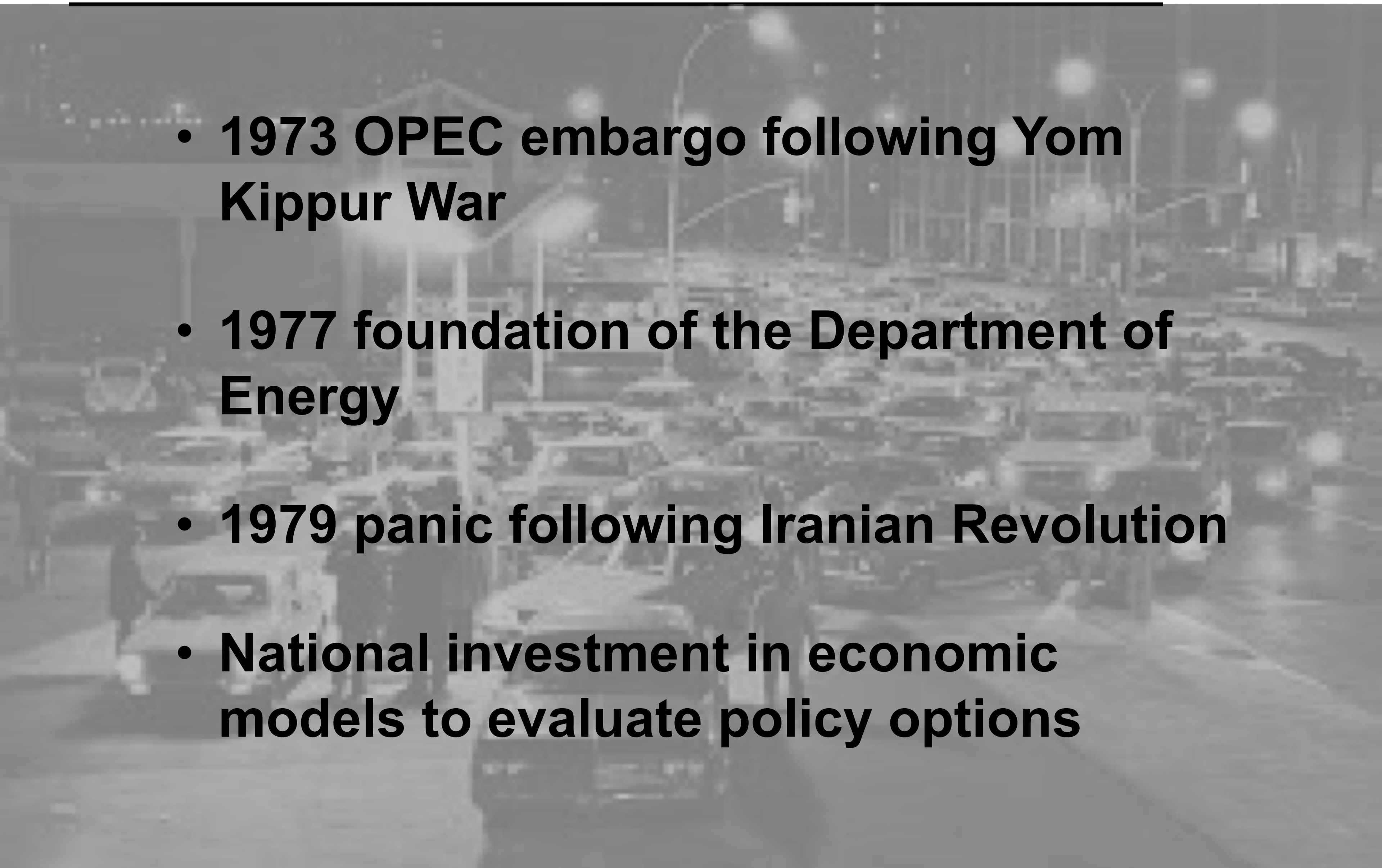
The Oil Crises of the 1970s



- **1973 OPEC embargo following Yom Kippur War**
- **1977 foundation of the Department of Energy**
- **1979 panic following Iranian Revolution**
- **National investment in economic models to evaluate policy options**

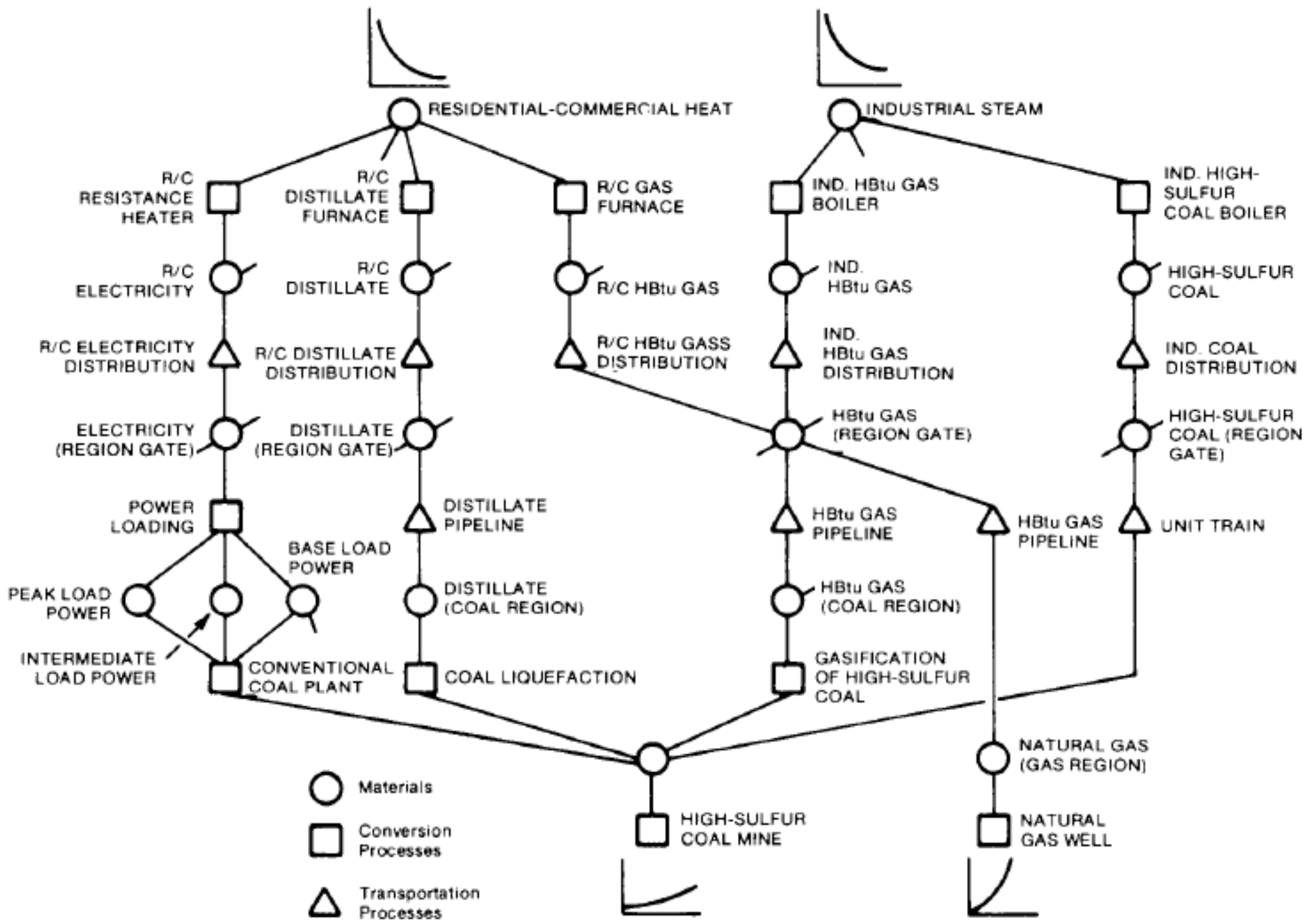
The Generalized Equilibrium Modeling System (GEMS)

- Developed as a methodology initially by SRI (Cazalet, et al.) to support analysis of synthetic natural gas investment decisions.

- Commercialized as a software system to enable rapid development of partial-equilibrium models.

- Still in use today in the North American Regional Gas (NARG) Model.

GEMS: A network modeling methodology

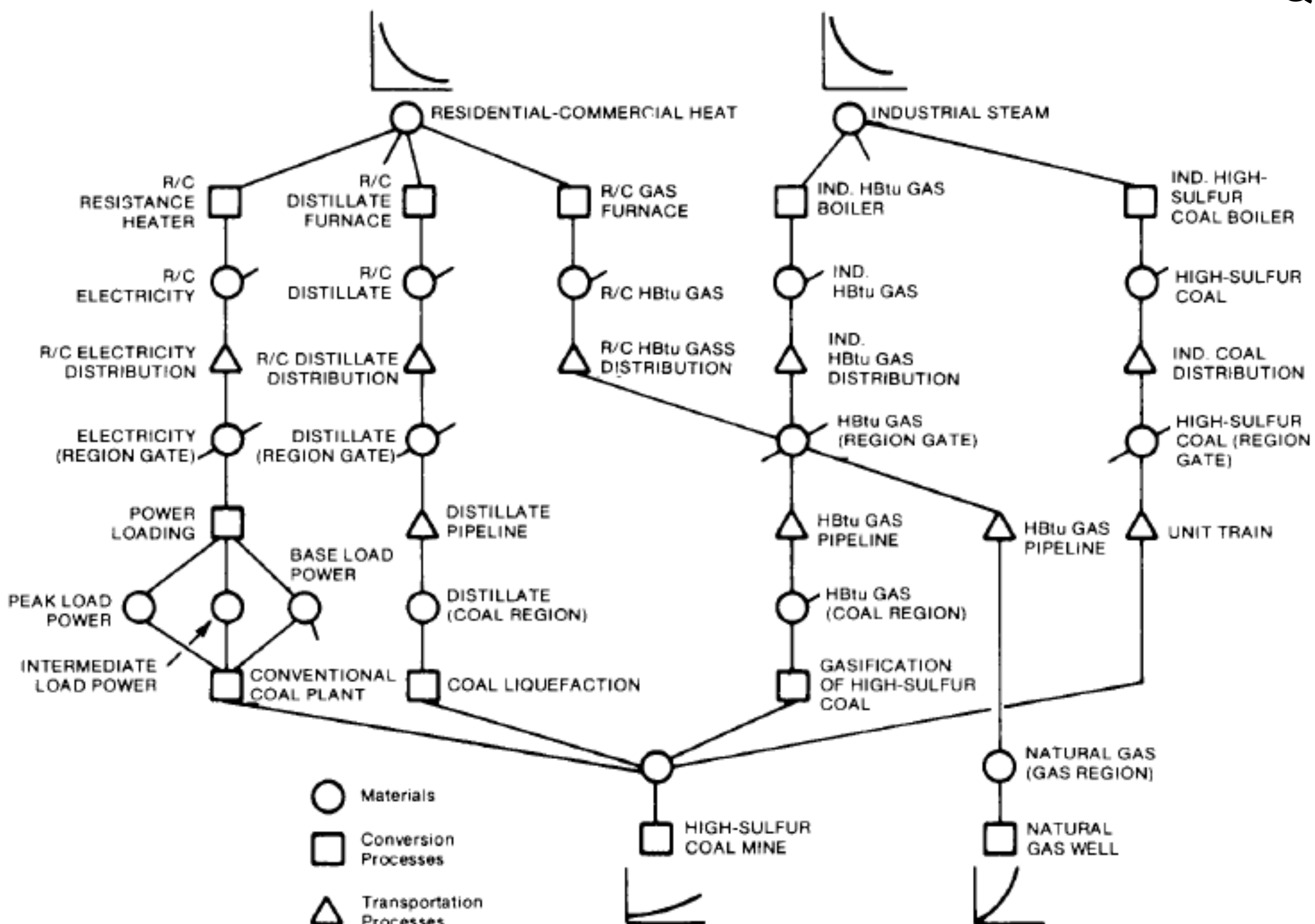


Research questions

- Existence and uniqueness of equilibria for GE models.
- **Convergence of the Successive Under Relaxation (SUR) algorithm**
- Development of a faster (Hybrid Newton-Secant) algorithm

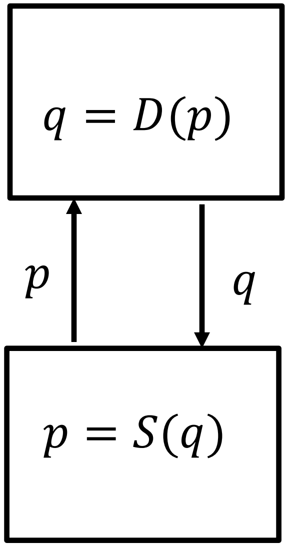
”Up-Down” calculation of prices and quantities

Calculate
Quantities



Calculate
Prices

Successive under-relaxation



- Find fixed point to composite mapping $p^* = S[D(p^*)]$.
- SUR Iteration $p^{k+1} = (1-\alpha)p^k + \alpha S\left[D\left(p^k\right)\right]$
- $0 < \alpha \leq 1$ is a *relaxation parameter.*
- S is continuous, bounded and monotonic.
- D is continuous, bounded and *antitonic.*
- *Theorem 1:* Under these conditions, the mapping $H = DS$ has a unique fixed point p^*.
- *Theorem 2:* There exists an $\varepsilon > 0$ such that SUR will converge for any $0 < \alpha < \varepsilon$.

Airline deregulation

- 1979 “Big Bang” deregulation in the United States
- Rise of ”first wave” low-cost carriers, notably People Express
- How can legacy carriers possibly compete given higher cost structure?

Airline customer segments

Leisure Traveler

- Very Price-Sensitive
- Books Earlier
- Not Schedule Sensitive
- Willing to accept restrictions (Saturday night stay, etc.)

Business Traveler

- Less Price-Sensitive
- Books Later
- Schedule Sensitive
- Unwilling to accept restrictions

Airline fare classes

Discount Fares

- Early booking
- Restricted
- Cheaper

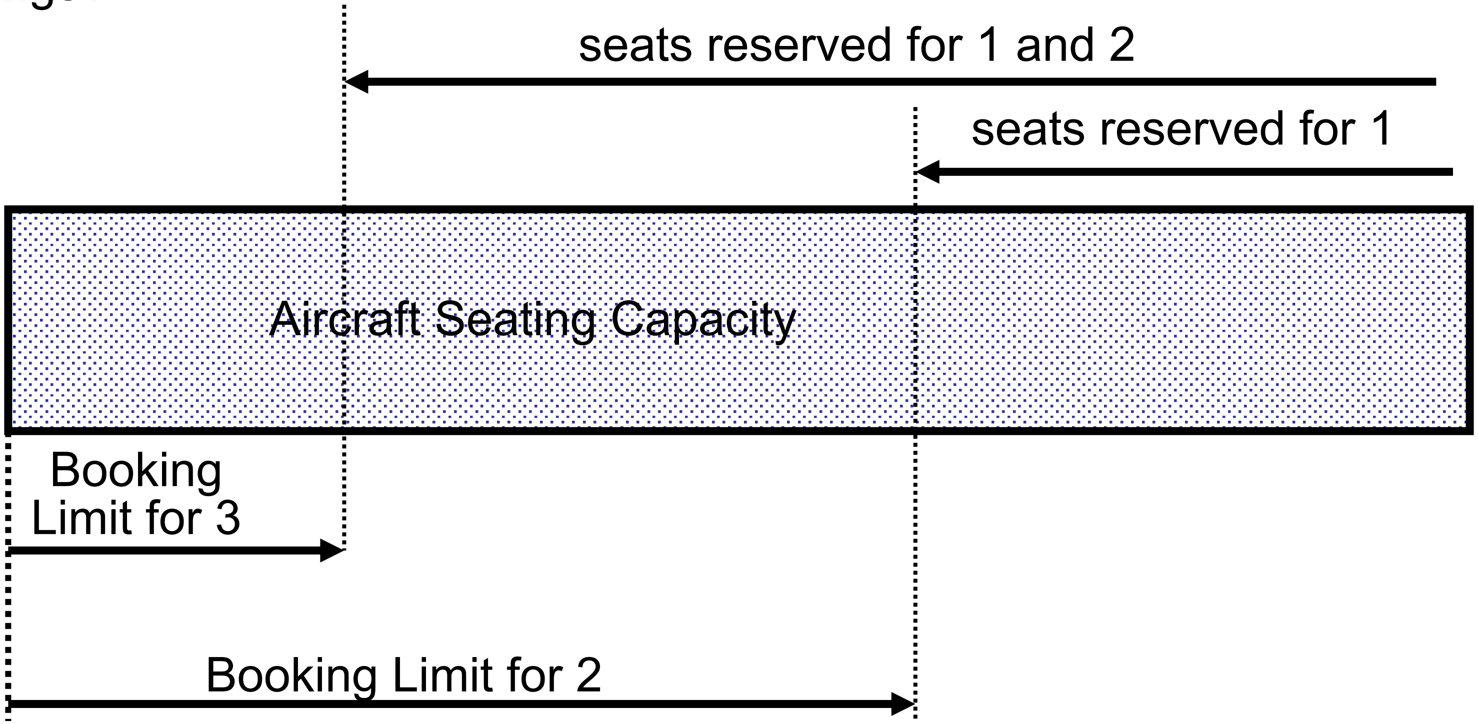
Full Fare

- Late booking
- Unrestricted
- More Expensive

Revenue Management issue: how many seats to save for late-booking, higher-paying passengers.

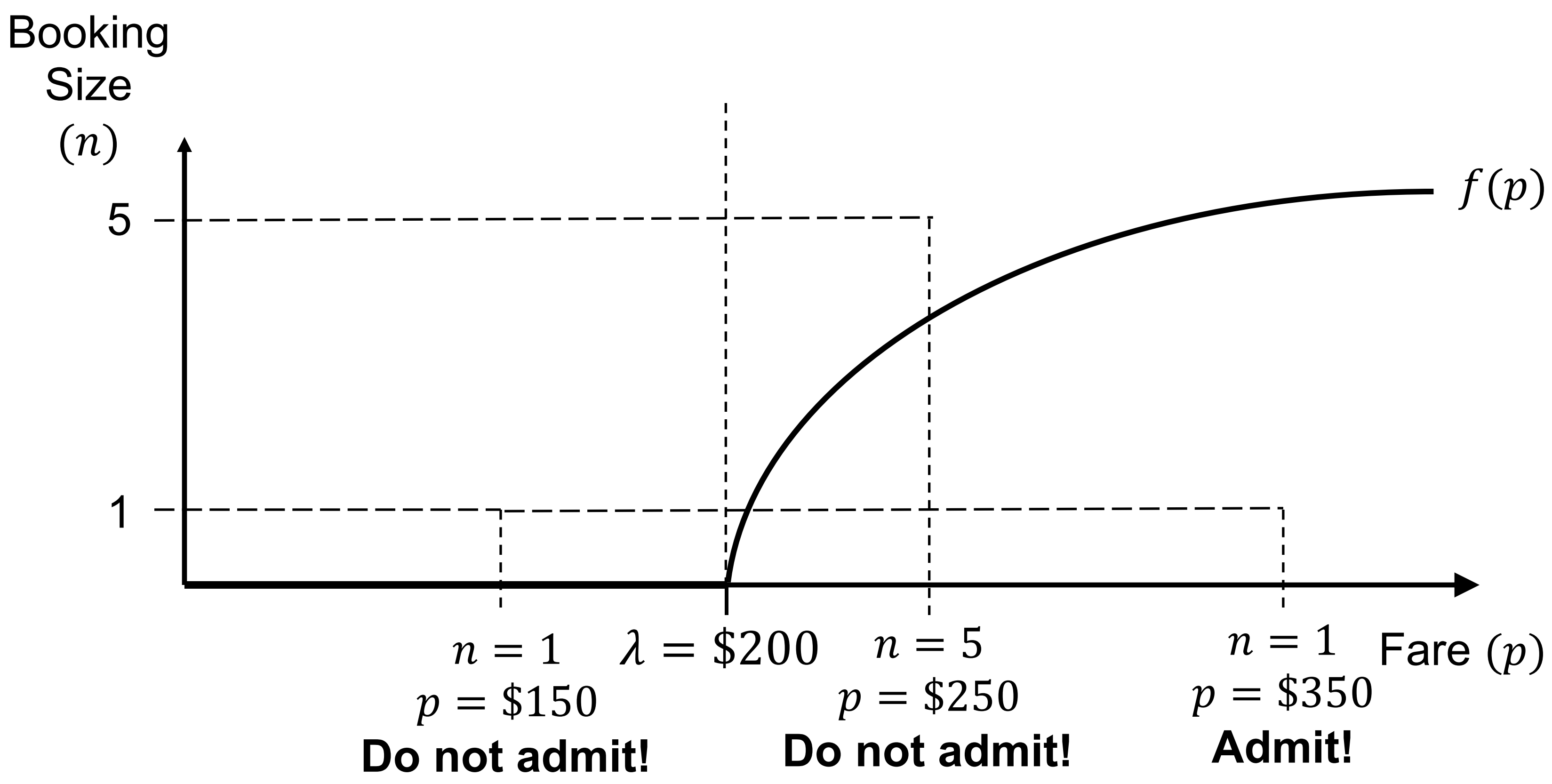
The multi-fare class problem

Bookings occur over time with increasing fares. How many seats do we allow to book at the current fare versus saving for potential future higher-fare bookings?



Three classes booking in order with $f_1 > f_2 > f_3$.

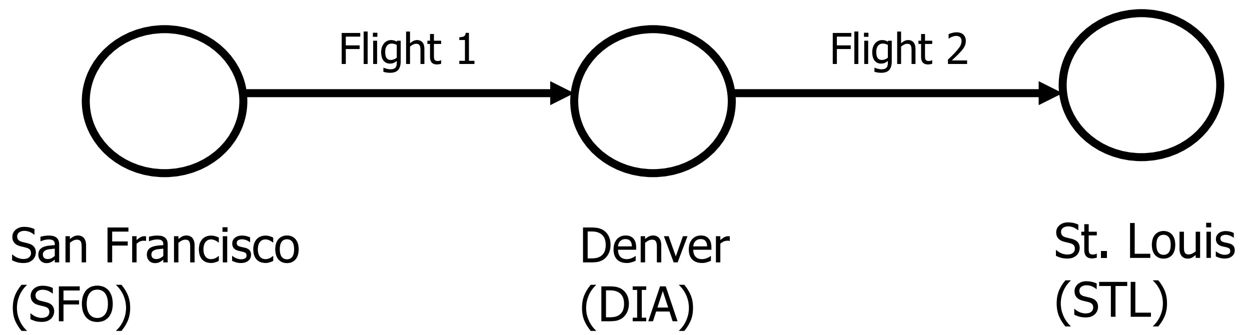
Dynamic single-leg admittance control



Admittance Rule: Admit a booking request only if $n < f(p)$

An airline network

The network management problem

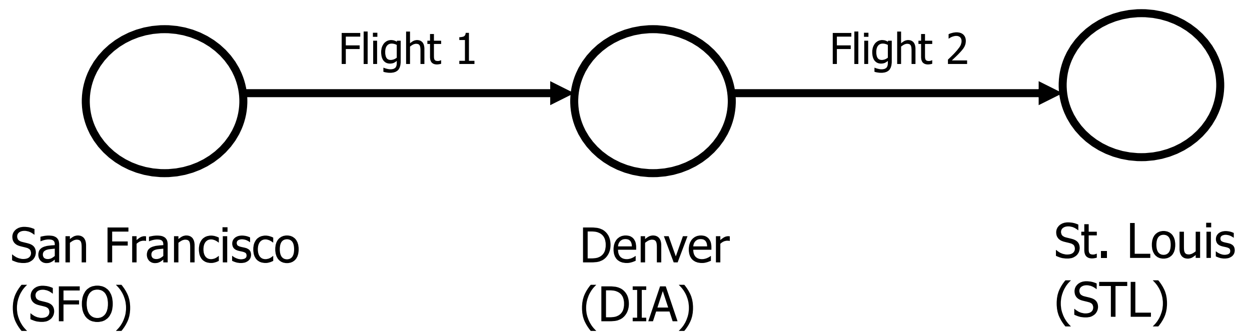


SFO – STL Fare = $400
SFO – DIA Fare = $200
DIA – STL Fare = $250

One seat left on each flight: who do we accept?

Which passengers we want to accept depends upon expected demands for all products. Sometimes we prefer SFO-STL pax, sometimes we prefer SFO-DIA or DIA-STL pax.

Why network management is hard



Market	Y-Class	M-Class	B-Class	G-Class
SFO-STL	$600	$400	$300	$250
SFO-DIA	$280	$200	$150	$140
DIA-STL	$350	$250	$180	$110

Network admittance control

Assume that every booking request is for a single seat.

1. At time *t,* for each leg i in the network, calculate a “bid price”, λ_{it}.

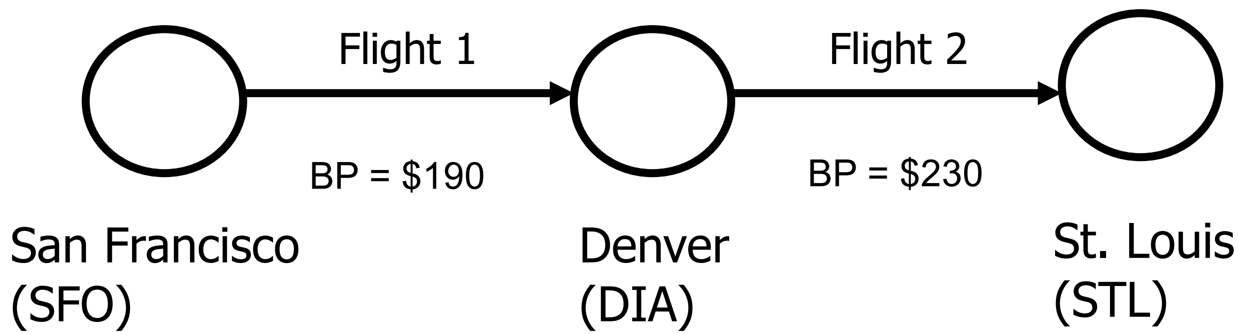
2. Assume a booking request of type *j* arrives at time *t* with associated fare p_j. Let $\sigma_{ij} = 1$ if leg *i* is part of booking request *j* and 0 otherwise.

3. If $p_j \geq \sum_i \sigma_{ij}\lambda_{it}$, accept the booking. Otherwise reject it.

4. Set $t \rightarrow t + \Delta$ and go to 1.

This approach is called “bid pricing” and it is widely used in network management.

Bid-price example



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Cool! But how do we calculate bid prices?

An SUR-based approach

$$\begin{tikzcd} \boxed{q = D(\lambda)} \arrow[d, shift left=2, "q"] \\ \boxed{\lambda = S(q)} \arrow[u, shift left=2, "\lambda"] \end{tikzcd}$$

- Find fixed point to composite mapping $p^* = S[D(\lambda^*)]$.
- SUR Iteration $\lambda^{k+1} = (1-\alpha)\lambda^k + \alpha S\left[D\left(\lambda^k\right)\right]$
- Can incorporate "consumer-choice" models in which booking choice depends on which set of booking classes are available on each itinerary.

Tens of billions of $ of airline, hotel and rental car bookings have been managed using this approach.

US Patent 5,918,209 (Campbell, Phillips, et al. 1999)

Method and system for determining marginal values for use in a revenue management system

Abstract

A method and system for determining marginal values for perishable resources expiring at a future time, for example, an airline seat, hotel room night, rental car day or the like, for use in a perishable resource revenue management system. Data for the perishable resources and composite resources is loaded from the perishable resource revenue management system into the marginal value system. Internal data structures are constructed for linking each of the perishable resources to their associated composite resources and for linking each of the composite resources to their associated perishable resources. The marginal values for the perishable resources are determined using a continuous optimization function using interdependencies among the perishable resources and the composite resources in the internal data structures. The marginal values are stored from the marginal value system into the perishable resource revenue management system.

Lessons Learned

1. Your dissertation research may have applications far removed for its original domain!
2. If you have a hammer, keep it. You are likely to run into a nail at some point!
3. Make sure that you have

a great advisor!

Thanks, Shmuel!

Discussion, Questions, and (Maybe) Answers