

Written HW4 Solutions

Surface $S: \frac{y^2}{9} + z^2 = 9x^2 \quad z \geq 0$

Curve $C: \vec{r}(t) = \langle \frac{1}{3}t, 3t\cos t, t\sin t \rangle \quad 0 \leq t \leq 2\pi$

2D-vector field $\vec{F} = \nabla f = \langle f_x, f_y \rangle$ where $S = f(x, y)$ is the surface

① Level curves:

$$z=0 \quad 9x^2 - \frac{y^2}{9} = 0 \Leftrightarrow x = \pm \frac{1}{3}y$$

$$z=1 \quad 9x^2 - \frac{y^2}{9} = 1$$

$$z=2 \quad 9x^2 - \frac{y^2}{9} = 4$$

:

$$\frac{y^2}{9} + z^2 = 9x^2 \Leftrightarrow z^2 = 9x^2 - \frac{y^2}{9}$$

$$S = f(x, y) = \sqrt{9x^2 - \frac{y^2}{9}}$$

$$\vec{F} = \nabla S = \langle f_x, f_y \rangle = \left\langle \frac{9x}{\sqrt{9x^2 - \frac{y^2}{9}}}, -\frac{\frac{y}{9}}{\sqrt{9x^2 - \frac{y^2}{9}}} \right\rangle$$

\vec{F} is orthogonal to level curves C

② C lies on the surface because $\vec{r}(t)$ satisfies equation $\frac{y^2}{9} + z^2 = 9x^2$

$$\underbrace{\frac{(3t\cos t)^2}{9}}_{\frac{y^2}{9}} + \underbrace{(t\sin t)^2}_{z^2} = t^2 \stackrel{?}{=} 9 \cdot \left(\frac{1}{3}t\right)^2 = t^2$$

The projection of C on xy -plane is $\vec{r}_1(t) = \langle \frac{1}{3}t; 3t\cos t \rangle$

③ $\int_C \vec{F} \cdot d\vec{r} = 0$ since \vec{F} is orthogonal to the tangent direction of level curve, which is $d\vec{r}$, i.e. $\vec{F} \cdot d\vec{r} = 0$

④ Since $\vec{F} = \nabla S$, \vec{F} is conservative.

$$\begin{aligned} \int_C \vec{F} \cdot d\vec{r} &= S(B) - S(A) & B: t=2\pi &< \frac{2\pi}{3}, 2\pi \cos 2\pi \rangle = \langle \frac{2\pi}{3}, 6\pi \rangle \\ &= \sqrt{9 \cdot \left(\frac{2\pi}{3}\right)^2 - \frac{16\pi^2}{9}} - 0 & A: t=0 &< 0, 0 \rangle \\ &= 0 - 0 = 0 \end{aligned}$$

⑤ The normal vector of the surface is $\vec{n} = \langle 18x, -\frac{2y}{9}, -2z \rangle$

$$\text{At point } (\frac{\pi}{6}, 0, \frac{\pi}{2}) \quad \vec{n} = \langle 3\pi, 0, -\pi \rangle$$

Any vector that is orthogonal to \vec{n} is tangent to surface S

e.g. $\langle 0, 1, 0 \rangle, \langle 1, 0, 3 \rangle$ This method work for all surfaces #

