

## Quiz 8

MATH 2162.02

RECITATION TIME:

NAME:

**Problem 1.** (5 points) Consider the vector field  $\mathbf{F} = \langle z^2 \sin y, xz^2 \cos y, 2xz \sin y \rangle$ .

(a) (3 pts) Compute the curl  $\nabla \times \mathbf{F}$ .

Answer.  $\nabla \times \vec{F} = \begin{vmatrix} i & j & k \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ z^2 \sin y & xz^2 \cos y & 2xz \sin y \end{vmatrix}$

$$= \left[ \frac{\partial}{\partial y}(2xz \sin y) - \frac{\partial}{\partial z}(xz^2 \cos y) \right] \vec{i} - \left[ \frac{\partial}{\partial x}(2xz \sin y) - \frac{\partial}{\partial z}(z^2 \sin y) \right] \vec{j}$$

$$+ \left[ \frac{\partial}{\partial x}(xz^2 \cos y) - \frac{\partial}{\partial y}(z^2 \sin y) \right] \vec{k}$$

$$= (2xz \cos y - 2xy \cos y) \vec{i} - (2z \sin y - 2z \sin y) \vec{j} + (z^2 \cos y - z^2 \cos y) \vec{k}$$

$$= \langle 0, 0, 0 \rangle$$

(b) (2 pts) Let  $C$  be the curve given by  $\mathbf{r}(t) = \langle \cos t, \sin t, 5 \rangle$  for  $0 \leq t \leq 2\pi$ .

Use part (a) to evaluate the following line integral. Do NOT use Stoke's theorem.

$$\int_C \mathbf{F} \cdot d\mathbf{r}$$

Answer: Since  $\nabla \times \vec{F} = \vec{0}$ ,  $\vec{F}$  is a conservative vector field.

$\vec{r}(t)$  is a closed curve.

$$\int_C \vec{F} \cdot d\vec{r} = \oint_C \vec{F} \cdot d\vec{r} = 0$$

**Problem 2.** (5 points) Consider the vector field  $\mathbf{F} = \frac{\langle -y, x \rangle}{x^2 + y^2}$ .

(a) (2 pts) Compute the two-dimensional curl of  $\mathbf{F}$ .

$$\begin{aligned}\text{Answer: } 2D\text{curl } \vec{F} &= \frac{\partial g}{\partial x} - \frac{\partial f}{\partial y} \\ &= \frac{1 \cdot (x^2+y^2) - 2x \cdot x}{(x^2+y^2)^2} - \frac{-1(x^2+y^2) + 2y \cdot y}{(x^2+y^2)} \\ &= \frac{y^2-x^2-(y^2-x^2)}{x^2+y^2} \\ &= \frac{0}{x^2+y^2} \\ &= 0\end{aligned}$$

(b) (2 pts) Use the parametrization  $\mathbf{r}(t) = (\cos t, \sin t)$ ,  $0 \leq t \leq 2\pi$ , to compute circulation of  $\mathbf{F}$  along the unit circle centered at the origin.

$$\begin{aligned}\text{Answer: } \int_C \vec{F} \cdot d\vec{r} &= \int_0^{2\pi} \vec{F}(t) \cdot \vec{r}'(t) dt \\ \vec{F}(t) &= \frac{\langle -\sin t, \cos t \rangle}{(\cos t)^2 + (\sin t)^2} = \langle -\sin t, \cos t \rangle \\ \vec{r}'(t) &= \langle -\sin t, \cos t \rangle \\ \vec{F}(t) \cdot \vec{r}'(t) &= \sin^2 t + \cos^2 t = 1 \\ \int_C \vec{F} \cdot d\vec{r} &= \int_0^{2\pi} 1 dt = 2\pi\end{aligned}$$

(c) (1 pt) Explain why (a) and (b) do not contradict Green's Theorem.

**Answer:**

$R$  is the region:  $\{(x,y) | 0 < x^2 + y^2 \leq 1\}$

The boundary of  $R$  is  $C$ .

$R$  is not simply connected.

We cannot apply Green's thm. i.e.  $\oint_C \vec{F} \cdot d\vec{r} \neq \iint_R 2D\text{curl } \vec{F} dA$

② (b) do not contradict Green's thm

