

Math 2162.02

Spring 2017

Final Exam

5/1/2017

Time Limit: 1 hour, 45 Minutes

Name: _____

Name.##: _____

Lecturer: _____

TA: _____

Recitation Time: _____

This exam contains 11 pages (including this cover page) and 10 questions. The total number of points is 200. Calculators are not permitted, and the exam is closed book and closed notes.

Do not open the exam until instructed to do so. Show all your work; solutions without appropriate supporting details may not be given credit. Simplify your answers as much as possible, but not more. Do not cheat.

Question	Points	Score
1	20	
2	22	
3	24	
4	22	
5	20	
6	24	
7	20	
8	18	
9	15	
10	15	
Total:	200	

A collection of potentially useful information:

The Divergence Theorem:

$$\iiint_R \nabla \cdot \mathbf{F} \, dV = \iint_S \mathbf{F} \cdot \mathbf{n} \, dS.$$

Stokes' Theorem:

$$\iint_S (\nabla \times \mathbf{F}) \cdot \mathbf{n} \, dS = \oint_C \mathbf{F} \cdot d\mathbf{r}$$

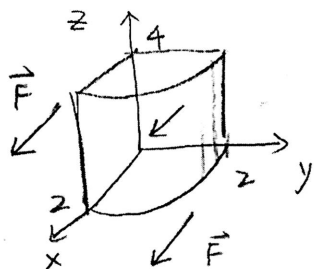
Taylor's Theorem:

$$R_n(x) = \frac{f^{(n+1)}(c)}{(n+1)!} (x-a)^{n+1}$$

$$\cos^2(x) = \frac{1+\cos(2x)}{2}$$

1. (20 points) Consider the vector field $\vec{F}(x, y, z) = \langle x, 0, 0 \rangle$ and let S be the part of the cylinder of radius 2 and height 4 which lies in the first octant. That is, $S : x^2 + y^2 = 4$, $0 \leq z \leq 4$, $x, y \geq 0$. Let R be the solid in the first octant bounded by S and four other faces.

- (a) (4 points) Sketch the surface S and the field \vec{F} .



$\vec{F} \parallel x\text{-axis}$

\vec{F} points toward positive direction of x -axis in the first octant. The magnitude of \vec{F} increases as \vec{F} moves away from origin

- (b) (4 points) Do you expect the flux outward across the boundary of S to be positive, negative, or zero? Explain briefly.

Answer The flux outward across the boundary of S is positive, since the angle between \vec{F} and outward normal vector \vec{n} is less than $\pi/2$, i.e. $\vec{F} \cdot \vec{n} > 0$

- (c) (12 points) Compute the flux outward across S , or

$$\iint_S \vec{F} \cdot \vec{n} dS.$$

Hint: You may find the Divergence Theorem helpful.

Answer:

Divergence thm can be applied to a solid enclosed by an oriented surface. S is only part of the boundary surface of the cylinder in the first octant

$$\iint_S \vec{F} \cdot \vec{n} dS + \iint_{\substack{4 \text{ other} \\ \text{faces}}} \vec{F} \cdot \vec{n} dS = \iiint_D \nabla \cdot \vec{F} dV$$

$$\iint_{\substack{4 \text{ other} \\ \text{faces}}} \vec{F} \cdot \vec{n} dS = 0 \text{ for the following reasons}$$

① $\vec{F} = \langle 0, 0, 0 \rangle$ across the face on yz -plane

② $\vec{F} \perp \vec{n}$ on the other three faces

$$\iiint_D \nabla \cdot \vec{F} dV = \iiint_D \left(\frac{\partial x}{\partial x} + \frac{\partial 0}{\partial y} + \frac{\partial 0}{\partial z} \right) dV = \iiint_D 1 dV = \text{volume } D = 4\pi$$

$$\text{Hence } \iint_S \vec{F} \cdot \vec{n} dS = 4\pi$$

2. (22 points) Consider the function $f(x, y) = \frac{1}{3}x^3 - 4xy + \frac{2}{3}y^3 + 7$. Find the point (x_0, y_0) such that the rate of change of $f(x, y)$ in the direction of the vector $\vec{i} + \vec{j}$ is the smallest. Then find the rate of change at that point.

Answer:

The rate of change of f in the direction of $\vec{u} = \langle 1, 1 \rangle$ is

$$\begin{aligned} D_{\vec{u}} f &= \nabla f \cdot \frac{\vec{u}}{|\vec{u}|} = \langle f_x, f_y \rangle \cdot \frac{\vec{u}}{|\vec{u}|} \\ &= \langle x^2 - 4y, -4x + 2y^2 \rangle \cdot \frac{\langle 1, 1 \rangle}{\sqrt{2}} \\ &= \frac{1}{\sqrt{2}} \cdot (x^2 - 4y - 4x + 2y^2) \\ &= \frac{1}{\sqrt{2}} (x^2 - 4x + 2y^2 - 4y) \\ &= \frac{1}{\sqrt{2}} [(x-2)^2 + 2(y-1)^2 - 6] \end{aligned}$$

At $(2, 1)$, f has smallest rate of change along $\langle 1, 1 \rangle$ and the rate of change is $-\frac{6}{\sqrt{2}}$.

3. (24 points) Let $f(x) = e^x \sin(x)$ on the interval $[-\frac{\pi}{2}, \frac{\pi}{2}]$.

(a) (12 points) Find a polynomial of degree 3 which approximates f .

Answer:

$$P_3(x) = f(x_0) + f'(x_0)(x-x_0) + \frac{f''(x_0)}{2!}(x-x_0)^2 + \frac{f'''(x_0)}{3!}(x-x_0)^3$$

$$x_0 = 0 \text{ and}$$

$$f'(x) = e^x \sin x + e^x \cos x \quad f'(0) = e^0 \sin 0 + e^0 \cos 0 = 1$$

$$f''(x) = e^x \sin x + 2e^x \cos x - e^x \sin x \quad f''(0) = 2e^0 \cos 0 = 2$$

$$f'''(x) = 2e^x \cos x - 2e^x \sin x \quad f'''(0) = 2e^0 \cos 0 = 2$$

$$f(0) = e^0 \sin 0 = 0$$

$$P_3(x) = 0 + 1 \cdot x + \frac{2}{2} \cdot x^2 + \frac{2}{6} x^3$$

$$P_3(x) = x + x^2 + \frac{1}{3} x^3$$

(b) (12 points) Estimate the maximum absolute error in using your approximation in place of f on the interval $[-\frac{\pi}{2}, \frac{\pi}{2}]$. Is the error less than 0.01?

$$R_3(x) = \frac{f^{(4)}(c)}{4!}(x-x_0)^4$$

$$f^{(4)}(x) = 2e^x \cos x - 4e^x \sin x - 2e^x \cos x = -4e^x \sin x$$

$$\text{On } [-\frac{\pi}{2}, \frac{\pi}{2}]$$

$$|f^{(4)}(c)| \leq 4e^{\frac{\pi}{2}} \cdot 1$$

$$|x-0|^4 \leq \left(\frac{\pi}{2}\right)^4$$

$$\text{Error} \leq \frac{4e^{\frac{\pi}{2}}}{4!} \left(\frac{\pi}{2}\right)^4$$

4. (22 points) A roller coaster car's acceleration is given by $\vec{a}(t) = \langle e^{3t}, t^2, \sin(t) \rangle$ for t in the interval $[0, \pi]$. At time $t = 0$, the location of the car is the origin, $\langle 0, 0, 0 \rangle$, and the velocity of the car is $\langle 1, 0, 0 \rangle$.

(a) (16 points) Find the velocity and position functions for the roller coaster car.

Answer:

$$\vec{v}(t) = \int \vec{a}(t) dt + \vec{C} \text{ and use } \vec{v}(0) \text{ to determine } \vec{C}$$

$$\vec{v}(t) = \langle \int e^{3t} dt, \int t^2 dt, \int \sin t dt \rangle + \vec{C}$$

$$= \langle \frac{1}{3}e^{3t}, \frac{1}{3}t^3, -\cos t \rangle + \vec{C}$$

$$\vec{v}(0) = \langle \frac{1}{3}e^0, \frac{1}{3} \cdot 0, -\cos 0 \rangle + \vec{C} = \langle 1, 0, 0 \rangle$$

$$\Rightarrow \vec{C} = \langle \frac{2}{3}, 0, 1 \rangle$$

$$\vec{v}(t) = \langle \frac{1}{3}e^{3t} + \frac{2}{3}, \frac{1}{3}t^3, 1 - \cos t \rangle$$

$$\vec{r}(t) = \int \vec{v}(t) dt + \vec{C} \text{ and use } \vec{r}(0) \text{ to determine } \vec{C}$$

$$\vec{r}(t) = \langle \int \frac{1}{3}e^{3t} + \frac{2}{3} dt, \int \frac{1}{3}t^3 dt, \int 1 - \cos t dt \rangle + \vec{C}$$

$$= \langle \frac{1}{9}e^{3t} + \frac{2}{3}t, \frac{1}{12}t^4, t - \sin t \rangle + \vec{C}$$

$$\vec{r}(0) = \langle \frac{1}{9}e^0 + 0, \frac{1}{12} \cdot 0, 0 - \sin 0 \rangle + \vec{C} = \langle 0, 0, 0 \rangle$$

$$\vec{C} = \langle -\frac{1}{9}, 0, 0 \rangle$$

$$\vec{r}(t) = \langle \frac{1}{9}e^{3t} + \frac{2}{3}t - \frac{1}{9}, \frac{1}{12}t^4, t - \sin t \rangle$$

- (b) (6 points) Set up, but do not evaluate, an integral which would compute the length of the car's trajectory.

Answer:

$$S = \int_0^\pi |\vec{r}'(t)| dt = \int_0^\pi |\vec{v}(t)| dt$$

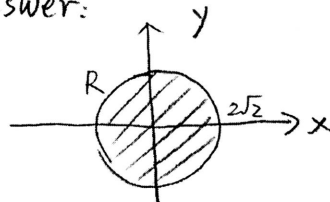
$$|\vec{v}(t)| = \sqrt{\left(\frac{1}{3}e^{3t} + \frac{2}{3}\right)^2 + \left(\frac{1}{3}t^3\right)^2 + (1 - \cos t)^2}$$

5. (20 points) Consider the solid cone whose boundary is given by $\frac{z^2}{2} = x^2 + y^2$ with $0 \leq z \leq 4$. Suppose also that the cone has a constant density of 10 kg/m^3 .

(a) (10 points) What is the mass of this cone? You might find the following formula helpful.

$$M = \iiint_R \rho \, dV$$

Answer:



Projection on xy-plane

$$z = 4 \Rightarrow$$

$$\frac{4^2}{2} = x^2 + y^2$$

$$x^2 + y^2 = 8$$

$$\begin{aligned} M &= \iint_R \int_0^4 \frac{1}{\sqrt{2(x^2+y^2)}} \rho \, dz \, dA \\ &= \int_0^{2\pi} \int_0^{2\sqrt{2}} \int_{\frac{\sqrt{2}}{2}r}^4 10 \, dz \cdot r \, dr \, d\theta \\ &= \int_0^{2\pi} \int_0^{2\sqrt{2}} 10(4 - \frac{\sqrt{2}}{2}r) \cdot r \, dr \, d\theta \end{aligned}$$

$$\begin{aligned} &= \int_0^{2\pi} \left[20r^2 - \frac{10\sqrt{2}}{3} r^3 \right]_0^{2\sqrt{2}} d\theta \\ &= \int_0^{2\pi} 20 \times 8 - \frac{10\sqrt{2}}{3} \cdot 2\sqrt{2} \times 8 \, d\theta \\ &= \int_0^{2\pi} 160 - \frac{320}{3} \, d\theta \\ &= \frac{160}{3} \times 2\pi \end{aligned}$$

(b) (10 points) What is the z-coordinate of the center of mass of this cone? You might find the following formula helpful.

$$M_{xy} = \iiint_R \rho z \, dV$$

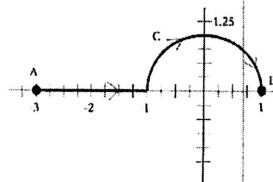
Answer:

$$\begin{aligned} M_{xy} &= \int_0^{2\pi} \int_0^{2\sqrt{2}} \int_{\frac{\sqrt{2}}{2}r}^4 10 \cdot z \, dz \cdot r \, dr \, d\theta \\ &= \int_0^{2\pi} \int_0^{2\sqrt{2}} \left(5z^2 \Big|_{\frac{\sqrt{2}}{2}r}^4 \right) r \, dr \, d\theta \\ &= \int_0^{2\pi} \int_0^{2\sqrt{2}} (80 - 10r^2) r \, dr \, d\theta \\ &= \int_0^{2\pi} \left[40r^2 - \frac{10}{4} r^4 \right]_0^{2\sqrt{2}} d\theta \\ &= \int_0^{2\pi} 40 \times 8 - \frac{10}{3} (\sqrt{2})^2 \times 8 \, d\theta \\ &= 160 \times 2\pi \end{aligned}$$

$$\bar{z} = \frac{M_{xy}}{M} = 3$$

$$\bar{x} = \bar{y} = 0$$

6. (24 points) Let the curve C be that drawn below, consisting of a straight segment on the x -axis between $-3 \leq x \leq -1$ and a semi-circular segment for $-1 \leq x \leq 1$. Let $\vec{F} = \langle -y, x \rangle$ and $\vec{G} = \langle \cos(x) + y, x - 1 \rangle$.



- (a) (8 points) Is either \vec{F} or \vec{G} conservative? Explain.

Answer: For \vec{F} ,

$$\frac{\partial q}{\partial x} = 1 \neq \frac{\partial t}{\partial y} = -1 \quad \vec{F} \text{ is Not conservative}$$

For \vec{G}

$$\frac{\partial q}{\partial x} = 1 = \frac{\partial t}{\partial y} \quad \vec{G} \text{ is conservative}$$

- (b) (8 points) Compute the work done by \vec{F} along C .

Answer: $\text{work} = \int_C \vec{F} \cdot d\vec{r}$

$$\int_C \vec{F} \cdot d\vec{r} = \int_{C_1} \vec{F} \cdot d\vec{r} + \int_{C_2} \vec{F} \cdot d\vec{r}$$

$$C_1: \vec{r}(t) = \langle t, 0 \rangle \quad -3 \leq t \leq -1$$

$$\int_{C_1} \vec{F} \cdot d\vec{r} = \int_{-3}^{-1} \langle 0, t \rangle \cdot \langle 1, 0 \rangle dt = 0$$

↙ clockwise

$$C_2: \vec{r}(t) = \langle \sin t, \cos t \rangle \quad 0 \leq t \leq \pi$$

$$\int_{C_2} \vec{F} \cdot d\vec{r} = \int_0^\pi \langle -\cos t, \sin t \rangle \cdot \langle \cos t, -\sin t \rangle dt$$

$$= \int_0^\pi -1 dt$$

$$= -\pi$$

$$\text{Work} = 0 - \pi = -\pi$$

- (c) (8 points) Compute the work done by \vec{G} along C .

Answer: Method I. use similar method in (b)

Method II Find ϕ such that $\nabla \phi = \vec{G}$ and $\int_C \vec{G} \cdot d\vec{r} = \phi(B) - \phi(A)$

$$\vec{G} = \langle \cos x + y, x - 1 \rangle = \nabla \phi = \langle \phi_x, \phi_y \rangle$$

Step 1. $\phi_x = \cos x + y$

$$\phi = \int (\cos x + y) dx + C(y) = \sin x + xy + C(y)$$

Step 2. $\phi_y = (\sin x + xy + C(y))' = x + C'(y) = x - 1$

$$C'(y) = -1 \Rightarrow C(y) = -y + C$$

$$\phi = \sin x + xy - y + C$$

$$\text{Work} = \phi(B) - \phi(A) = \sin 1 - \sin(-3)$$

7. (20 points) For each of the following series, state:
 (I) whether the series converges or diverges and
 (II) what test you used to come to this conclusion.

(a) (4 points) $\sum_{k=1}^{\infty} \frac{1}{k^2}$

Converges.

p-series test and $p = 2 > 1$

(b) (4 points) $\sum_{k=1}^{\infty} \frac{(-1)^k}{k^2 - 3}$ Converges

Limit Comparison Test

Alternating series test also works

compare with $\sum_{k=1}^{\infty} \frac{1}{k^2}$

$$\lim_{k \rightarrow \infty} \left| \frac{(-1)^k}{k^2 - 3} \right| / \frac{1}{k^2} = 1 \quad \text{They have the same property.}$$

(c) (4 points) $\sum_{k=2}^{\infty} \frac{4}{k \ln k}$ Diverges

Integral test

$$\int_2^{\infty} \frac{1}{x \ln x} dx = \lim_{b \rightarrow \infty} \int_2^b \frac{1}{x \ln x} dx \quad \text{Let } u = \ln x \quad du = \frac{1}{x} dx$$

$$\int \frac{1}{x \ln x} dx = \int \frac{1}{u} du = \ln u = \ln(\ln x) \rightarrow \infty \text{ as } x \rightarrow \infty$$

(d) (4 points) $\sum_{k=1}^{\infty} (-1)^{k+1} e^{k-3}$ Diverges.

Geometric Series test

$$\lim_{k \rightarrow \infty} (-1)^{k+1} e^{k-3} \neq 0$$

$$\sum_{k=1}^{\infty} (-1)^{k+1} e^{k-3} = \sum_{k=1}^{\infty} (-1)^{k+1} e^{-3} (-e)^k$$

Divergence test also works
(Better)

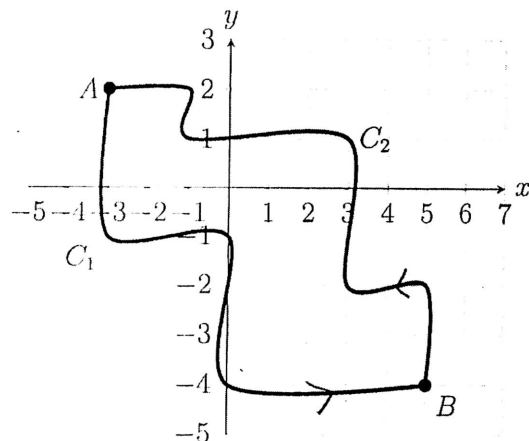
(e) (4 points) $\sum_{k=1}^{\infty} \frac{k!}{k^k}$ Converges

Ratio Test

$$\frac{a_{k+1}}{a_k} = \frac{(k+1)!}{(k+1)^{k+1}} \cdot \frac{k^k}{k!} = \frac{k^k}{(k+1)^k} = \left(\frac{k}{k+1}\right)^k = \left(1 + \frac{1}{k}\right)^{-k}$$

$$\lim_{k \rightarrow \infty} \left(1 + \frac{1}{k}\right)^{-k} = e^{-1} < 1$$

8. (18 points) Consider the following graph:



and let us remind you of Green's Theorem:

$$\iint_R 2D \operatorname{curl}(\vec{F}) dA = \int_C \vec{F} \cdot d\vec{r}$$

(a) (6 points) Let $C = C_1 \cup C_2$ and let \vec{r} parameterize C in a counterclockwise direction.

If $\vec{F} = \langle -2y \sin(2xy), -2x \sin(2xy) \rangle$, estimate: $\oint_C \vec{F} \cdot d\vec{r}$

Answer: $\oint_C \vec{F} \cdot d\vec{r} = \iint_R \left(\frac{\partial f}{\partial x} - \frac{\partial f}{\partial y} \right) dA = 0$

$$\frac{\partial f}{\partial x} = -2 \sin(2xy) - 4xy \cos(2xy)$$

$$\frac{\partial f}{\partial y} = -2 \sin(2xy) - 4xy \cos(2xy)$$

(b) (6 points) Let $C = C_1 \cup C_2$ and let \vec{r} parameterize C in a counterclockwise direction.

If $\vec{F} = \langle -2y, y^2 \rangle$, estimate: $\oint_C \vec{F} \cdot d\vec{r}$

Answer: $\oint_C \vec{F} \cdot d\vec{r} = \iint_R \left(\frac{\partial g}{\partial x} - \frac{\partial f}{\partial y} \right) dA = \iint_R (0 - 2) dA = -2 \iint_R 1 dA$

$$= -2 \cdot \text{Area of } R$$

$$\approx -2 \cdot 27 = -54$$

(c) (6 points) If $\vec{F} = \langle e^x, 2x \rangle$, C_1 runs from A to B , C_2 runs from B to A , and

$$\int_{C_1} \vec{F} \cdot d\vec{r} = 33$$

estimate:

Answer: $\oint_C \vec{F} \cdot d\vec{r} = \int_{C_1} \vec{F} \cdot d\vec{r} + \int_{C_2} \vec{F} \cdot d\vec{r} = 33 + \int_{C_2} \vec{F} \cdot d\vec{r}$

$$\oint_C \vec{F} \cdot d\vec{r} = \iint_R \left(\frac{\partial g}{\partial x} - \frac{\partial f}{\partial y} \right) dA = \iint_R 2 dA = 2 \cdot 27 = 54$$

$$\int_{C_2} \vec{F} \cdot d\vec{r} = 54 - 33 = 21$$

9. (15 points) Multiple Choice: circle the correct answer for each question. Make sure your choice is clearly indicated.

B (a) (3 points) The line $\vec{r}(t) = \langle t+1, 2t-1, -3t+6 \rangle$ is perpendicular to which of the following planes?

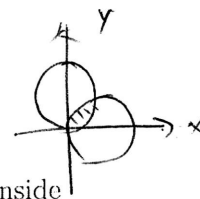
- A. $z = x + y + 1$ $\vec{n} = \langle 1, 1, -1 \rangle$
 B. $-x - 2y + 3z = 11$ $\vec{n} = \langle -1, -2, 3 \rangle$
 C. $x - y + 6z = 5$ $\vec{n} = \langle 1, -1, 6 \rangle$
 D. $z = 0$ $\vec{n} = \langle 0, 0, 1 \rangle$

$$\vec{v} \parallel \vec{n}$$

D (b) (3 points) Which of the following describes a sphere in cylindrical coordinates?

- A. $x^2 + y^2 + z^2 = 15$ \times
 B. $r^2 = 9$ cylinder
 C. $0 \leq \theta \leq 2\pi$ R^3
 D. $z^2 = 8 - r^2$ sphere

$$(r, \theta, z)$$



A (c) (3 points) Which of the following integrals represents the area of the region inside both of the circles $r = 2 \cos(\theta)$ and $r = 2 \sin(\theta)$?

- A. $\int_0^{\pi/4} \int_0^{2 \sin(\theta)} r dr d\theta + \int_{\pi/4}^{\pi/2} \int_0^{2 \cos(\theta)} r dr d\theta$
 B. $\int_0^{\pi/2} (2 \cos(\theta) - 2 \sin(\theta)) d\theta$
 C. $\int_0^{2\pi} \int_{2 \sin(\theta)}^{2 \cos(\theta)} r dr d\theta$
 D. $2 \int_{\pi/4}^{\pi/2} \int_{2 \sin(\theta)}^{2 \cos(\theta)} r dr d\theta$

$$2 \cos \theta = 2 \sin \theta$$

$$\tan \theta = 1$$

$$\theta = \frac{\pi}{4} \quad \frac{5\pi}{4}$$

$$\text{Area} = \iint_R 1 dA$$

$$= \int_{\alpha}^{\beta} \int_{f_1}^{f_2} r dr d\theta$$

$$= \frac{1}{2} \int_{\alpha}^{\beta} f_2^2 - f_1^2 d\theta$$

B (d) (3 points) The parametric function given by $x(t) = 3t - \frac{2}{3}$ and $y(t) = 9t^2 - 4$ is

- A. a straight line.
 B. a parabola.
 C. a circle.
 D. an ellipse.

Eliminate t

$$0 \Rightarrow t = \frac{x+2}{3}$$

$$\text{plug in (2)} \quad y = 9 \cdot \left(\frac{x+2}{3}\right)^2 - 4$$

A (e) (3 points) The unit tangent vector \vec{T} and the principal unit normal vector \vec{N} for the curve $\vec{r}(t) = \langle 4 \cos(t), 4 \sin(t), 3t \rangle$ at $t = \frac{\pi}{2}$ are

- A. $\vec{T} = \langle -\frac{4}{5}, 0, \frac{3}{5} \rangle$ and $\vec{N} = \langle 0, -1, 0 \rangle$.
 B. $\vec{T} = \langle -\frac{4}{5}, 0, \frac{3}{5} \rangle$ and $\vec{N} = \langle -\frac{3}{5}, 0, -\frac{4}{5} \rangle$.
 C. $\vec{T} = \langle 1, 0, 0 \rangle$ and $\vec{N} = \langle 0, 1, 0 \rangle$.
 D. $\vec{T} = \langle \frac{4}{5}, \frac{3}{5}, 0 \rangle$ and $\vec{N} = \langle \frac{3}{5}, -\frac{4}{5}, 0 \rangle$.

$$\vec{T} = \frac{\vec{r}'(t)}{|\vec{r}'(t)|} = \frac{\langle -4 \sin t, 4 \cos t, 3 \rangle}{\sqrt{16 + 9}}$$

$$= \frac{1}{5} \langle -4 \sin t, 4 \cos t, 3 \rangle$$

$$\vec{N} = \frac{d\vec{T}/dt}{|d\vec{T}/dt|}$$

10. (15 points) For each of the following statements, circle T if the statement is true and F if the statement is false. You do not need to show any explanations for this problem.

F (a) (3 points) T / F
 $\int_C \nabla(\varphi) \cdot d\vec{r} = 0$ for any curve C .

It is true, if C is closed. ($\vec{F} = \nabla\varphi$ is conservative)

T (b) (3 points) T / F
 For any closed and bounded region R in the plane and any continuous function f , the function f must have an absolute maximum on R .

Extreme value theorem.

T (c) (3 points) T / F
 For any vector fields \vec{F} and \vec{G} , $\nabla \times (\vec{F} + \vec{G}) = \nabla \times \vec{F} + \nabla \times \vec{G}$.

F (d) (3 points) T / F
 If $\lim_{n \rightarrow \infty} a_n = 0$ then it must be true that $\sum_{n=0}^{\infty} a_n$ converges.

Counterexample : $\sum_{n=1}^{\infty} \frac{1}{n}$ diverges, but $\lim_{n \rightarrow \infty} \frac{1}{n} = 0$

F (e) (3 points) T / F
 The curve $\vec{r}(t) = \langle t^2, t^2 + 1, t^2 + 3 \rangle$ is parameterized by arc length.

$$|\vec{r}'(t)| = \langle 2t, 2t, 2t \rangle$$

$$|\vec{r}'(t)| = \sqrt{(2t)^2 \times 3} \neq 1$$

