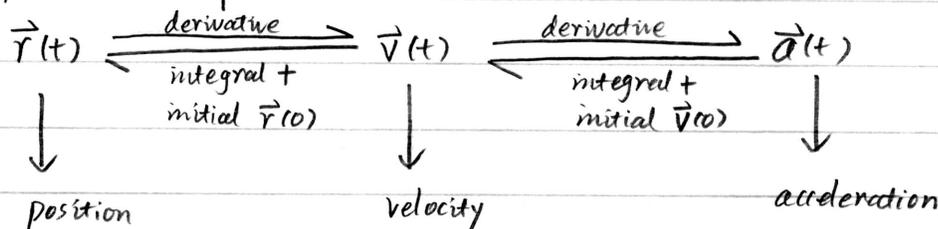


§ 12.7 Motion in space:



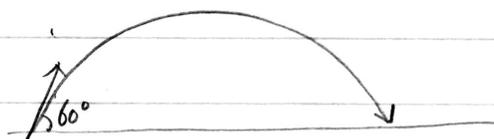
Example. HW6 # 28

$\vec{r}(t)$ is a 2D trajectory, assuming no forces other than gravity. $\vec{r}(0) = \langle 0, 0 \rangle$

$|\vec{v}(0)| = 150 \text{ m/s}$ and launch angle 60°

Q: The time in air, range of object, max height

Answer:



$$\vec{a}(t) = \langle 0, -g \rangle \quad g = 9.8 \text{ m/s}^2$$

$$\vec{v}(0) = \langle 150 \cos 60^\circ, 150 \sin 60^\circ \rangle = \langle 75, 75\sqrt{3} \rangle$$

$$\vec{v}(t) = \int_0^t \vec{a} dt + C = \langle 0, -gt \rangle + C$$

$$\text{Then use } \vec{v}(0) = \langle 0, 0 \rangle + C = \langle 75, 75\sqrt{3} \rangle \Rightarrow C = \langle 75, 75\sqrt{3} \rangle$$

$$\Rightarrow \vec{v}(t) = \langle 75, -gt + 75\sqrt{3} \rangle$$

$$\begin{aligned}
 \vec{r}(t) &= \int \vec{v}(t) dt + C = \langle \int 75 dt, \int -gt + 75\sqrt{3} dt \rangle + C \\
 &= \langle 75t, -\frac{1}{2}gt^2 + 75\sqrt{3}t \rangle + C
 \end{aligned}$$

$$\text{Then use } \vec{r}(0) = \langle 0, 0 \rangle + C = \langle 0, 0 \rangle \Rightarrow C = \langle 0, 0 \rangle$$

$$\Rightarrow \vec{r}(t) = \langle 75t, -\frac{1}{2}gt^2 + 75\sqrt{3}t \rangle$$

① Time to hit the ground? vertical component of $\vec{r}(t) = 0$, i.e.

$$-\frac{1}{2}gt^2 + 75\sqrt{3}t = 0 \Leftrightarrow t = 0 \text{ and } \boxed{t = \frac{150\sqrt{3}}{g}}$$

② Range: The horizontal component of $\vec{r}(t)$ when it hits ground,

$$\text{Plug } t = \frac{150\sqrt{3}}{g} \text{ in } 75t \Leftrightarrow \boxed{75 \times \frac{150\sqrt{3}}{g}}$$

③ Max height? When vertical component of $\vec{v}(t)$ becomes 0. i.e.

$$-gt + 75\sqrt{3} = 0 \Rightarrow t = \frac{75\sqrt{3}}{g}$$

Then plug $t = \frac{75\sqrt{3}}{g}$ in $-\frac{1}{2}gt^2 + 75\sqrt{3}t$ (vertical component of $\vec{r}(t)$)

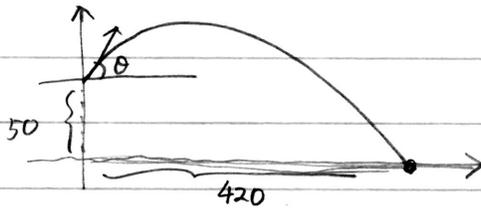
i.e. $-\frac{1}{2}g\left(\frac{75\sqrt{3}}{g}\right)^2 + 75\sqrt{3}\left(\frac{75\sqrt{3}}{g}\right)$

#

Example: HW 6 # 31

A golfer stands 420 ft horizontally from the hole and 50 ft above the hole. Assuming initial speed of the ball is 120 ft/s. At what angle should it be hit to land in hole.

Answer:



$$\vec{a} = \langle 0, -g \rangle$$

$$\vec{v}(0) = \langle 120\cos\theta, 120\sin\theta \rangle$$

$$\vec{r}(0) = \langle 0, 50 \rangle$$

$$\text{Range} = 420$$

$$\vec{v}(t) = \langle 120\cos\theta, -gt + 120\sin\theta \rangle$$

← check!

$$\vec{r}(t) = \langle 120\cos\theta t, -\frac{1}{2}gt^2 + 120\sin\theta t + 50 \rangle$$

Hit the hole \Leftrightarrow when $\vec{r}(t)_y = 0$, $\vec{r}(t)_x = 420$

$$120\cos\theta t = 420 \Rightarrow t = \frac{420}{120\cos\theta}$$

$$-\frac{1}{2}g\left(\frac{420}{120\cos\theta}\right)^2 + 120\sin\theta \cdot \frac{420}{120\cos\theta} + 50 = 0 \quad \text{solve for } \theta$$

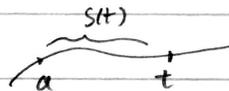
Use Wolfram alpha to find zero point.

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Similar problem HW 6 # 29

§ 12.8 Length of curves

$$d) \vec{r}(t) = \langle x(t), y(t), z(t) \rangle$$



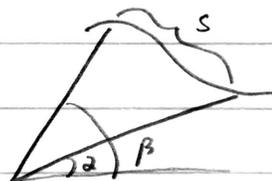
$$s(t) = \int_a^t |\vec{r}'(t)| dt = \int_a^t \sqrt{x'(t)^2 + y'(t)^2 + z'(t)^2} dt$$

Remark: if $|\vec{r}'(t)| \equiv 1$, then the parameter t corresponds to arclength s , i.e.

$\vec{r}(t)$ is parameterized by arc length too

$$\langle 2 \rangle \quad \gamma = f(\theta)$$

$$L = \int_a^b \sqrt{f'(\theta)^2 + g'(\theta)^2} d\theta$$



Example: Old Midterm.

Two trajectories for a certain rocket are modeled by $\vec{r}_1(t)$ and $\vec{r}_2(t)$.

Which is longer from 0 to $t=3$.

$$\vec{r}_1(t) = \langle t, 2t-5 \rangle \quad \vec{r}_2(t) = \langle t^2, \frac{t^3}{3} - 4 \rangle$$

Answer: $L = \int_a^b |\vec{r}'(t)| dt$

$$\vec{r}_1'(t) = \langle 1, 2 \rangle \quad |\vec{r}_1'(t)| = \sqrt{1+2^2} = \sqrt{5}$$

$$\vec{r}_2'(t) = \langle 2t, t^2 \rangle \quad |\vec{r}_2'(t)| = \sqrt{4t^2+t^4}$$

$$L_1 = \int_0^3 \sqrt{5} dt = \sqrt{5} \cdot t \Big|_0^3 = 3\sqrt{5} \approx 6.7$$

$$L_2 = \int_0^3 \sqrt{4t^2+t^4} dt = \int_0^3 t\sqrt{4+t^2} dt$$

$$\text{Let } y = 4+t^2 \quad dy = 2t dt$$

$$\int t\sqrt{4+t^2} dt = \int \sqrt{y} \cdot \frac{1}{2} dy = \frac{1}{2} \int y^{\frac{1}{2}} dy = \frac{1}{2} \frac{1}{\frac{1}{2}} y^{\frac{1}{2}+1} = \frac{1}{3} y^{\frac{3}{2}}$$

$$L_2 = \frac{1}{3} (4+t^2)^{\frac{3}{2}} \Big|_0^3 = \frac{1}{3} (4+9)^{\frac{3}{2}} = \frac{1}{3} 4^{\frac{3}{2}} \approx 13$$

$$L_2 > L_1$$

#

Example HW6 # 41

Determine whether the following curve uses arc length as a parameter. If not, find a description that uses arc length as a parameter.

$$\vec{r}(t) = \langle t^2, 8t^2, 2\sqrt{6}t^2 \rangle \quad 1 \leq t \leq 4$$

Answer: $s(t) = \int_1^t |\vec{r}'(t)| dt$

$$\vec{r}'(t) = \langle 2t, 16t, 4\sqrt{6}t \rangle$$

$$|\vec{r}'(t)| = \sqrt{4t^2 + 256t^2 + 16 \cdot 26t^2} = 26t$$

$$s(t) = \int_1^t 26t dt = 13t^2 \Big|_1^t = 13t^2 - 13$$

$$t^2 = \frac{s+13}{13} = 1 + \frac{s}{13}$$

$$\vec{r}(s) = \langle \frac{s}{13} + 1, 8 \cdot (1 + \frac{s}{13}), 2\sqrt{6} \cdot (1 + \frac{s}{13}) \rangle$$

#

Problems in Polar coordinates?

Practice HW6 # 37-39

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