

Sep. 21

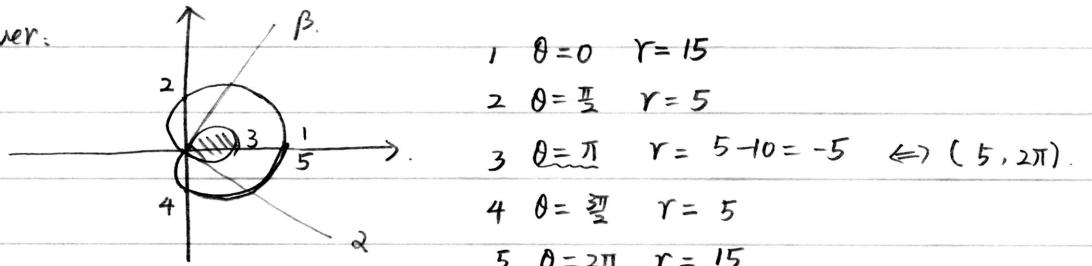
S 11.3

$$\text{Recall Area} = \frac{1}{2} \int_{\alpha}^{\beta} f(\theta)^2 - g(\theta)^2 d\theta$$

Examples HW5

7. Find the area inside the inner loop of limacon $r = 5 + 10\cos\theta$.

Answer:



Answer: Solve for α and β

$$r = 0 \iff 5 + 10\cos\theta = 0 \iff \cos\theta = -\frac{1}{2} \quad \theta = \frac{2\pi}{3}, \frac{4\pi}{3}$$

$$\text{Area} = \int_{\frac{2\pi}{3}}^{\frac{4\pi}{3}} \frac{1}{2} (5 + 10\cos\theta)^2 d\theta$$

$$= \int_{\frac{2\pi}{3}}^{\frac{4\pi}{3}} \frac{1}{2} (25 + 100\cos^2\theta + 100\cos\theta) d\theta$$

$$= \frac{25}{2} \cdot \left(\frac{4\pi}{3} - \frac{2\pi}{3} \right) + \frac{1}{2} \int_{\frac{2\pi}{3}}^{\frac{4\pi}{3}} 100 \frac{1+\cos 2\theta}{2} d\theta + 50 \int_{\frac{2\pi}{3}}^{\frac{4\pi}{3}} \cos\theta d\theta$$

$$= \frac{25}{2} \cdot \frac{2\pi}{3} + 25 \left(\frac{4\pi}{3} - \frac{2\pi}{3} \right) + \frac{25}{2} \sin 2\theta \Big|_{\frac{2\pi}{3}}^{\frac{4\pi}{3}} + 50 \sin\theta \Big|_{\frac{2\pi}{3}}^{\frac{4\pi}{3}}$$

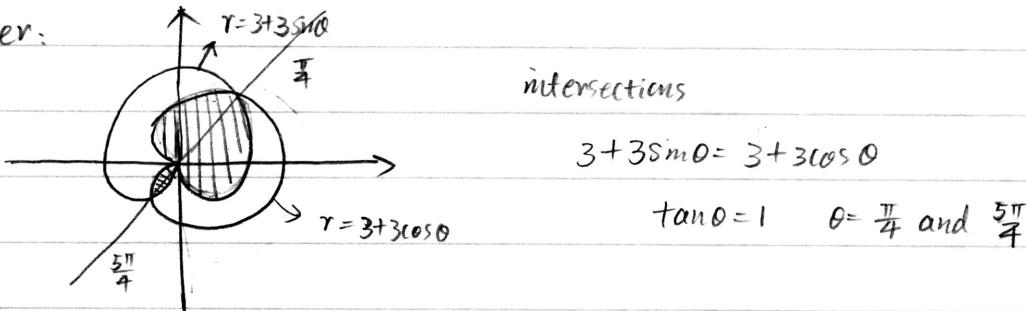
$$= \frac{25\pi}{3} + \frac{50\pi}{3} + \frac{25\sqrt{3}}{2} - 50\sqrt{3}$$

$$= 25\pi - \frac{25\sqrt{3}}{2}$$

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#6. $r = 3 + 3\sin\theta$ $r = 3 + 3\cos\theta$. Area lies within both curves

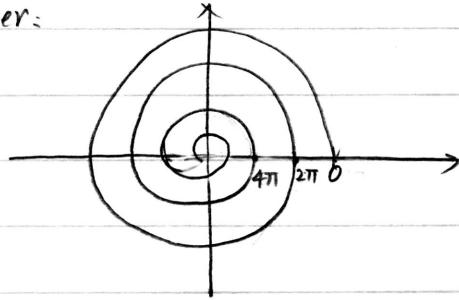
Answer:



$$\text{Area} = \frac{1}{2} \int_{-\frac{\pi}{2}}^{\frac{\pi}{4}} (3 + 3\sin\theta)^2 d\theta + \int_{\frac{\pi}{4}}^{\frac{\pi}{2}} (3 + 3\cos\theta)^2 d\theta + \frac{1}{2} \int_{\frac{3\pi}{4}}^{\frac{5\pi}{4}} (3 + 3\sin\theta)^2 d\theta$$

#8 Let R_n be the region bdd by the n th turn and $(n+1)$ th turn of $r = e^{-\theta}$ in 1st and 2nd quadrants for $\theta \geq 0$. Find A_n , $\lim_{n \rightarrow \infty} A_n$ and $\lim_{n \rightarrow \infty} \frac{A_{n+1}}{A_n}$.

Answer:



$$A_n = \frac{1}{2} \int_0^{\pi} [e^{-(\theta+(n-1) \cdot 2\pi)]^2 - [e^{-(\theta+n \cdot 2\pi)}]^2] d\theta$$

$$= \frac{1}{2} \int_0^{\pi} e^{-2\theta - 4\pi(n-1)} - e^{-2\theta - 4n\pi} d\theta$$

$$= \frac{1}{2} e^{-4\pi(n-1)} \int_0^{\pi} e^{-2\theta} d\theta - \frac{1}{2} e^{-4n\pi} \int_0^{\pi} e^{-2\theta} d\theta$$

$$= \frac{1}{2} e^{-4\pi(n-1)} e^{4\pi} [-\frac{1}{2} e^{-2\theta}]_0^{\pi} - \frac{1}{2} e^{-4n\pi} [-\frac{1}{2} e^{-2\theta}]_0^{\pi}$$

$$= \frac{1}{2} e^{-4\pi(n-1)} e^{4\pi} [-\frac{1}{2} e^{-2\pi} + \frac{1}{2}] - \frac{1}{2} e^{-4n\pi} [-\frac{1}{2} e^{-2\pi} + \frac{1}{2}]$$

$$= \frac{1}{4e^{4\pi(n-1)}} [e^{4\pi} e^{-2\pi} + e^{4\pi} + e^{-2\pi} - 1]$$

$$= \frac{1}{4e^{4\pi n}} [-e^{2\pi} + e^{4\pi} + e^{-2\pi} - 1] \quad \#$$

$$\lim_{n \rightarrow \infty} A_n = 0$$

$$\lim_{n \rightarrow \infty} \frac{A_{n+1}}{A_n} = \lim_{n \rightarrow \infty} \frac{4e^{4\pi n}}{4e^{4(n+1)\pi}} = \lim_{n \rightarrow \infty} \frac{1}{e^{4\pi}} = \frac{1}{e^{4\pi}} \quad \#$$

§ 12.1 - 2

Recall $P(x_1, y_1, z_1)$, $Q(x_2, y_2, z_2)$

$$\vec{PQ} = \langle x_2 - x_1, y_2 - y_1, z_2 - z_1 \rangle$$

\rightarrow HW 5

$$|\vec{PQ}| = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}$$

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Unit vectors parallel to \vec{PQ} are $\pm \frac{\vec{PQ}}{|\vec{PQ}|}$

HW 5 #20

Example: Determine values of x and y such that $(1, 2, 3)$, $(5, 6, 1)$ and (x, y, z) collinear

Answer: Collinear $\Leftrightarrow \vec{AB} \parallel \vec{Ac}$ $\Leftrightarrow \vec{AB} = \lambda \vec{Ac}$ and λ is a constant

$$\vec{AB} = \langle 5-1, 6-2, 1-3 \rangle = \langle 4, 4, -2 \rangle$$

$$\vec{Ac} = \langle x-1, y-2, z-3 \rangle = \langle x-1, y-2, -1 \rangle$$

$$\vec{AB} = \lambda \vec{Ac} \Leftrightarrow \frac{x-1}{4} = \frac{y-2}{4} = \frac{-1}{2} = \lambda$$

$$\Rightarrow x=3 \quad y=4$$

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Old Midterm

Find two different sets of parametric equations for $f(x) = x^3 + 2 \quad 1 \leq x \leq 3$

① First parametric equation

Answer: $x = t$
 $y = t^3 + 2 \quad 1 \leq t \leq 3$

② Second parametric equation

(Answer is not unique)

Answer: $x = t-1 \quad 2 \leq t \leq 4$
 $y = (t-1)^3 + 2$

③ Verify that at $(2, 10)$ on the graph, each of your equations gives correct slope of tangent line

Answer: Slope of tangent line at $(2, 10)$ is $f'(2) = 3 \cdot 2^2 = 3 \cdot 4 = 12$

Verify:

① $\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}} = \frac{3t^2}{1} \quad \text{Plug in } t=2 \quad \frac{dy}{dx} = 12$

② $\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}} = \frac{3(t-1)^2}{1} \quad \text{Plug in } t=3 \quad \frac{dy}{dx} = 12$

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